

KHE24: Towards a Practical Solver for Nurse Rostering (Abstract)

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Abstract Nurse rostering—assigning nurses to the shifts of a hospital ward—is one of the most studied timetabling problems. This paper describes ongoing work on the KHE24 nurse rostering solver, an improved version of the KHE18 solver published previously. It uses a time sweep algorithm to find an initial solution, followed by repair using two methods (ejection chains, and optimal reassignment by dynamic programming). Results are presented for several well-known data sets. They show that KHE24 is making progress towards success in practice, taking running time and breadth of application into account as well as solution cost.

Keywords Nurse rostering · Time sweep · Ejection chains · XESTT

1 Introduction

Nurse rostering—assigning nurses to the shifts of a hospital ward—is one of the most studied timetabling problems. It is an NP-complete problem, and exact algorithms are out of reach in general, although many smaller instances have been solved to optimality using integer programming [4, 36].

Many inexact methods have been tried. Recent work covers a wide range, including integer programming [15, 29, 32, 36, 40], weighted maxSAT [10, 11], simulated annealing [9, 38], hyper-heuristics [2, 16, 33], variable neighbourhood search [39], and constraint programming [34]. For older work, see [41].

The solver presented here, KHE24, is the 2024 version of the main solver built by the author on his KHE solver platform [20]. It is an improved version of the KHE18 solver described in [22]. It runs in polynomial time and aims to find competitive but not optimal solutions quickly, across a wide range of instances. It finds an initial solution using a time sweep algorithm (Section 3.1). This

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timetables the first day of the cycle, then the second, and so on. It then tries two repair methods: ejection chains (Section 3.3) and optimal reassignment using dynamic programming (Section 3.4).

The author’s intention is to test KHE24 on four well-known data sets, to demonstrate its ability to solve a wide range of practical instances. At present, however, only two data sets and half of a third have been tested (Section 4). Although KHE24 has not found any new best solutions, on its own terms (that is, considering running time and breadth of application as well as cost) it promises to be successful. The work is ongoing.

2 The nurse rostering problem and its XESTT formulation

Nurse rostering is the problem of assigning shifts to the nurses of a hospital ward. Hospitals operate 24 hours a day, so there are usually at least three types of shifts: morning, afternoon, and night. Each shift demands a certain number of nurses, often with specific skills. There may be some flexibility in how many nurses to assign, and the number typically changes from day to day.

Perhaps the most characteristic feature of the problem is the large array of requirements that each nurse’s timetable must satisfy. In addition to workload limits, there are rules such as ‘a nurse must have a day off after a sequence of night shifts’, ‘a nurse may work at most four days in a row’, and so on. These requirements are different at different institutions.

Instead of the usual formulas, this paper’s formal definition of the nurse rostering problem is supplied by the XESTT nurse rostering data format [23]. XESTT is an XML format which is capable of representing the instances found in all the well-known data sets. It is based on the XHSTT high school timetabling format [30,31]; the name ‘XESTT’ was chosen to be reminiscent of ‘XHSTT’, with ‘employee scheduling’ replacing ‘high school’. Full details of XESTT appear online [17] and will not be repeated here. Instead, this section offers an overview, and explains the importance of XESTT to the present work.

An XESTT instance consists of the *cycle* (the sequence of *times* that events may be assigned); a set of *resources* (entities that attend events); a set of *events* (meetings); and a set of *constraints*, specifying conditions that solutions should satisfy, and penalties to impose when they don’t.

Each event contains a *starting time*, which may either be preassigned a time or left open for a solver to assign; a *duration*, which is a fixed positive integer giving the number of consecutive times, starting at the starting time, that the event is running; an optional *workload*, which is a fixed non-negative integer representing the workload of the event in arbitrary units, for example in minutes; and any number of *event resources*, each specifying one resource which attends the event for the full duration, which may either be preassigned a resource or left open for a solver to assign.

In nurse rostering instances, each event represents one shift. Each event has duration 1; its actual duration in minutes can be expressed as a workload, if needed. Its starting time is preassigned to a time unique to the shift. For

example, if on each day there is a morning, afternoon, and night shift, then each day will contain three times, one for each shift. This arrangement is somewhat artificial, but, as [23] explains, most nurse rostering constraints concern shifts, not workload in minutes, and this works well in practice. Within an event, each event resource represents a demand for one nurse.

Sets of times, resources, and events may be defined, called *time groups*, *resource groups* and *event groups*. Each resource has one *resource type*, saying what type of resource it is. In nurse rostering there is just one type, **Nurse**.

XESTT offers 18 constraint types, but 9 are not used in nurse rostering, mainly because all the events have preassigned times. Of the 9 types that are used, 3 are *cover constraints*, specifying the number of resources that should attend each event, and the skills those resources should have. The other 6, called *resource constraints* here, constrain the timetables of individual resources, specifying unavailable times, workload limits, unwanted patterns (such as a day shift immediately following a night shift), and so on.

Each constraint contains a Boolean *required* flag indicating whether it is *hard* or *soft*, and an integer *weight*. When a constraint is violated, the degree of violation is multiplied by the weight to give a cost. Algorithms aim to minimize firstly the total cost of hard constraints (the *hard cost*), and secondly the total cost of soft constraints (the *soft cost*). In nurse rostering, solutions with non-zero hard cost are usually considered to be *infeasible*, that is, useless.

XESTT is important here for two reasons. First, it makes it easy to test KHE24 on a wide range of instances, because all these instances have been converted from their original formats to XESTT.

Second, XESTT uses just 9 types of constraints to represent all of the constraints found in other models. It can do this because its constraints accept arbitrary time groups, allowing the same type of constraint to deal with days, night shifts, weekends, and so on. Algorithms like the ejection chain algorithm in this paper, which handle each constraint type explicitly, have only 9 types to handle. Without XESTT or something like it, the number of constraint types would be much larger, and such an approach would hardly be feasible.

3 The KHE24 solver

The KHE24 solver presented here is built on the author's KHE solver platform and is available from the KHE web site [20]. It is an improved version of the KHE18 nurse rostering solver [22] which itself was descended from the KHE14 high school timetabling solver [19]. Full details appear in the KHE documentation [20]; this section covers the main points.

KHE24 is in fact a general timetabling solver, but in nurse rostering the times of all events are preassigned, so KHE24's time assignment part does nothing during nurse rostering except convert the time preassignments in the instances into time assignments in the solutions. This takes almost no time.

After time assignment comes resource assignment—the assignment of nurses to shifts. KHE24 first carries out a *construction phase* which builds an initial

assignment, then continues with a *repair phase* which tries several kinds of repairs that improve that assignment. The repair phase is run twice, to enable repairs of one kind to open the way to repairs of other kinds.

In the KHE platform, instances and solutions are distinct objects. Instances are immutable after creation, whereas solutions change as solving proceeds. A solution consists of a set of *meets*, each representing one event. Within each meet there are *tasks*, one for each event resource of the meet's event. Each task is, in effect, a variable to which one resource may be assigned; it represents an indivisible piece of work for one resource. We will use 'task' in the following in preference to 'shift', because 'shift' can mean both 'task' and 'meet'.

KHE24 has many parameters (called *options*), settable from the command line, determining such things as how to divide up the available running time, the maximum number of days to include when swapping the timetables of two resources, and so on. However, in this paper, the default values of all these options are used. There is no tuning of parameters for different instances, or even different data sets. The only options passed to the calls to KHE24 reported on here are those that set its time limit, the number of solutions to make for each instance, and the number of those solves to run in parallel; and those options (except the time limit) are the same for all runs. This is an important aspect of our claim that KHE24 could become a *practical* solver.

3.1 Construction using time sweep

Because of the high density of constraints in nurse rostering, it may be easier to avoid introducing a problem than to remove it later. So it makes sense for the construction phase to try hard to produce a very good solution [27].

Many nurse rostering constraints concern what happens over consecutive days within nurses' timetables. This suggests that the initial solution should be constructed day by day—the tasks of the first day assigned first, then the tasks of the second day, and so on. As each day is assigned, these kinds of constraints can often be satisfied. KHE24 uses this *time sweep* method, employing weighted bipartite matching to assign each day. This method was presented in detail in the earlier paper [22], so we omit the details here.

The only other use of time sweep known to the author is [27], which finds initial solutions one week at a time in chronological order. It cites an aircrew scheduling paper [35] as its inspiration. The time sweep idea seems to be well known, however. A referee mentioned a similar algorithm from vehicle routing, which sweeps through drop-off points in order of their angle from the base.

3.2 Task grouping

The KHE platform allows tasks to be *grouped*. Assigning a resource to one element of a group assigns it to all. This was included to support high school timetabling: the lessons of one course should be assigned a common teacher. It has also proved useful in nurse rostering.

For example, in instance `COI-GPost` (Section 4.1), a nurse who takes a Friday night task should also take the following Saturday and Sunday night tasks, because constraints penalize Friday night tasks before free weekends, incomplete weekends (working on Saturday or Sunday but not both), and day tasks after night tasks. Then the Monday and Tuesday night tasks can be grouped, as can the Wednesday and Thursday night tasks, owing to limits (minimum 2 and maximum 3) on the number of consecutive night tasks.

KHE24 has a grouping phase which precedes time sweep. It runs quickly but is general enough to be widely useful. For example, it groups the tasks of solutions of instance `COI-GPost` following the line of argument just given. Full details appear in the earlier paper [22].

A grouping will occasionally be inadvisable for some unexpected reason. So KHE24 applies the groupings during the construction and first repair phases, but then removes them, so that if something else is needed there is a chance to find it during the second repair phase. The author has observed a few cases where omitting to remove groupings led to infeasible solutions.

3.3 Repair using ejection chains

After constructing an initial solution using time sweep, KHE24 repairs it using two methods. The most effective of these is *ejection chain repair*. Although KHE18 also used ejection chain repair, the details were not settled and the published description was quite sketchy. KHE24's version is much more settled, so this section presents it in detail.

A *defect* in a solution is one violation of a constraint. For example, if nurse `N2` should work at most two weekends but in fact works three, that is a defect.

A *repair* is a change to a solution which removes a defect. For example, unassigning one of nurse `N2`'s busy weekends repairs the defect just described.

An ejection chain is like a path in a graph where each node represents one defect and each edge represents one repair. Starting from some defect, the first repair removes that defect but introduces one new defect. The second repair removes that defect but introduces another new defect, and so on. If some repair removes a defect without introducing a new defect, then the chain ends successfully: the solution has been improved. Or if the repair of one defect introduces two or more new defects, then the chain ends unsuccessfully: the repair has to be undone, with no improvement. (More precisely, a chain ends successfully whenever the current solution cost is less than it was at the start of the chain; new defects are acceptable, if their cost is low. A chain may be extended whenever a repair introduces a new defect whose removal would make the current solution cost less than it was at the start of the chain.)

There are usually several ways to repair a defect. For example, nurse `N2`'s defect can be repaired by unassigning any one of the three busy weekends. So finding a successful chain involves a search tree: if the first repair does not begin a successful chain, then the second is tried, and so on recursively.

The main loop of the ejection chain repair algorithm visits each defect of the current solution and attempts to remove it by searching a tree of ejection chains, stopping at the first successful chain, if any. It cycles around the defects until a complete cycle of attempts has failed, at which point it terminates.

There are several ways to limit the method to polynomial time. The usual one, which KHE24 uses, is to refuse to visit the same part of the solution twice while repairing a given defect [18, 19]. There is also a time limit (Section 3.5).

Each type of constraint gives rise to one type of defect (or two: maximum and minimum limit violations are repaired differently). For each defect, a set of repairs suited to its type is tried, making the chains *polymorphic*.

At the lowest level, just one basic operation can change a solution: the *task move*, which changes the assignment of task s from resource r_1 to resource r_2 , where $r_1 \neq r_2$. Here r_1 may be NULL, in which case the task move is also a *task assignment*; or r_2 may be NULL, making it also a *task unassignment*. One repair is (can only be) a set of these basic operations.

Several authors (see below) use a swap of the timetables of two resources over several consecutive days as their main repair operation. KHE24 also does this; the maximum number of days is 16. It also tries assignments to a resource r_2 over several days, and unassignments of a resource r_1 over several days. In these cases, the maximum number of days is much smaller, just 4.

Unlike metaheuristics, which choose repairs at random, KHE24's ejection chain algorithm is quite focused. It finds all small repairs (typically extending over one or two days) which would repair the current defect, and then for each small repair it tries a set of alternative repairs by extending that small repair over various sets of adjacent days. Simple checks ensure that the same large repair is almost never tried twice when repairing a given defect.

It is not hard to find suitable small repairs, as was done above for nurse N2's busy weekends. If a resource r is overloaded during some set of times (one day, weekend, or whatever), all small repairs are tried that either unassign r during those times or swap its timetable on those times' days with another resource which is free then. If r is underloaded during some set of times, all small repairs are tried that either assign r to some unassigned task during those times or swap its timetable on the relevant days with some other resource which is busy then. These two kinds of repairs handle all kinds of resource constraints. For example, if r has too many consecutive night shifts, that is treated as r being overloaded during the times of the two endpoints of the over-long sequence.

Cover defects are handled similarly. For example, a task needing assignment is handled by finding all small repairs that assign a suitable resource to the task, then expanding that assignment over adjacent days.

It is common for several tasks to be equivalent, in the sense that the effect of assigning any given resource to any one of them is the same. For example, if the Thursday night shift demands one senior nurse and three ordinary nurses, there will be one task requiring a senior nurse which is not equivalent to any other task, and three equivalent tasks demanding ordinary nurses. If the demand is for two or three ordinary nurses, then the three tasks are still equivalent but there is a preference to assign resources to the first two rather

than to the third. In such cases the ejection chain solver will try one repair per equivalence class, not one repair per task, saving time.

The author knows of three other papers that use ejection chains in nurse rostering. The first two are fairly old and use data sets that are not in use today, making their results hard to evaluate. One [12] includes a repair which swaps the timetables of two resources over one week. The other [28] uses chains of task moves in a tabu search framework. The chains are rather different from those used here: each is generated at random in a way that preserves coverage, but without checking other costs until the whole chain is generated. Reference [28] cites [1] as a source for these chains—a very early use. The third previous use of ejection chains [3,4,7] is much more recent. Its basic repair swaps the timetables of two resources over a variable number of consecutive days.

3.4 Repair using optimal reassignment

There is a dynamic programming algorithm, well known to those who use column generation to solve nurse rostering problems, for finding an optimal assignment of tasks to a single nurse, given fixed timetables for the other nurses. The author has recently generalized this algorithm to find an optimal reassignment of an arbitrary subset of the nurses on an arbitrary subset of the days of the cycle, leaving the other nurses and the other days fixed [25].

The algorithm reported on in [25] was deficient in one respect: it did not support the XESTT limit workload constraint, which limits total workload measured in arbitrary units, such as minutes. This deficiency is now fixed.

Analysis shows that this algorithm is polynomial in the number of days it reassigns, but exponential in the number of resources it reassigns [25]. The author's practical experience with the algorithm bears this out: it costs very little to reassign a few more days, but reassigning just one more resource can increase the running time dramatically. Disappointingly, despite the author's best attempts at optimization, he has been able to reliably reassign up to only three resources, although for a practically unlimited number of days.

Using this algorithm, the author has implemented a VLSN search which chooses 3 resources and 28 consecutive days at random, optimally reassigns those resources over those days, and then repeats using new random choices until time runs out or 50 consecutive attempts produce no improvement. It is nowhere near as effective as ejection chains, but it operates on quite different principles and occasionally makes a contribution.

3.5 Making good use of available running time

Running time is a major issue for the larger instances, so it must be used well. Each of KHE24's phases yields diminishing returns as its running time increases, so it would be a mistake to spend all of the available time on one phase and have nothing left for the others.

KHE24 limits each day during time sweep (including repairs associated with that day) to 3 seconds. Two-thirds of the remaining time is then given to the first repair phase, and one-third to the second. Within these phases, the two repair methods (ejection chains and optimal reassignment) get equal time. If ejection chains finish early, optimal reassignment gets the surplus.

The ejection chain algorithm of [7] imposes a time limit on each search for a chain that repairs one defect. KHE24 imposes a limit of 120 on the number of recursive calls when repairing one defect, which has a similar effect.

Actual running time need not match the time limit exactly. On the one hand, a phase will often end of its own accord well before its time limit. On the other, the time limits are *soft*: instead of being interrupted, each phase consults wall clock time periodically and decides for itself whether to stop. In practice, KHE24 never significantly overruns its time limit, as the actual running times reported in the results tables of Section 4 will show.

3.6 Randomization

Randomization is not stressed in algorithmic solvers like it is in, for example, metaheuristic ones. Still, including some randomization allows the algorithm to be run multiple times with different seeds to obtain different results. Doing this and keeping the best solution is a simple way to utilize multiple cores and trade off solution quality and running time.

For example, ejection chain repair randomizes by trying alternative repairs starting at a random point in the sequence of possible repairs. Such methods are not deep, but they seem to work, judging from the spread of solution costs they produce.

4 Results

This section presents the results of running KHE24 on several well-known sets of instances, after conversion to XESTT by the NRConv program [21, 23], with comparisons with previous results. The converted instances and results are available, in the form of XESTT archive files, at [21].

Our aim here is not to find new best solutions, but rather to achieve success in practice, which means to find good solutions to a wide variety of real-world instances quickly. This idea has been formalized by the author [26] as follows:

A solver is successful in practice if, on every instance that is likely to be encountered in practice, it finds a solution whose cost is within 10% of the best known when run for 5 minutes, and within 5% of the best known when run for 60 minutes.

A detailed justification appears in [26], but just briefly, a solver satisfying these conditions can be used by a practitioner for testing possible scenarios, and also for finding solutions for actual use which are indistinguishable to

casual inspection from the best known solutions. Our task, then, is to evaluate the KHE24 solver against this standard. As a proxy for ‘every instance that is likely to be encountered in practice’, we intend to test the solver on four widely used data sets, although in this paper we only test on two data sets and part of a third. And for ‘best known solutions’ we will use sets of solutions from other authors which are either the best, or among the best known.

KHE24 (as described in Section 3) is run 24 times on each instance, with a different random seed for each run, and the best of the 24 solutions is kept. We call this version of the solver KHE24x24. Twelve threads are used, running on the author’s 12-core Intel i7-12700 processor. (The Intel web site gives several clock frequencies, from 1.6GHz to 4.9GHz, chosen depending on various factors including temperature, so it’s hard to be definite about processor speed.) Each individual solve is given a time limit which is half the overall time limit (half of either 5 minutes or 60 minutes). For each instance, the reported running time is the wall clock time in seconds from the start of the first solve to the end of the last one. It does not include file read and write times; they are negligible.

Each result table was generated by the author’s HSEval program from an XESTT archive file and included with no hand editing. A blank entry indicates that there is no solution for the instance of its row in the solution group of its column. If there are multiple solutions, one with minimum running time among all solutions with minimum cost is shown. The minimum solution costs in each row appear in bold. Any solutions with non-zero hard cost are shown as ‘inf.’ (infeasible). No costs are reported on trust; all are calculated from the solutions in the archive file, and hence verified, by HSEval.

Adjacent to each cost c , in the ‘Rel.’ column, is a relative cost: the value of c divided by the best cost in the row, when the best cost is non-zero. When KHE24x24 is run for 5 minutes, a relative cost of up to 1.10 represents success in practice; when it is run for 60 minutes, 1.05 represents success in practice.

Running times are shown (in seconds) where present in the archive. HSEval cannot verify them. KHE24x24 running times are as defined above.

Care is needed with the average costs at the foot of each table, since costs are sensitive to the scale of the constraint weights. Constraints that measure workload in minutes may produce costs in the thousands.

4.1 The Curtois original instances

The Curtois original instances are the instances available online at [8] under the heading ‘Original instances.’ Their XESTT versions appear in XESTT archive file `COI.xml` at [21], along with the solutions posted with the instances. Nearly all of these solutions are optimal, according to Table 3 of [4].

Table 1 compares KHE24x24’s solutions with the solutions from [8]. It is important to analyse what is happening, and not simply take these results at face value. For example, KHE’s high relative costs for the Post and Ikegami instances suggest that KHE is performing poorly on those instances, but the actual costs tell a different story. Also, in every solution of `COI-WHPP` with cost

Table 1 Results of running KHE24x24 on the Curtois original instances. Column Misc shows the best solutions from XESTT archive file `COI.xml`, taken from Curtois’ web site [8]. The other columns show the results for KHE24x24, running for 5 minutes and 60 minutes. Here and elsewhere, running times are in seconds. This table is derived from XESTT archive file `KHE24-2024-03-07-COI.xml`, available at [21].

Instances (27)	Misc			KHE24x24-05			KHE24x24-60		
	Cost	Rel.	Time	Cost	Rel.	Time	Cost	Rel.	Time
COI-Ozkarahan	0	1.00	-	0	1.00	0.1	0	1.00	0.1
COI-Musa	175	1.00	-	175	1.00	15.9	175	1.00	16.9
COI-Millar-2.1	0	1.00	1.0	0	1.00	6.8	0	1.00	7.3
COI-Millar-2.1.1	0	1.00	-	0	1.00	0.0	0	1.00	0.0
COI-LLR	301	1.00	10.0	301	1.00	46.5	301	1.00	50.8
COI-Azaiez	0	1.00	600.0	0	1.00	300.1	0	1.00	1800.0
COI-GPost	5	1.00	-	8	1.60	143.5	8	1.60	139.4
COI-GPost-B	3	1.00	-	5	1.67	157.5	5	1.67	1800.0
COI-QMC-1	13	1.00	-	16	1.23	70.6	16	1.23	62.5
COI-QMC-2	29	1.00	-	30	1.03	44.1	30	1.03	43.5
COI-WHPP	5	1.00	-	2001	400.20	300.1	2001	400.20	3600.1
COI-BCV-3.46.2	894	1.00	17840.0	896	1.00	300.1	894	1.00	3600.1
COI-BCV-4.13.1	10	1.00	-	10	1.00	300.1	10	1.00	3600.1
COI-SINTEF	0	1.00	-	0	1.00	6.9	0	1.00	7.4
COI-ORTEC01	270	1.00	105.0	300	1.11	174.3	295	1.09	211.4
COI-ORTEC02	270	1.00	-	295	1.09	154.3	295	1.09	158.8
COI-ERMUGH	779	1.00	124.0	1481	1.90	301.7	882	1.13	3600.4
COI-CHILD	149	1.00	-	2824	18.95	324.8	261	1.75	3600.3
COI-ERRVH	2001	1.00	-	6739	3.37	307.7	2219	1.11	3829.4
COI-HED01	136	1.00	-	138	1.01	254.3	136	1.00	1824.7
COI-Valouxis-1	20	1.00	-	100	5.00	300.0	100	5.00	866.1
COI-Ikegami-2.1	0	1.00	13.0	0	1.00	150.0	0	1.00	1800.1
COI-Ikegami-3.1	2	1.00	21600.0	10	5.00	300.1	9	4.50	3600.3
COI-Ikegami-3.1.1	3	1.00	2820.0	14	4.67	300.2	15	5.00	3600.5
COI-Ikegami-3.1.2	3	1.00	2820.0	15	5.00	300.1	10	3.33	3600.4
COI-BCDT-Sep	100	1.00	-	230	2.30	300.1	210	2.10	3600.3
COI-MER	7081	1.00	36002.7	56048	7.92	406.4	13840	1.95	3616.5
Average	454	1.00		2653	17.52	195.0	804	16.47	1801.4

below 1000, some nurses must take night shifts only, and the rest must take non-night shifts only—hardly a real-world scenario.

In summary, KHE24 is successful or almost successful in practice on most of the Curtois original instances, the main exceptions being the larger ones, such as `COI-ERMUGH`, `COI-CHILD`, `COI-ERRVH`, and `COI-MER`. The large differences in cost between the 5-minute and 60-minute runs on these instances suggest that their large size is overwhelming the solver, although further analysis is needed. The Curtois original instances are rarely used for testing these days, which is a pity, considering how interesting and varied they are.

Table 2 Results for the Long and Medium instances of the First International Timetabling Competition. The GOAL column shows the solutions from the GOAL research group web site [37], which the GOAL group has shown to be virtually optimal. The other columns show the results for KHE24x24, running for 5 minutes and 60 minutes. This table is derived from XESTT archive file KHE24-2024-03-07-INRC1-Long-And-Medium.xml, available at [21].

Instances (30)	GOAL		KHE24x24-05			KHE24x24-60		
	Cost	Rel.	Cost	Rel.	Time	Cost	Rel.	Time
INRC1-L01	197	1.00	199	1.01	222.4	199	1.01	405.5
INRC1-L02	219	1.00	227	1.04	219.9	225	1.03	775.3
INRC1-L03	240	1.00	240	1.00	226.0	240	1.00	480.4
INRC1-L04	303	1.00	305	1.01	219.1	304	1.00	740.4
INRC1-L05	284	1.00	285	1.00	218.4	285	1.00	686.9
INRC1-LH01	346	1.00	367	1.06	237.4	354	1.02	2471.2
INRC1-LH02	89	1.00	95	1.07	249.3	93	1.04	2192.8
INRC1-LH03	38	1.00	44	1.16	235.8	42	1.11	1779.7
INRC1-LH04	22	1.00	33	1.50	229.8	30	1.36	1741.4
INRC1-LH05	41	1.00	52	1.27	224.4	47	1.15	1957.4
INRC1-LL01	235	1.00	252	1.07	235.9	246	1.05	2451.3
INRC1-LL02	229	1.00	250	1.09	239.2	238	1.04	2455.2
INRC1-LL03	220	1.00	262	1.19	248.3	244	1.11	2445.9
INRC1-LL04	222	1.00	256	1.15	235.3	237	1.07	2443.1
INRC1-LL05	83	1.00	87	1.05	245.0	84	1.01	1908.4
INRC1-M01	240	1.00	243	1.01	179.2	243	1.01	186.1
INRC1-M02	240	1.00	244	1.02	182.2	244	1.02	214.9
INRC1-M03	236	1.00	239	1.01	180.2	239	1.01	197.4
INRC1-M04	237	1.00	240	1.01	182.3	240	1.01	198.8
INRC1-M05	303	1.00	308	1.02	209.3	308	1.02	285.8
INRC1-MH01	111	1.00	140	1.26	219.2	132	1.19	1471.3
INRC1-MH02	221	1.00	237	1.07	226.9	231	1.05	1040.9
INRC1-MH03	34	1.00	41	1.21	208.9	41	1.21	301.0
INRC1-MH04	78	1.00	85	1.09	218.8	85	1.09	458.2
INRC1-MH05	119	1.00	130	1.09	222.7	132	1.11	1102.9
INRC1-ML01	157	1.00	170	1.08	234.0	163	1.04	1079.7
INRC1-ML02	18	1.00	26	1.44	139.6	26	1.44	241.2
INRC1-ML03	29	1.00	35	1.21	125.7	35	1.21	141.5
INRC1-ML04	35	1.00	41	1.17	183.5	41	1.17	330.8
INRC1-ML05	107	1.00	114	1.07	217.6	114	1.07	1309.5
Average	164	1.00	175	1.11	213.9	171	1.09	1116.5

4.2 The First International Nurse Rostering Competition

The First International Nurse Rostering Competition [14] has published many instances, still available from the competition web site [13]. XESTT versions are available in files `INRC1-Long-And-Medium.xml` and `INRC1-Sprint.xml` [21], along with the GOAL research group’s virtually optimal solutions [37].

Tables 2 and 3 show KHE24’s results. Running times are not given for the GOAL solutions because the GOAL solution files do not contain any; but the GOAL web site has a table of running times. About 10 instances, from the Long and Medium sets, have running times of about 4 hours. Many others have running times under one minute, often well under.

Table 3 Results for the Sprint instances of the First International Timetabling Competition. This table is derived from XESTT archive file KHE24-2024-03-07-INRC1-Sprint.xml, available at [21]. Other details as for Table 2.

Instances (30)	GOAL		KHE24x24-05			KHE24x24-60		
	Cost	Rel.	Cost	Rel.	Time	Cost	Rel.	Time
INRC1-S01	56	1.00	56	1.00	42.0	56	1.00	48.1
INRC1-S02	58	1.00	58	1.00	38.5	58	1.00	38.1
INRC1-S03	51	1.00	51	1.00	50.2	51	1.00	50.4
INRC1-S04	59	1.00	59	1.00	44.1	59	1.00	44.9
INRC1-S05	58	1.00	58	1.00	49.7	58	1.00	50.4
INRC1-S06	54	1.00	54	1.00	52.0	54	1.00	50.5
INRC1-S07	56	1.00	56	1.00	46.9	56	1.00	45.3
INRC1-S08	56	1.00	56	1.00	40.4	56	1.00	40.4
INRC1-S09	55	1.00	55	1.00	59.1	55	1.00	61.5
INRC1-S10	52	1.00	52	1.00	46.8	52	1.00	46.7
INRC1-SH01	32	1.00	34	1.06	31.6	34	1.06	31.4
INRC1-SH02	32	1.00	32	1.00	30.0	32	1.00	29.8
INRC1-SH03	62	1.00	62	1.00	42.7	62	1.00	42.3
INRC1-SH04	66	1.00	67	1.02	58.6	67	1.02	58.1
INRC1-SH05	59	1.00	59	1.00	54.2	59	1.00	54.2
INRC1-SH06	130	1.00	134	1.03	59.8	134	1.03	58.7
INRC1-SH07	153	1.00	153	1.00	53.0	153	1.00	52.6
INRC1-SH08	204	1.00	206	1.01	124.2	206	1.01	127.4
INRC1-SH09	338	1.00	338	1.00	149.7	338	1.00	223.6
INRC1-SH10	306	1.00	306	1.00	72.3	306	1.00	70.0
INRC1-SL01	37	1.00	38	1.03	50.5	38	1.03	50.2
INRC1-SL02	42	1.00	43	1.02	37.4	43	1.02	36.8
INRC1-SL03	48	1.00	49	1.02	51.5	49	1.02	53.9
INRC1-SL04	73	1.00	73	1.00	139.8	73	1.00	231.6
INRC1-SL05	44	1.00	45	1.02	48.9	45	1.02	46.0
INRC1-SL06	42	1.00	42	1.00	20.2	42	1.00	20.0
INRC1-SL07	42	1.00	43	1.02	35.8	43	1.02	34.4
INRC1-SL08	17	1.00	17	1.00	19.4	17	1.00	19.8
INRC1-SL09	17	1.00	17	1.00	19.0	17	1.00	18.8
INRC1-SL10	43	1.00	43	1.00	31.0	43	1.00	32.5
Average	78	1.00	79	1.01	53.3	79	1.01	58.9

Another source of virtually optimal solutions to these instances is the branch and price algorithm whose results are reported in Table 5 of [4]. The author has not tried to obtain these solutions. Their reported running times are better than the GOAL ones, never exceeding about 10 minutes.

KHE24 is generally successful in practice on these instances, although once again there are some exceptions, including INRC1-LH04, INRC1-LH05, INRC1-MH01, and INRC1-ML02. Analysis of these cases is needed. On the Sprint instances, running for 60 minutes is never better than running for 5.

4.3 The Second International Nurse Rostering Competition

The Second International Nurse Rostering Competition [5,6] was notable for requiring its instances to be solved week by week, as occurs in the real world.

Table 4 Results for the Second International Timetabling Competition 4-week instances. The LOR17 column shows the solutions obtained from the authors of [27]. This table is derived from XESTT archive file `KHE24-2024-03-07-INRC2-4.xml`, available at [21].

Instances (20)	LOR17		KHE24x24-05			KHE24x24-60		
	Cost	Rel.	Cost	Rel.	Time	Cost	Rel.	Time
INRC2-4-030-1-6291	1695	1.00	2040	1.20	300.1	1865	1.10	3600.1
INRC2-4-030-1-6753	1890	1.00	2230	1.18	300.1	2005	1.06	3600.2
INRC2-4-035-0-1718	1425	1.00	1835	1.29	300.1	1590	1.12	3600.2
INRC2-4-035-2-8875	1155	1.00	1535	1.33	300.1	1360	1.18	3600.1
INRC2-4-040-0-2061	1685	1.00	2125	1.26	300.1	1875	1.11	3600.1
INRC2-4-040-2-6106	1890	1.00	2360	1.25	300.1	2020	1.07	3600.1
INRC2-4-050-0-0487	1505	1.00	2015	1.34	300.1	1745	1.16	3600.1
INRC2-4-050-0-7272	1500	1.00	1975	1.32	300.1	1690	1.13	3600.1
INRC2-4-060-1-6115	2505	1.00	3335	1.33	300.2	2840	1.13	3600.2
INRC2-4-060-1-9638	2750	1.00	3785	1.38	300.2	3165	1.15	3600.2
INRC2-4-070-0-3651	2435	1.00	3225	1.32	300.1	2850	1.17	3600.1
INRC2-4-070-0-4967	2175	1.00	3035	1.40	300.1	2565	1.18	3600.1
INRC2-4-080-2-4333	3340	1.00	4015	1.20	300.1	3775	1.13	3600.3
INRC2-4-080-2-6048	3260	1.00	4230	1.30	300.1	3750	1.15	3600.2
INRC2-4-100-0-1108	1245	1.00	2100	1.69	300.2	1670	1.34	3600.2
INRC2-4-100-2-0646	1950	1.00	2700	1.38	300.2	2275	1.17	3600.2
INRC2-4-110-0-1428	2440	1.00	3245	1.33	300.2	2755	1.13	3600.2
INRC2-4-110-0-1935	2560	1.00	3550	1.39	300.2	3015	1.18	3600.2
INRC2-4-120-1-4626	2170	1.00	3060	1.41	300.2	2600	1.20	3600.3
INRC2-4-120-1-5698	2220	1.00	3140	1.41	300.2	2650	1.19	3600.2
Average	2090	1.00	2777	1.34	300.1	2403	1.15	3600.2

However, here we focus on a set of conventional 4-week and 8-week instances from the competition that have been tackled by several authors [27, 38]. Their converted versions appear in files `INRC2-4.xml` and `INRC2-8.xml` at [21], along with some solutions produced by the authors of [27].

The results for the 4-week instances appear in Table 4. There is no running time information in the solution files obtained from other authors. Reference [27] gives a formula that was used to determine the running time limit; it is about 20 minutes for the largest 4-week instances. Reference [38] used a running time limit of 2750 seconds (about 45 minutes).

These results are disappointing, relative to the best known solutions and to the author’s own testing. For example, the cost of the 5-minute solution to instance `INRC2-4-030-1-6291` reported here is 2040 (Table 4), but in other 5-minute tests the author has obtained solutions whose cost is as low as 1785. This problem will probably be easy to fix, but the paper submission deadline intervened before the the author had time to fix it.

Sadly, the publication deadline has prevented the author from tackling the 8-week instances with `KHE24x24`. A few preliminary tests have revealed that `KHE24x24`’s solutions are not currently competitive. The author’s experience with other data sets suggests that detailed analysis will reveal concrete areas in which the algorithm is performing poorly, whose improvement will lead to more competitive results. However this is speculation at present.

4.4 The 2014 Curtois and Qu instances

The instances here are the 24 instances published in 2014 by Curtois and Qu [7,8]. Converted versions appear in file `CQ14.xml`, which also holds four sets of solutions received from Curtois via private correspondence.

Again, the publication deadline has prevented the author from presenting results for this data set. He does not have even preliminary results to report.

There are more recent results for these instances [8,10,11]. Reference [8] has slightly better results than the ones in file `CQ14.xml`, with lower bounds. The bounds show that many of the results are optimal, although the last five instances, which are much larger than the others, still have significant optimality gaps. We regard those last five instances as not relevant to the concerns of this paper, since they have cycles of 26 and 52 weeks, much longer than is encountered in practice.

5 Conclusion

This paper has presented KHE24, a nurse rostering solver which aims to find very good but not optimal solutions quickly across a wide range of instances. Polynomial-time methods are used: time sweep for the initial assignment, and ejection chains and optimal reassignment using dynamic programming for repair. No parameter tuning is needed.

The results so far give hope that the solver will eventually be successful in practice (that is, taking cost, running time, and breadth of application into account). At the time of writing, however, many tests are missing and for some of those the author's preliminary results are uncompetitive. The author is actively continuing this work.

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