The KHE High School Timetabling Engine

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Part A

The Platform

Chapter 1. Introduction

Some instances of high school timetabling problems, taken from institutions in several countries and specified formally in an XML format called XHSTT, have recently become available [11]. For the first time, the high school timetabling problem can be studied in its full generality.

KHE is an open-source ANSI C library, released under the GNU public licence, which aims to provide a fast and robust foundation for solving instances of high school timetabling problems expressed in the XHSTT format. Users of KHE may read and write XML files, create solutions, and add and change time and resource assignments using any algorithms they wish. The cost of the current solution is always available, kept up to date by a hand-coded constraint propagation network. KHE also offers features inherited from the author's KTS system [6, 8], notably layer trees and matchings, and solvers for several major sub-tasks.

KHE is intended for production use, but it is also a research vehicle, so new versions will not be constrained by backward compatibility. Please report bugs to me at *jeff@it.usyd.edu.au.* I will release a corrected version within a few days of receiving a bug report, wherever possible.

This introductory chapter explains how to install and use KHE, surveys its data types, and describes some operations common to many types.

1.1. Installation and use

KHE has a home page, at

```
http://www.it.usyd.edu.au/~jeff/khe/
```

The current version of KHE is a gzipped tar file in that directory. The current version of this documentation (a PDF file) is also stored there. The names of these files change with each release; they are most easily downloaded using links on the home page.

The version number of a KHE release is its date of release, in the format *yyyy_mm_dd*. For example, the first release was on 9 August 2010, so its version number is 2010_08_09. Its files' names are `khe-2010_08_09.tar.gz` and `khe_guide-2010_08_09.pdf`. The version number also appears in a preprocessor definition in file `khe.h`, like this for example:

```
#define KHE_VERSION "2010_08_09"
```

To install KHE, download a release and unpack it using `gunzip` and `tar xf` as usual. The resulting directory, `khe`, contains the source files of KHE, a makefile, and a `doc` subdirectory containing the source files of this documentation. Typing `make` in directory `khe` compiles KHE, producing a set of `.o` files and an executable called `khe` which may be used for testing.

Run `khe` with no command line arguments to get a usage message. It is capable of reading an XML archive, solving each of its instances, and writing out the archive with the solutions added as a new solution group.

More commonly, it is desired to use KHE within a larger program. A simple way to

incorporate KHE into a larger C program is as follows. Run make as before, then move directory khe to be subdirectory khe of the main source code directory of the larger C program. Add

```
#include "khe.h"
```

at the top of each source file of the larger program that requires access to KHE. To ensure that the C compiler can find file khe.h, add -I khe to the command which invokes the C compiler. Add khe/*.o to the list of files that are to be linked together to form the executable. Remove executable khe, and also remove object file khe_main.o, since it contains an unwanted main().

It is necessary to add -lm to the main linker command, to gain access to the mathematical functions, and also -lexpat, because KHE relies on the well-known Expat library for reading XML. Expat offers a choice of encodings for the characters it reads. However, this choice must be made at compile time, and since the precompiled version of Expat on the author's computer returns UTF-8 characters, UTF-8 is used uniformly throughout KHE, represented by the C char type. Users who want other encodings will have to convert to and from UTF-8.

You may already have Expat on your system, since a lot of software that reads XML uses it. If not, you can get it from SourceForge. The author's experience was that his system's package installer did not install the required expat.h include file, but downloading from SourceForge and following the basic install procedure described in the distribution worked fine.

KHE uses Posix threads to implement solving in parallel (function KheParallelSolve from Section 8.2), so the compiler and linker commands need the -pthread flag. If you don't have Posix threads, the makefile documents a workaround. The only difference will be that parallel solvers will do their solving sequentially rather than in parallel.

Another possible porting problem arises in those parts of KHE which consult the system to find out how much time has been consumed while solving. Again, there is a workaround for this in the makefile, which if taken will cause all time measurements to be 0.

1.2. The data types of KHE

This section is an overview of KHE's data types. The following chapters have the details.

Type KHE_ARCHIVE represents one archive, that is, a collection of instances plus a collection of solution groups. Type KHE_SOLN_GROUP represents one solution group, that is, a set of solutions of the instances of the archive it lies in. The word 'solution' is abbreviated to 'soln' wherever it appears in the KHE interface. Use of these types is optional: instances do not have to lie in archives, and solutions do not have to lie in solution groups.

Type KHE_INSTANCE represents one instance of the high school timetabling problem. KHE_TIME_GROUP represents a set of times; KHE_TIME represents one time. KHE_RESOURCE_TYPE represents a resource type (typically *Teacher*, *Room*, *Class*, or *Student*); KHE_RESOURCE_GROUP represents a set of resources of one type; and KHE_RESOURCE represents one resource.

Type KHE_EVENT_GROUP represents a set of events; KHE_EVENT represents one event, including all information about its time. Type KHE_EVENT_RESOURCE represents one resource element within an event. Type KHE_CONSTRAINT represents one constraint. This could have any of the constraint types of the XML format (it is their abstract supertype).

Type KHE_SOLN represents one solution, complete or partial, of a given instance, optionally

lying within a solution group. Type KHE_MEET represents one meet (KHE's commendably brief name for what the XML format calls a solution event, split event, or sub-event: one event as it appears in a solution), including all information about its time. Type KHE_TASK represents one piece of work for a resource to do: one resource element within a meet.

KHE supports multi-threading by ensuring that each instance and its components (of type KHE_INSTANCE, KHE_TIME_GROUP, and so on) is immutable after loading of the instance is completed, and that operations applied to one solution object do not interfere with operations applied simultaneously to another.[1] Thus, after instance loading is completed, it is safe to create multiple threads with different KHE_SOLN objects in each thread, all referring to the same instance, and operate on those solutions in parallel. No such guarantees are given for operating on the same solution from different threads.

1.3. Common operations

This section describes some miscellaneous operations that are common to many data types.

Use of KHE often involves creating objects that contain references to KHE entities (objects of types defined by KHE) alongside other information. Sometimes it is necessary to go backwards, from a KHE entity to a user-defined object. Accordingly, each KHE entity contains a *back pointer* which the user is free to set and retrieve, using calls which look generically like this:

```
void KheEntitySetBack(KHE_ENTITY entity, void *back);
void *KheEntityBack(KHE_ENTITY entity);
```

All back pointers are initialized to NULL. In general, KHE itself does not set back pointers. The exception is that some solvers packaged with KHE set the back pointers of the solution entities they deal with. This is documented where it occurs.

Timetables often contain symmetries of various kinds. In high school timetabling, the student group resources of one form are often symmmetrical: they attend the same kinds of events over the course of the cycle.

Knowledge of similarity can be useful when solving. For example, it might be useful to timetable similar events attended by student group resources of the same form at the same time. Accordingly, several KHE entities offer an operation of the form

```
bool KheEntitySimilar(KHE_ENTITY e1, KHE_ENTITY e2);
```

which returns true if KHE considers that the two entities are similar. If they are the exact same entity, they are always considered similar. In other cases, the definition of similarity varies with the kind of entity, although it follows a common pattern: evidence both in favour of similarity and against it is accumulated, and there needs to be a significant amount of evidence in favour, and more evidence in favour than against. For example, an event containing no event resources will never be considered similar to any event except itself, since positive evidence, such as requests for the same kinds of teachers, is lacking.

[1] Assuming that KHE is linked to an implementation of malloc() suited to multiple threads, such as the Linux glibc implementation by Doug Lea and W. Gloger. KHE does not leak memory, although, since garbage collection is not standard in C, the user must indicate when major objects, such as instances and solutions, are no longer required.

Similarity is not a transitive relation in general. In other words, if `e1` and `e2` are similar, and `e2` and `e3` are similar, that does not imply that `e1` and `e3` are similar. There is a heuristic aspect to it that seems inevitable, although the intention is to stay on the safe side: to declare two entities to be similar only when they clearly are similar.

Another operation that applies to many entities, albeit a humble one, is printing the current state of the entity as an aid to debugging. The KHE operations for this mostly take the form

```
void KheEntityDebug(KHE_ENTITY entity, int verbosity,
  int indent, FILE *fp);
```

They produce a debug print of `entity` onto `fp`.

The `verbosity` parameter controls how much detail is printed. Any value is acceptable. A zero or negative value always prints nothing. Every positive value prints something, and as the value increases, more detail is printed, depending, naturally, on the kind of entity. Value 1 tries to print the minimum amount of information needed to identify the entity, often just its name.

If `indent` is non-negative, a multi-line format is used in which each line begins with at least `indent` spaces. If `indent` is negative, the print appears on one line with no indent and no concluding newline. Since space is limited, verbosity may be reduced when `indent` is negative.

Many entities are organized hierarchically. Depending on the verbosity, printing an entity may include printing its descendants. Their debug functions are passed a value for `indent` which is 2 larger than the value received (when non-negative), so that the hierarchy is represented in the debug output by indenting. The debug print of one entity usually begins with `[` and ends with a matching `]`, making it easy to move around the printed hierarchy using a text editor.

Chapter 2. Archives and Solution Groups

This chapter describes the KHE_ARCHIVE and KHE_SOLN_GROUP data types, representing archives and solution groups as in the XML format. Their use is optional, since instances are not required to lie in archives, and solutions are not required to lie in solution groups.

2.1. Archives

An archive is defined in the XML format to be a collection of instances together with groups of solutions to those instances. There may be any number of instances and solution groups. To create a new, empty archive, call

```
KHE_ARCHIVE KheArchiveMake(char *id, KHE_ARCHIVE_METADATA md);
```

Both parameters are optional (may be NULL); id is an identifier for the archive, and md is metadata, which can be created by KheArchiveMetaDataMake below. Functions

```
char *KheArchiveId(KHE_ARCHIVE archive);
KHE_ARCHIVE_METADATA KheArchiveMetaData(KHE_ARCHIVE archive);
```

return these two attributes. To set and retrieve the back pointer (Section 1.3), call

```
void KheArchiveSetBack(KHE_ARCHIVE archive, void *back);
void *KheArchiveBack(KHE_ARCHIVE archive);
```

Archive metadata may be created by calling

```
KHE_ARCHIVE_METADATA KheArchiveMetaDataMake(char *name,
  char *contributor, char *date, char *description, char *remarks);
```

where remarks, being optional, may be NULL. The attributes may be retrieved by calling

```
char *KheArchiveMetaDataName(KHE_ARCHIVE_METADATA md);
char *KheArchiveMetaDataContributor(KHE_ARCHIVE_METADATA md);
char *KheArchiveMetaDataDate(KHE_ARCHIVE_METADATA md);
char *KheArchiveMetaDataDescription(KHE_ARCHIVE_METADATA md);
char *KheArchiveMetaDataRemarks(KHE_ARCHIVE_METADATA md);
```

Initially an archive contains no instances and no solution groups. Solution groups are added automatically as they are created, because every solution group lies in exactly one archive. An instance may be added to an archive by calling

```
bool KheArchiveAddInstance(KHE_ARCHIVE archive, KHE_INSTANCE ins);
```

KheArchiveAddInstance returns true if it succeeds in adding ins to archive, and false otherwise, which can only be because archive already contains an instance with the same Id as

`ins`. The instance will appear after any instances already present. An instance may be deleted from an archive (but not destroyed) by calling

```
void KheArchiveDeleteInstance(KHE_ARCHIVE archive, KHE_INSTANCE ins);
```

`KheArchiveDeleteInstance` aborts if `ins` is not in `archive`. If there are any solutions for `ins` in `archive`, they are deleted too. The gap left by deleting the instance is filled by shuffling subsequent instances up one place.

To visit the instances of an archive, call

```
int KheArchiveInstanceCount(KHE_ARCHIVE archive);
KHE_INSTANCE KheArchiveInstance(KHE_ARCHIVE archive, int i);
```

The first returns the number of instances in `archive`, and the second returns the `i`'th of those instances, counting from 0 as usual in C. There is also

```
bool KheArchiveRetrieveInstance(KHE_ARCHIVE archive, char *id,
  KHE_INSTANCE *ins);
```

If `archive` contains an instance with the given `id`, this function sets `ins` to that instance and returns `true`; otherwise it leaves `*ins` untouched and returns `false`. In the same way,

```
int KheArchiveSolnGroupCount(KHE_ARCHIVE archive);
KHE_SOLN_GROUP KheArchiveSolnGroup(KHE_ARCHIVE archive, int i);
bool KheArchiveRetrieveSolnGroup(KHE_ARCHIVE archive, char *id,
  KHE_SOLN_GROUP *soln_group);
```

visit the solution groups of an archive, and retrieve a solution group by `id`.

2.2. Solution groups

A solution group is a set of solutions to instances of its archive. To create a solution group, call

```
bool KheSolnGroupMake(KHE_ARCHIVE archive, char *id,
  KHE_SOLN_GROUP_METADATA md, KHE_SOLN_GROUP *soln_group);
```

Parameter `archive` is compulsory. The solution group will be added to the archive. Parameters `id` and `md` are the Id and MetaData attributes from the XML file; both are optional, with `NULL` meaning absent, although they are compulsory if `archive` is to be written later. If the operation is successful, then `true` is returned with `*soln_group` set to the new solution group; if it is unsuccessful (which can only be because `id` is already the Id of a solution group of `archive`), then `false` is returned with `*soln_group` set to `NULL`.

To set and retrieve the back pointer (Section 1.3) of a solution group, call

```
void KheSolnGroupSetBack(KHE_SOLN_GROUP soln_group, void *back);
void *KheSolnGroupBack(KHE_SOLN_GROUP soln_group);
```

as usual. To retrieve the other attributes, call

```
KHE_ARCHIVE KheSolnGroupArchive(KHE_SOLN_GROUP soln_group);
char *KheSolnGroupId(KHE_SOLN_GROUP soln_group);
KHE_SOLN_GROUP_METADATA KheSolnGroupMetaData(KHE_SOLN_GROUP soln_group);
```

Solution group metadata may be created by calling

```
KHE_SOLN_GROUP_METADATA KheSolnGroupMetaDataMake(char *contributor,
  char *date, char *description, char *publication, char *remarks);
```

where `publication` and `remarks`, being optional, may be `NULL`. The attributes are retrieved by

```
char *KheSolnGroupMetaDataContributor(KHE_SOLN_GROUP_METADATA md);
char *KheSolnGroupMetaDataDate(KHE_SOLN_GROUP_METADATA md);
char *KheSolnGroupMetaDataDescription(KHE_SOLN_GROUP_METADATA md);
char *KheSolnGroupMetaDataPublication(KHE_SOLN_GROUP_METADATA md);
char *KheSolnGroupMetaDataRemarks(KHE_SOLN_GROUP_METADATA md);
```

Initially a solution group has no solutions. These are added and deleted by calling

```
void KheSolnGroupAddSoln(KHE_SOLN_GROUP soln_group, KHE_SOLN soln);
void KheSolnGroupDeleteSoln(KHE_SOLN_GROUP soln_group, KHE_SOLN soln);
```

A solution can only be added when its instance lies in the solution group's archive.

To visit the solutions of a solution group, call

```
int KheSolnGroupSolnCount(KHE_SOLN_GROUP soln_group);
KHE_SOLN KheSolnGroupSoln(KHE_SOLN_GROUP soln_group, int i);
```

as usual. Solutions have no Id attributes, so there is no `KheSolnGroupRetrieveSoln` function. When solution `i` is deleted, `KheSolnGroupSolnCount` decreases by 1, solution `i + 1` becomes solution `i`, and so on.

2.3. Reading archives

KHE reads and writes archives in a standard XML format [11]. To read an archive, call

```
bool KheArchiveRead(FILE *fp, KHE_ARCHIVE *archive, KML_ERROR *ke,
  bool infer_resource_partitions, bool allow_invalid_solns,
  char **leftover, int *leftover_len, FILE *echo_fp);
```

File `fp` must be open for reading UTF-8, and it remains open after the call returns. If, starting from its current position, `fp` contains a legal XML archive, then `KheArchiveRead` sets `*archive` to that archive and `*ke` to `NULL` and returns `true` with the current position of `fp` moved to after the archive. If there was a problem reading the file, then it sets `*archive` to `NULL` and `*ke` to an error object and returns `false`. Any reports in the archive are discarded without checking.

Type `KML_ERROR` is from the KML module packaged with KHE. A full description of the KML module appears in Appendix A.4. Given an object of type `KML_ERROR`, operations

```
int KmlErrorLineNum(KML_ERROR ke);
int KmlErrorColNum(KML_ERROR ke);
char *KmlErrorString(KML_ERROR ke);
```

return the line number, the column number, and a string description of the error.

KheArchiveRead builds the archive object by calling only functions described in this guide; there is nothing special about the archive it makes. Parameter infer_resource_partitions is passed on to the calls to KheInstanceMakeEnd (Section 3.1). KheArchiveRead builds complete representations of the solutions it reads, by calling KheSolnMakeCompleteRepresentation, KheSolnAssignPreassignedTimes, and KheSolnAssignPreassignedResources (Section 4.3); but it does not call KheSolnMatchingBegin or KheSolnEvennessBegin (Chapter 7).

Usually, if there are errors in the file, KheArchiveRead returns false and sets *ke to the first error. But if allow_invalid_solns is true, then some errors lying in solutions are handled differently: the erroneous solutions are converted to invalid placeholders (Section 4.11). Each invalid placeholder solution contains its first error, and none of its errors cause false to be returned or *ke to be set. Not all errors, not even all errors lying in solutions, can be handled in this way; those that cannot cause KheArchiveRead to return false and set *ke as usual.

KheArchiveRead calls KmlRead (Appendix A.4.3), passing leftover, leftover_len, and echo_fp to it, and setting its end_label parameter to "</HighSchoolTimetableArchive>" if leftover is non-NULL, and to NULL if leftover is NULL. Appendix A.4.3 has the details, but just briefly, leftover and leftover_len should be NULL when the archive occupies fp from its current position to the end, and non-NULL when other material may follow the archive in fp; and echo_fp would normally be NULL.

To create an archive by reading an XML string, call

```
bool KheArchiveReadFromString(char *str, KHE_ARCHIVE *archive,
  KML_ERROR *ke, bool infer_resource_partitions, bool allow_invalid_solns);
```

This is just like KheArchiveRead except that the archive lies in str instead of fp, and is expected to occupy the entire string.

2.4. Reading archives incrementally

A large archive may have to be read one solution at a time. For this, call

```
bool KheArchiveReadIncremental(FILE *fp, KHE_ARCHIVE *archive,
  KML_ERROR *ke, bool infer_resource_partitions, bool allow_invalid_solns,
  char **leftover, int *leftover_len, FILE *echo_fp,
  KHE_ARCHIVE_FN archive_begin_fn, KHE_ARCHIVE_FN archive_end_fn,
  KHE_SOLN_GROUP_FN soln_group_begin_fn,
  KHE_SOLN_GROUP_FN soln_group_end_fn, KHE_SOLN_FN soln_fn, void *impl);
```

The return value and the first eight parameters, to echo_fp inclusive, are as for KheArchiveRead. The next five parameters are callback functions, and the last parameter, impl, is not used by KHE but is instead passed through to the calls on the callback functions. Any or all of the callback functions may be NULL, in which case the corresponding callbacks are not made.

Callback function archive_begin_fn is called by KheArchiveReadIncremental at the start of the archive. It must be written by the user like this:

```
void archive_begin_fn(KHE_ARCHIVE archive, void *impl)
{
  ...
}
```

Its archive parameter is set to the archive that KheArchiveReadIncremental will eventually build, the one it returns in its *archive parameter; its impl parameter contains the value of the impl parameter of KheArchiveReadIncremental. At the time of this call, archive contains its Id and metadata attributes, but no instances and no solution groups.

Callback function archive_end_fn is called at the end of the archive, just before KheArchiveReadIncremental itself returns:

```
void archive_end_fn(KHE_ARCHIVE archive, void *impl)
{
  ...
}
```

When this function is called, archive contains all of its instances and solution groups. If KheArchiveReadIncremental returns true, there has been one callback to archive_begin_fn and one to archive_end_fn, if non-NULL.

Callback function soln_group_begin_fn is called at the start of each solution group:

```
void soln_group_begin_fn(KHE_SOLN_GROUP soln_group, void *impl)
{
  ...
}
```

Its soln_group parameter is set to one of the solution groups that the final archive will eventually contain, and its impl parameter is as before. At the time of this call, soln_group contains its Id and MetaData, and KheSolnGroupArchive(soln_group) returns the enclosing archive, but there are no solutions in soln_group.

Callback function soln_group_end_fn is called at the end of each solution group:

```
void soln_group_end_fn(KHE_SOLN_GROUP soln_group, void *impl)
{
  ...
}
```

At the time of this call, soln_group contains all its solutions.

Finally, callback function soln_fn is called after each solution is read:

```
void soln_fn(KHE_SOLN soln, void *impl)
{
  ...
}
```

The solution is complete, and `KheSolnSolnGroup(soln)` returns the enclosing solution group.

The purpose of incremental reading is to process the solutions as they are read, so that they can be discarded and their memory reclaimed. One way to save memory is to replace each solution by a placeholder. This can be done by passing `NULL` for all callbacks except `soln_fn`, which would be defined like this:

```
void soln_fn(KHE_SOLN soln, void *impl)
{
  if( !KheSolnIsPlaceholder(soln) )
    KheSolnReduceToPlaceholder(soln);
}
```

The test is needed only if `allow_invalid_solns` is `true`. As Section 4.11 explains, `KheSolnReduceToPlaceholder` reclaims most of the memory of `soln`, leaving just the `soln` object itself and a few key attributes, including its cost. This memory will then be recycled for holding other solutions. In this way, the total memory cost is reduced to not much more than the memory needed to hold the instances, but enough information is retained to support operations which (for example) print tables of solutions and their costs.

Other applications might process `soln` in some way (print timetables, for example) before finishing with a call to `KheSolnReduceToPlaceholder`, or even `KheSolnDelete`.

2.5. Writing archives and solution groups

To write an archive to a file, call

```
void KheArchiveWrite(KHE_ARCHIVE archive, bool with_reports, FILE *fp);
```

File `fp` must be open for writing UTF-8 characters, and it remains open after the call returns. If `with_reports` is `true`, each written solution contains a `Report` section evaluating the solution.

Ids, names, and meta-data are optional in KHE but compulsory when writing XML: if any are missing, `KheArchiveWrite` writes an incomplete file and aborts with an error message. They will all be present when `archive` was produced by `KheArchiveRead`.

When an event has a preassigned time, there is a problem if one of its meets is not assigned that time. If the meet is assigned some other time (which is possible in KHE, although not easy), then writing that time will cause the solution to be declared invalid when it is re-read. If the meet is not assigned any time, then, whether or not the preassigned time is written, the meaning is that the preassigned time is assigned, which is not the true state of the solution. The same problem arises with preassigned event resources whose tasks are not assigned the preassigned resource.

Accordingly, `KheArchiveWrite` also writes an incomplete file and aborts with an error message when it encounters a meet (or task) derived from a preassigned event (or event resource) whose assigned time (or resource) is unequal to the preassigned time (or resource).

When writing solutions, `KheArchiveWrite` writes as little as possible. It does not write an unassigned or preassigned task. It does not write a meet if its duration equals the duration of the corresponding event, its time is unassigned or preassigned, and its tasks are not written according to the rule just given (see also Section 4.3).

A similar function is

```
void KheArchiveWriteSolnGroup(KHE_ARCHIVE archive,
  KHE_SOLN_GROUP soln_group, bool with_reports, FILE *fp);
```

It also writes `archive`, omitting all its solution groups except `soln_group`. Also,

```
void KheArchiveWriteWithoutSolnGroups(KHE_ARCHIVE archive, FILE *fp);
```

writes `archive` omitting all solution groups.

Chapter 3. Instances

An *instance* is a particular case of the high school timetabling problem, for a particular term or semester of a particular school. This chapter describes the `KHE_INSTANCE` data type, which represents instances as defined in the XML format.

3.1. Creating instances

To make a new, empty instance, call

```
KHE_INSTANCE KheInstanceMakeBegin(char *id, KHE_INSTANCE_METADATA md);
```

Parameters `id` and `md` are the Id and MetaData attributes from the XML file; both are optional, with `NULL` meaning absent. Functions

```
char *KheInstanceId(KHE_INSTANCE ins);
char *KheInstanceName(KHE_INSTANCE ins);
KHE_INSTANCE_METADATA KheInstanceMetaData(KHE_INSTANCE ins);
```

may be called to retrieve these attributes. `KheInstanceName` is a convenience function that calls `KheInstanceMetaDataName` below.

For the convenience of functions that reorganize archives, an instance may lie in any number of archives. To add an instance to an archive and delete it from an archive, call functions `KheArchiveAddInstance` and `KheArchiveDeleteInstance` from Section 2.1. To visit the archives containing a given instance, call

```
int KheInstanceArchiveCount(KHE_INSTANCE ins);
KHE_ARCHIVE KheInstanceArchive(KHE_INSTANCE ins, int i);
```

in the usual way.

To set and retrieve the back pointer of `ins`, call

```
void KheInstanceSetBack(KHE_INSTANCE ins, void *back);
void *KheInstanceBack(KHE_INSTANCE ins);
```

as usual.

After the instance has been completed, using functions still to be defined, call

```
void KheInstanceMakeEnd(KHE_INSTANCE ins, bool infer_resource_partitions);
```

This must be done, single-threaded, before any solution is created. It checks the instance and initializes various constant data structures used to speed the solution process. Parameter `infer_resource_partitions` is the subject of Section 3.5.5.

Instance metadata may be created by calling

```
KHE_INSTANCE_METADATA KheInstanceMetaDataMake(char *name,
  char *contributor, char *date, char *country,
  char *description, char *remarks);
```

where `remarks`, being optional, may be `NULL`. The attributes may be retrieved by calling

```
char *KheInstanceMetaDataName(KHE_INSTANCE_METADATA md);
char *KheInstanceMetaDataContributor(KHE_INSTANCE_METADATA md);
char *KheInstanceMetaDataDate(KHE_INSTANCE_METADATA md);
char *KheInstanceMetaDataCountry(KHE_INSTANCE_METADATA md);
char *KheInstanceMetaDataDescription(KHE_INSTANCE_METADATA md);
char *KheInstanceMetaDataRemarks(KHE_INSTANCE_METADATA md);
```

`KheInstanceMetaDataRemarks` may return `NULL`.

An instance may contain any number of time groups, times, resource types, event groups, events, and constraints. These are added by the functions that create them, to be given later.

3.2. Visiting and retrieving the components of instances

To visit all the time groups of an instance, or retrieve a time group by `id`, call

```
int KheInstanceTimeGroupCount(KHE_INSTANCE ins);
KHE_TIME_GROUP KheInstanceTimeGroup(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveTimeGroup(KHE_INSTANCE ins, char *id,
  KHE_TIME_GROUP *tg);
```

The first returns the number of time groups in `ins`. The second returns the `i`'th time group, counting from 0 as usual in C. The third searches for a time group of `ins` with the given `id`; if found, it sets `*tg` to it and returns `true`, otherwise it leaves `*tg` unchanged and returns `false`.

Only time groups created by calls to `KheTimeGroupMake` (Section 3.4.1) made by the user may be accessed by calling `KheInstanceTimeGroupCount`, `KheInstanceTimeGroup`, and `KheInstanceRetrieveTimeGroup`. Some other time groups are created automatically by KHE, but they are accessed in other ways. They include one time group for each time, holding just that time; a time group holding the full set of times of the instance; and an empty time group. These last two are returned by

```
KHE_TIME_GROUP KheInstanceFullTimeGroup(KHE_INSTANCE ins);
KHE_TIME_GROUP KheInstanceEmptyTimeGroup(KHE_INSTANCE ins);
```

Time groups may also be created during solving (Section 4.4). Those too are not accessible via `KheInstanceTimeGroupCount`, `KheInstanceTimeGroup`, or `KheInstanceRetrieveTimeGroup`.

To visit all the times of an instance, or retrieve a time by Id, call

```
int KheInstanceTimeCount(KHE_INSTANCE ins);
KHE_TIME KheInstanceTime(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveTime(KHE_INSTANCE ins, char *id, KHE_TIME *t);
```

These work in the same way as the functions above for visiting and retrieving time groups.

To visit all the resource types of an instance, or retrieve a resource type by `id`, call

```
int KheInstanceResourceTypeCount(KHE_INSTANCE ins);
KHE_RESOURCE_TYPE KheInstanceResourceType(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveResourceType(KHE_INSTANCE ins, char *id,
  KHE_RESOURCE_TYPE *rt);
```

These work in the same way as the corresponding functions for visiting and retrieving time groups and times. Resource types have operations which give access to their resource groups and resources. For convenience there are also operations

```
bool KheInstanceRetrieveResourceGroup(KHE_INSTANCE ins, char *id,
  KHE_RESOURCE_GROUP *rg);
bool KheInstanceRetrieveResource(KHE_INSTANCE ins, char *id,
  KHE_RESOURCE *r);
```

which search all the resource types of `ins` for a resource group or resource with the given `id`. It is also possible to bypass resource types and visit all resources directly, by calling

```
int KheInstanceResourceCount(KHE_INSTANCE ins);
KHE_RESOURCE KheInstanceResource(KHE_INSTANCE ins, int i);
```

in the usual way. The resources will be visited in the order they were created.

To visit all the event groups of an instance, or to retrieve an event group by `id`, call

```
int KheInstanceEventGroupCount(KHE_INSTANCE ins);
KHE_EVENT_GROUP KheInstanceEventGroup(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveEventGroup(KHE_INSTANCE ins, char *id,
  KHE_EVENT_GROUP *eg);
```

These work in the usual way. Some event groups are created automatically by KHE, including one event group for each event, holding just that event; an event group holding the full set of events of the instance; and an empty event group. These last two are returned by

```
KHE_EVENT_GROUP KheInstanceFullEventGroup(KHE_INSTANCE ins);
KHE_EVENT_GROUP KheInstanceEmptyEventGroup(KHE_INSTANCE ins);
```

Automatically defined event groups are not visited by `KheInstanceEventGroupCount` and `KheInstanceEventGroup`. Even more event groups may be created during solving. Those also do not appear in the list of event groups of the original instance.

To visit the events of an instance, or to retrieve an event by `id`, call

```
int KheInstanceEventCount(KHE_INSTANCE ins);
KHE_EVENT KheInstanceEvent(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveEvent(KHE_INSTANCE ins, char *id, KHE_EVENT *e);
```

To visit the event resources of an instance, call

```
int KheInstanceEventResourceCount(KHE_INSTANCE ins);
KHE_EVENT_RESOURCE KheInstanceEventResource(KHE_INSTANCE ins, int i);
```

The event resources may also be visited via their events.

To visit all the constraints of an instance, or to retrieve a constraint by `id`, call

```
int KheInstanceConstraintCount(KHE_INSTANCE ins);
KHE_CONSTRAINT KheInstanceConstraint(KHE_INSTANCE ins, int i);
bool KheInstanceRetrieveConstraint(KHE_INSTANCE ins, char *id,
  KHE_CONSTRAINT *c);
```

These operations work in the usual way.

3.3. Constraint density

Within a given instance, the *density* of a given kind of constraint is the number of applications of constraints of that kind, divided by the number of places where constraints of that kind could apply. The density is a floating-point number, usually between 0 and 1, although it can exceed 1, since nothing prevents several constraints of the same kind from applying at one place.

In support of this concept KHE offers functions

```
int KheInstanceConstraintDensityCount(KHE_INSTANCE ins,
  KHE_CONSTRAINT_TAG constraint_tag);
int KheInstanceConstraintDensityTotal(KHE_INSTANCE ins,
  KHE_CONSTRAINT_TAG constraint_tag);
```

returning the number of applications of constraints of kind `constraint_tag` in `ins` (the *density count*), and the number of places where constraints of that kind could apply in `ins` (the *density total*). The density is the quotient of these two quantities, unless the density total is 0, in which case the density is undefined, although it may be reported as 0.0 in that case. Precise definitions of the density count and density total are given for each kind of constraint in Section 3.7.

The first time either of these functions is called for any value of `constraint_tag`, the results of both functions are calculated for all values of `constraint_tag` and stored in `ins`. So multi-threaded calls on these functions are only safe if one single-threaded call is made first.

3.4. Times

3.4.1. Time groups

A time group, representing a set of times, is created and added to an instance by calling

```
bool KheTimeGroupMake(KHE_INSTANCE ins, KHE_TIME_GROUP_KIND kind,
  char *id, char *name, KHE_TIME_GROUP *tg);
```

This works like all creations of named objects do in KHE: if `id` is non-`NULL` and `ins` already contains a time group with this `id`, it returns `false` and creates nothing; otherwise it creates a new time group, sets `*tg` to point to it, and returns `true`.

Parameter `kind` has type

```
typedef enum {
  KHE_TIME_GROUP_KIND_ORDINARY,
  KHE_TIME_GROUP_KIND_WEEK,
  KHE_TIME_GROUP_KIND_DAY
} KHE_TIME_GROUP_KIND;
```

`KHE_TIME_GROUP_KIND_ORDINARY` is the usual kind. The XML format allows some time groups to be referred to as Weeks and Days, although they do not differ from other time groups in any other way. Values `KHE_TIME_GROUP_KIND_WEEK` and `KHE_TIME_GROUP_KIND_DAY` record this usage; they matter only when reading and writing XML files, not when solving.

The `id` and `name` parameters may be `NULL`; they are used only when writing XML, when they represent the compulsory Id and Name attributes of the time group. Irrespective of the order time groups are created in, to conform with the XML rules, when writing time groups KHE writes days first, then weeks, then ordinary time groups; it does not write predefined time groups.

To set and retrieve the back pointer of `tg`, call

```
void KheTimeGroupSetBack(KHE_TIME_GROUP tg, void *back);
void *KheTimeGroupBack(KHE_TIME_GROUP tg);
```

in the usual way. The other attributes may be retrieved by calling

```
KHE_INSTANCE KheTimeGroupInstance(KHE_TIME_GROUP tg);
KHE_TIME_GROUP_KIND KheTimeGroupKind(KHE_TIME_GROUP tg);
char *KheTimeGroupId(KHE_TIME_GROUP tg);
char *KheTimeGroupName(KHE_TIME_GROUP tg);
```

Initially the time group is empty. There are several operations for changing its set of times:

```
void KheTimeGroupAddTime(KHE_TIME_GROUP tg, KHE_TIME t);
void KheTimeGroupSubTime(KHE_TIME_GROUP tg, KHE_TIME t);
void KheTimeGroupUnion(KHE_TIME_GROUP tg, KHE_TIME_GROUP tg2);
void KheTimeGroupIntersect(KHE_TIME_GROUP tg, KHE_TIME_GROUP tg2);
void KheTimeGroupDifference(KHE_TIME_GROUP tg, KHE_TIME_GROUP tg2);
```

These add a time to `tg`, remove a time, replace `tg`'s set of times with its union or intersecton with the set of times of `tg2`, and with the difference of `tg`'s times and `tg2`'s times. The first two operations are treated as set operations, so `KheTimeGroupAddTime` does nothing if `t` is already present, and `KheTimeGroupSubTime` does nothing if `t` is not already present.

Changes to the time groups of an instance are not allowed after `KheInstanceMakeEnd` is called, since instances are immutable after that point. However, solutions may construct time groups for their own use (Section 4.4).

There are also predefined time groups, for the full set of times of the instance and for the empty set of times (Section 3.2), and one for each time of the instance, containing just that time (Section 3.4). These time groups have `KHE_TIME_GROUP_KIND_ORDINARY` for kind and `NULL` for Id and Name. Their times may not be changed. They are never read or written; if time groups with their values are wanted in an instance, the user must define them.

The times of any time group are visited by

```
int KheTimeGroupTimeCount(KHE_TIME_GROUP tg);
KHE_TIME KheTimeGroupTime(KHE_TIME_GROUP tg, int i);
```

These work in the same way as the visit functions for instances above. And

```
bool TimeGroupContains(KHE_TIME_GROUP tg, KHE_TIME t);
bool KheTimeGroupEqual(KHE_TIME_GROUP tg1, KHE_TIME_GROUP tg2);
bool KheTimeGroupSubset(KHE_TIME_GROUP tg1, KHE_TIME_GROUP tg2);
bool KheTimeGroupDisjoint(KHE_TIME_GROUP tg1, KHE_TIME_GROUP tg2);
```

return `true` if `tg` contains `t`, if `tg1` and `tg2` contain the same times, if the times of `tg1` are a subset of the times of `tg2`, and if the times of `tg1` and `tg2` are disjoint. These tests use bit vectors, so are quite fast. There is nothing to prevent two distinct time groups from containing the same times, so the C equality operator should never be applied to time groups.

Here are some miscellaneous time group functions. Function

```
bool KheTimeGroupIsCompact(KHE_TIME_GROUP tg);
```

returns `true` when `tg` is *compact*: when it is empty or there are no gaps in its times, taken in chronological order. Function

```
int KheTimeGroupOverlap(KHE_TIME_GROUP tg, KHE_TIME time, int durn);
```

returns the number of times that a meet starting at `time` with duration `durn` would overlap with `tg`. And function

```
KHE_TIME_GROUP KheTimeGroupNeighbour(KHE_TIME_GROUP tg, int delta);
```

returns a predefined time group containing `tg`'s times shifted `delta` places, where `delta` may be any integer. The time group will be empty if `delta` is such a large (positive or negative) number that all the times are shifted off the cycle. For example, `KheTimeGroupNeighbour(tg, 0)` is `tg`, and `KheTimeGroupNeighbour(tg, -1)` holds the times that immediately precede `tg`'s.

As an aid to debugging, function

```
void KheTimeGroupDebug(KHE_TIME_GROUP tg, int verbosity,
  int indent, FILE *fp);
```

prints `tg` onto `fp` with the given verbosity and indent, as described for debug functions in general in Section 1.3. Verbosity 1 prints the Id of the time group in some cases, and the first and last time (at most) enclosed in braces in others.

3.4.2. Times

A time is created and added to an instance by calling

```
bool KheTimeMake(KHE_INSTANCE ins, char *id, char *name,
  bool break_after, KHE_TIME *t);
```

As usual, a `false` return value is only possible when `id` is non-`NULL` and already in use by another time object. Parameters `id` and `name` may be `NULL`, and are used only when writing XML.

Parameter `break_after` says that a break occurs after this time, so that, for example, an event of duration 2 could not begin here. This is not an XML feature; when representing XML this parameter should always be `false`. Within KHE itself it is used only by function `KheSolnSplitCycleMeet` and its associated operations (Section 4.8.3).

To set and retrieve the back pointer of a time, call functions

```
void KheTimeSetBack(KHE_TIME t, void *back);
void *KheTimeBack(KHE_TIME t);
```

as usual. The other attributes are retrieved by

```
KHE_INSTANCE KheTimeInstance(KHE_TIME t);
char *KheTimeId(KHE_TIME t);
char *KheTimeName(KHE_TIME t);
bool KheTimeBreakAfter(KHE_TIME t);
int KheTimeIndex(KHE_TIME t);
```

`KheTimeIndex` returns an automatically generated index number for `time`: 0 for the first time created, 1 for the second, and so on. The times of an instance form a sequence, not a set, and must be created in chronological order. This is unlike resources, events, etc., whose order of creation does not matter. The XML format requires times to appear in this same order. Function

```
bool KheTimeHasNeighbour(KHE_TIME t, int delta);
```

returns `true` when there is a time whose index is the index of `t` plus `delta`, where `delta` may be any integer, negative, zero, or positive. Function

```
KHE_TIME KheTimeNeighbour(KHE_TIME t, int delta);
```

returns this time when it exists, and aborts when it does not.

When calculating with the chronological ordering of time—deciding whether two meets are adjacent, and so on—it is often best to call `KheTimeIndex` to obtain the indexes of the times involved and work with them. However, these functions may help to avoid time indexes:

```
bool KheTimeLE(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
bool KheTimeLT(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
bool KheTimeGT(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
bool KheTimeGE(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
bool KheTimeEQ(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
bool KheTimeNE(KHE_TIME time1, int delta1, KHE_TIME time2, int delta2);
```

They return `true` when `KheTimeNeighbour(time1, delta1)`'s time index is less than or equal to `KheTimeNeighbour(time2, delta2)`'s, and so on. The neighbours need not exist; the functions simply convert times into indexes and perform the indicated integer operations. Also,

```
int KheTimeIntervalsOverlap(KHE_TIME time1, int durn1,
  KHE_TIME time2, int durn2);
```

takes two time intervals, one beginning at `time1` with duration `durn1`, the other beginning at `time2` with duration `durn2`, and returns the number of times lying in both intervals. For example,

the result will be 0 when either interval ends before the other begins. Similarly,

```
bool KheTimeIntervalsOverlapInterval(KHE_TIME time1, int durn1,
  KHE_TIME time2, int durn2, KHE_TIME *overlap_time, int *overlap_durn);
```

returns `true` when `KheTimeIntervalsOverlap` is non-zero, and sets `*overlap_time` and `*overlap_durn` to the starting time and duration of the overlap; otherwise it returns `false`.

For convenience, a time group is created for each time, holding just that time. Function

```
KHE_TIME_GROUP KheTimeSingletonTimeGroup(KHE_TIME t);
```

returns this predefined time group. It cannot be changed.

3.5. Resources

3.5.1. Resource types

A resource type, representing one broad category of resources, such as the teachers or rooms, is created and added to an instance in the usual way by the call

```
bool KheResourceTypeMake(KHE_INSTANCE ins, char *id, char *name,
  bool has_partitions, KHE_RESOURCE_TYPE *rt);
```

Attributes `id` and `name` represent the optional XML Id and Name attributes as usual. Its back pointer may be set and retrieved by

```
void KheResourceTypeSetBack(KHE_RESOURCE_TYPE rt, void *back);
void *KheResourceTypeBack(KHE_RESOURCE_TYPE rt);
```

as usual, and its other attributes may be retrieved by

```
KHE_INSTANCE KheResourceTypeInstance(KHE_RESOURCE_TYPE rt);
int KheResourceTypeIndex(KHE_RESOURCE_TYPE rt);
char *KheResourceTypeId(KHE_RESOURCE_TYPE rt);
char *KheResourceTypeName(KHE_RESOURCE_TYPE rt);
bool KheResourceTypeHasPartitions(KHE_RESOURCE_TYPE rt);
```

`KheResourceTypeIndex(rt)` returns the index of `rt` in the enclosing instance, that is, the value of `i` for which `KheInstanceResourceType` returns `rt`.

Attribute `has_partitions` is not an XML feature, and should be given value `false` when reading an XML instance. It indicates that there is a unique partitioning of the resources of this resource type, defined by a collection of specially marked resource groups called *partitions*. For example, the resources of a student groups resource type might be partitioned into forms, or the resources of a teachers resource type might be partitioned into faculties. When a resource type has partitions, each of its resources must lie in exactly one partition.

Each resource type contains an arbitrary number of resource groups, representing sets of resources of its type. Resource groups are added to a resource type automatically by the functions that create them. To visit all the resource groups of a given resource type, or to retrieve

a resource group with a given `id` from a given resource type, call

```
int KheResourceTypeResourceGroupCount(KHE_RESOURCE_TYPE rt);
KHE_RESOURCE_GROUP KheResourceTypeResourceGroup(KHE_RESOURCE_TYPE rt,
  int i);
bool KheResourceTypeRetrieveResourceGroup(KHE_RESOURCE_TYPE rt,
  char *id, KHE_RESOURCE_GROUP *rg);
```

These work in the usual way. The partitions of a resource type may be visited by

```
int KheResourceTypePartitionCount(KHE_RESOURCE_TYPE rt);
KHE_RESOURCE_GROUP KheResourceTypePartition(KHE_RESOURCE_TYPE rt, int i);
```

`KheResourceTypePartitionCount` returns 0 when `rt` does not have partitions.

Some resource groups are made automatically by KHE, including one resource group for each resource, holding just that resource; a resource group holding the full set of resources of the resource type; and an empty resource group. These last two are returned by

```
KHE_RESOURCE_GROUP KheResourceTypeFullResourceGroup(KHE_RESOURCE_TYPE rt);
KHE_RESOURCE_GROUP KheResourceTypeEmptyResourceGroup(KHE_RESOURCE_TYPE rt);
```

Automatically made resource groups are not visited by `KheResourceTypeResourceGroupCount` and `KheResourceTypeResourceGroup`. Even more resource groups may be created during solving, but those do not appear in the list of resource groups of the original instance.

To visit all the resources of a given resource type, or to retrieve a resource of a given resource type by `id`, call

```
int KheResourceTypeResourceCount(KHE_RESOURCE_TYPE rt);
KHE_RESOURCE KheResourceTypeResource(KHE_RESOURCE_TYPE rt, int i);
bool KheResourceTypeRetrieveResource(KHE_RESOURCE_TYPE rt,
  char *id, KHE_RESOURCE *r);
```

in the usual way.

Two functions, which should be called only after the instance is complete, are offered for summarising how complex the task of assigning resources of a given type is. The values of both functions are calculated as the instance is built and kept, so one call on either function costs practically nothing. The first is

```
bool KheResourceTypeDemandIsAllPreassigned(KHE_RESOURCE_TYPE rt);
```

It returns `true` if every event resource of type `rt` is preassigned. In practice this is always true for student group resource types, and often for teachers, but rarely for rooms. The second is

```
int KheResourceTypeAvoidSplitAssignmentsCount(KHE_RESOURCE_TYPE rt);
```

It returns the number of points of application of avoid split assignments constraints that constrain event resources of this type. The larger this number is, the more difficult the resource assignment problem for resources of this type is likely to be.

3.5.2. Resource groups

A resource group is created and added to a resource type by the call

```
bool KheResourceGroupMake(KHE_RESOURCE_TYPE rt, char *id, char *name,
  bool is_partition, KHE_RESOURCE_GROUP *rg)
```

This function returns `false` only when `id` is non-`NULL` and some other resource group of type `rt` has this `id`. The resource group lies in resource type `rt` with the usual `id` and `name` attributes. Attribute `is_partition` is not an XML feature, and should be given value `false` when reading an XML instance. It may be `true` only if attribute `has_partitions` of the resource group's resource type is `true`, in which case it indicates that this resource group is a partition, that is, one of those resource groups which define the unique partitioning of the resources of that type.

To set and retrieve the back pointer of a resource group, call

```
void KheResourceGroupSetBack(KHE_RESOURCE_GROUP rg, void *back);
void *KheResourceGroupBack(KHE_RESOURCE_GROUP rg);
```

as usual. The other attributes may be retrieved by calling

```
KHE_RESOURCE_TYPE KheResourceGroupResourceType(KHE_RESOURCE_GROUP rg);
KHE_INSTANCE KheResourceGroupInstance(KHE_RESOURCE_GROUP rg);
char *KheResourceGroupId(KHE_RESOURCE_GROUP rg);
char *KheResourceGroupName(KHE_RESOURCE_GROUP rg);
bool KheResourceGroupIsPartition(KHE_RESOURCE_GROUP rg);
```

`KheResourceGroupInstance` returns the resource group's resource type's instance.

Initially the resource group is empty. Several operations change its resources:

```
void KheResourceGroupAddResource(KHE_RESOURCE_GROUP rg, KHE_RESOURCE r);
void KheResourceGroupSubResource(KHE_RESOURCE_GROUP rg, KHE_RESOURCE r);
void KheResourceGroupUnion(KHE_RESOURCE_GROUP rg,
  KHE_RESOURCE_GROUP rg2);
void KheResourceGroupIntersect(KHE_RESOURCE_GROUP rg,
  KHE_RESOURCE_GROUP rg2);
void KheResourceGroupDifference(KHE_RESOURCE_GROUP rg,
  KHE_RESOURCE_GROUP rg2);
```

These add `r` to `rg`, remove `r`, replace `rg`'s set of resources with its union or intersecton with the set of resources of `rg2`, and with the difference of `rg`'s resources and `rg2`'s resources. All the resources and resource groups involved must be of the same type. The first two operations are treated as set operations, so `KheResourceGroupAddResource` does nothing if `r` is already present, and `KheResourceGroupSubResource` does nothing if `r` is not already present.

These functions may not be used to alter resource groups which define partitions. When a resource type has partitions, each of its resources is added to its partition when it is created.

Changes to the resource groups of an instance are not allowed after `KheInstanceMakeEnd` is called, since instances are immutable after that point. However, solutions may construct resource groups for their own use (Section 4.4).

There are also predefined resource groups, for the complete set of resources of each resource type and the empty set of resources of each type (see Section 3.5.1 for those), and one for each resource of the instance, containing just that resource (Section 3.5). The resources in predefined resource groups may not be changed.

The resources of any resource group are visited by

```
int KheResourceGroupResourceCount(KHE_RESOURCE_GROUP rg);
KHE_RESOURCE KheResourceGroupResource(KHE_RESOURCE_GROUP rg, int i);
```

These work in the usual way. And

```
bool KheResourceGroupContains(KHE_RESOURCE_GROUP rg, KHE_RESOURCE r);
bool KheResourceGroupEqual(KHE_RESOURCE_GROUP rg1,
  KHE_RESOURCE_GROUP rg2);
bool KheResourceGroupSubset(KHE_RESOURCE_GROUP rg1,
  KHE_RESOURCE_GROUP rg2);
bool KheResourceGroupDisjoint(KHE_RESOURCE_GROUP rg1,
  KHE_RESOURCE_GROUP rg2);
```

return `true` if `rg` contains `r`, if `rg1` and `rg2` contain the same resources, if the resources of `rg1` form a subset of the resources of `rg2`, and if the resources of `rg1` and `rg2` are disjoint. These tests use bit vectors, so are quite fast. Two distinct resource groups may contain the same resources, so it is best not to apply the C equality operator to resource groups.

After a resource group is finalized, function

```
KHE_RESOURCE_GROUP KheResourceGroupPartition(KHE_RESOURCE_GROUP rg);
```

may be called. If `rg` is non-empty and its resources share a partition, the result is that partition, otherwise the result is `NULL`. Since `KheResourceGroupPartition` is called when monitoring evenness, for efficiency the result is precomputed and stored in `rg` when it is finalized.

As an aid to debugging, function

```
void KheResourceGroupDebug(KHE_RESOURCE_GROUP rg, int verbosity,
  int indent, FILE *fp);
```

prints `rg` onto `fp` with the given verbosity and indent, as described for debug functions in general in Section 1.3. Verbosity 1 prints the Id of the resource group in some cases, and the first and last resource (at most) enclosed in braces in others.

3.5.3. Resources

A resource is created and added to its resource type by the call

```
bool KheResourceMake(KHE_RESOURCE_TYPE rt, char *id, char *name,
  KHE_RESOURCE_GROUP partition, KHE_RESOURCE *r);
```

A resource type is compulsory; `id` and `name` are the usual optional XML Id and Name.

Unlike `KheResourceGroupMake`, which returns `false` when its `id` parameter is non-`NULL` and some other resource group of the same resource type already has that Id, `KheResourceMake`

returns `false` and sets `*r` to `NULL` when its `id` parameter is non-`NULL` and some other resource *of any resource type* already has its Id. This is because predefined event resources are permitted to identify a resource by its Id alone, and so resource Ids must be unique among all the resources of the instance, not merely among resources of a given type.

The `partition` attribute is not an XML feature, and should be given value `NULL` when reading an XML instance. It must be non-`NULL` if and only if `rt`'s `has_partitions` attribute is `true`, in which case its value must be a resource group of type `rt` whose `is_partition` attribute is `true`, and it indicates that the new resource lies in the specified partition. The new resource will be added to the partition by this function, and no separate call to `ResourceGroupAddResource` to do this is necessary or even permitted.

To set and retrieve the back pointer of a resource, call

```
void KheResourceSetBack(KHE_RESOURCE r, void *back);
void *KheResourceBack(KHE_RESOURCE r);
```

as usual. The other attributes may be retrieved by the calls

```
KHE_INSTANCE KheResourceInstance(KHE_RESOURCE r);
int KheResourceInstanceIndex(KHE_RESOURCE r);
KHE_RESOURCE_TYPE KheResourceResourceType(KHE_RESOURCE r);
int KheResourceResourceTypeIndex(KHE_RESOURCE r);
char *KheResourceId(KHE_RESOURCE r);
char *KheResourceName(KHE_RESOURCE r);
KHE_RESOURCE_GROUP KheResourcePartition(KHE_RESOURCE r);
```

`KheResourceInstance` returns the enclosing instance, and `KheResourceInstanceIndex` returns `r`'s index in that instance (the value of `i` for which `KheInstanceResource(ins, i)` returns `r`). `KheResourceResourceType` returns the resource type of `r`, and `KheResourceResourceTypeIndex` returns `r`'s index in that resource type (the value of `i` for which `KheResourceTypeResource(rt, i)` returns `r`). Unlike the index numbers of times, which indicate chronological order, the index numbers of resources have no significance to the specification of the instance. They are made available only for convenience.

A resource group is created automatically for each resource `r`, holding just `r`. Function

```
KHE_RESOURCE_GROUP KheResourceSingletonResourceGroup(KHE_RESOURCE r);
```

returns this resource group. This resource group may not be changed.

The event resources that `r` is preassigned to are made available by calling

```
int KheResourcePreassignedEventResourceCount(KHE_RESOURCE r);
KHE_EVENT_RESOURCE KheResourcePreassignedEventResource(KHE_RESOURCE r,
  int i);
```

Naturally, the entire instance has to be loaded for these to work correctly. At present there is no way to visit events containing event resource groups containing a given resource.

Some constraints apply to resources. When these constraints are created, they are added to the resources they apply to. To visit all the constraints applicable to a given resource, call

```
int KheResourceConstraintCount(KHE_RESOURCE r);
KHE_CONSTRAINT KheResourceConstraint(KHE_RESOURCE r, int i);
```

There may be any number of avoid clashes constraints, avoid unavailable times constraints, limit idle times constraints, cluster busy times constraints, limit busy times constraints, and limit workload constraints, in any order. There are also

```
KHE_TIME_GROUP KheResourceHardUnavailableTimeGroup(KHE_RESOURCE r);
KHE_TIME_GROUP KheResourceHardAndSoftUnavailableTimeGroup(
  KHE_RESOURCE r);
```

`KheResourceHardUnavailableTimeGroup` returns the union of the domains of the required unavailable times constraints of `r`. `KheResourceHardAndSoftUnavailableTimeGroup` does the same, except that the domains of all unavailable times constraints are included. Both functions return the empty time group when there are no applicable constraints.

These two public functions are used by KHE when calculating lower bounds:

```
bool KheResourceHasAvoidClashesConstraint(KHE_RESOURCE r, KHE_COST cost);
int KheResourcePreassignedEventsDuration(KHE_RESOURCE r, KHE_COST cost);
```

`KheResourceHasAvoidClashesConstraint` returns `true` if some avoid clashes constraint of combined weight greater than `cost` applies to `r`; `KheResourcePreassignedEventsDuration` returns the total duration of events which are both preassigned `r` and either preassigned a time or subject to an assign time constraint of combined cost greater than `cost`.

As an aid to debugging, function

```
void KheResourceDebug(KHE_RESOURCE r, int verbosity,
  int indent, FILE *fp)
```

produces a debug print of resource `r` onto file `fp` with the given verbosity and indent, as described for debug functions in general in Section 1.3.

3.5.4. Resource layers

A *resource layer* is the set of events containing a preassignment of a given resource `r` which is the subject of a hard avoid clashes constraint. A resource layer's events may not overlap in time: they must spread horizontally across the timetable, hence the term 'layer'. Within a solution, the meets derived from the events of one resource layer form a *solution layer*, or just *layer*.

Layers are important in high school timetabling, at least for student group resources, since the total duration of their events is often close to the total duration of the cycle, and hence these events strongly constrain each other. The following operations are available on the layer of `r`:

```
int KheResourceLayerEventCount(KHE_RESOURCE r);
KHE_EVENT KheResourceLayerEvent(KHE_RESOURCE r, int i);
int KheResourceLayerDuration(KHE_RESOURCE r);
```

The first two work together in the usual way to return the events of the resource layer. They are sorted by increasing event index. If the resource is not preassigned to any events, or has no required avoid clashes constraint, then `KheResourceLayerEventCount` returns 0.

KheResourceLayerDuration returns the total duration of the events of the layer. In the unlikely case that r is assigned to the same event twice, the event still appears only once in the list of events of the layer, and contributes its duration only once to the layer duration.

3.5.5. Resource similarity and inferring resource partitions

Following the general approach introduced in Section 1.3, KHE offers function

```
bool KheResourceSimilar(KHE_RESOURCE r1, KHE_RESOURCE r2);
```

which returns true when resources r1 and r2 are similar: when they lie in similar resource groups and are preassigned to similar events. The exact definition is given below.

KheResourceSimilar often succeeds in recognising that student group resources from the same form are similar, and that teacher resources from the same faculty are similar. However, it needs positive evidence to work with. For example, when there are no student or teacher resource groups, and each event contains one preassigned student group resource, one preassigned teacher resource, and a request for one ordinary classroom, there is no basis for grouping the resources and each will be considered similar only to itself.

Resource partitions (Section 3.5.1) are not part of the XML format. But they are useful when solving, so KheInstanceMakeEnd has an infer_resource_partitions parameter which, when true, causes partitions to be added to each resource type rt that lacks them. Afterwards, KheResourceTypeHasPartitions(rt) will be true, KheResourceGroupIsPartition(rg) will be true for some of the resource groups of rt, and KheResourcePartition(r) will return a non-NULL partition for each resource r. All this is exactly as though the partitions had been entered explicitly, except that any specially created resource groups will not be visited by KheResourceTypeResourceGroupCount and KheResourceTypeResourceGroup.

The algorithm for inferring resource partitions is a simple application of resource similarity. Build a graph in which each node corresponds to one resource, and an edge joins two nodes when their resources are similar. The partitions are the connected components of this graph.

The details of how KheResourceSimilar works are not very important, but, for the record, here they are. To decide whether two resources are similar or not, two non-negative integers, the *positive evidence* and the *negative evidence*, are calculated as explained below. The two resources are similar if the positive evidence exceeds the negative evidence by at least two.

Evidence comes from two sources: the resource groups that the resources lie in, and the events that the resources are preassigned to. A resource group is *admissible* (i.e. admissible as evidence) if its number of resources is at least two and at most one third of the number of resources of its resource type. Inadmissible resource groups are considered to contain no useful information and are ignored. Each case of an admissible resource group containing both resources counts as two units of positive evidence, and each case of an admissible resource group containing one resource but not the other counts as one unit of negative evidence.

A definition of what it means for two events to be similar appears in Section 3.6.2. Each case of an event preassigned one resource being similar to an event preassigned the other counts as two units of positive evidence. Each case of an event preassigned one resource for which there is no similar event preassigned the other counts as one unit of negative evidence. The cases are distinct, in the sense that each event participates in at most one case.

3.6. Events

3.6.1. Event groups

An event group, representing a set of events, is created and added to an instance by calling

```
bool KheEventGroupMake(KHE_INSTANCE ins, KHE_EVENT_GROUP_KIND kind,
  char *id, char *name, KHE_EVENT_GROUP *eg);
```

As usual, it returns `false` only when `id` is non-`NULL` and `ins` already contains an event group with this `id`. To set and retrieve the back pointer, call

```
void KheEventGroupSetBack(KHE_EVENT_GROUP eg, void *back);
void *KheEventGroupBack(KHE_EVENT_GROUP eg);
```

as usual. The other attributes may be retrieved by the calls

```
KHE_INSTANCE KheEventGroupInstance(KHE_EVENT_GROUP eg);
KHE_EVENT_GROUP_KIND KheEventGroupKind(KHE_EVENT_GROUP eg);
char *KheEventGroupId(KHE_EVENT_GROUP eg);
char *KheEventGroupName(KHE_EVENT_GROUP eg);
```

The event group kind is a value of type

```
typedef enum {
  KHE_EVENT_GROUP_KIND_COURSE,
  KHE_EVENT_GROUP_KIND_ORDINARY
} KHE_EVENT_GROUP_KIND;
```

The XML format allows some event groups to be referred to as Courses, although they do not differ from other event groups in any other way. The `kind` attribute records this distinction; it is only used by KHE when reading and writing XML files, not when solving.

Irrespective of the order event groups are created in, to conform with the XML rules, when writing event groups KHE writes courses first, then ordinary event groups.

Initially the event group is empty. There are several operations for changing its events:

```
void KheEventGroupAddEvent(KHE_EVENT_GROUP eg, KHE_EVENT e);
void KheEventGroupSubEvent(KHE_EVENT_GROUP eg, KHE_EVENT e);
void KheEventGroupUnion(KHE_EVENT_GROUP eg, KHE_EVENT_GROUP eg2);
void KheEventGroupIntersect(KHE_EVENT_GROUP eg, KHE_EVENT_GROUP eg2);
void KheEventGroupDifference(KHE_EVENT_GROUP eg, KHE_EVENT_GROUP eg2);
```

These add an event to `eg`, remove an event, replace `eg`'s set of events with its union or intersecton with the set of events of `eg2`, and with the difference of `eg`'s events and `eg2`'s events. The first two operations are treated as set operations, so `KheEventGroupAddEvent` does nothing if `e` is already present, and `KheEventGroupSubEvent` does nothing if `e` is not already present.

Changes to the event groups of an instance are not allowed after `KheInstanceMakeEnd` is called, since instances are immutable after that point. However, solutions may construct event groups for their own use (Section 4.4).

There are also predefined event groups, for the complete set of events of the instance and for the empty set of events (Section 3), and one for each event of the instance, containing just that event (Section 3.6). The events in predefined event groups may not be changed.

To visit the events of an event group, functions

```
int KheEventGroupEventCount(KHE_EVENT_GROUP eg);
KHE_EVENT KheEventGroupEvent(KHE_EVENT_GROUP eg, int i);
```

are used in the usual way. And

```
bool KheEventGroupContains(KHE_EVENT_GROUP eg, KHE_EVENT e);
bool KheEventGroupEqual(KHE_EVENT_GROUP eg1, KHE_EVENT_GROUP eg2);
bool KheEventGroupSubset(KHE_EVENT_GROUP eg1, KHE_EVENT_GROUP eg2);
bool KheEventGroupDisjoint(KHE_EVENT_GROUP eg1, KHE_EVENT_GROUP eg2);
```

return `true` if `eg` contains `e`, if `eg1` and `eg2` contain the same events, if the events of `eg1` are a subset of the events of `eg2`, and if the events of `eg1` and `eg2` are disjoint. These tests use bit vectors, so are quite fast. There is nothing to prevent two distinct event groups from containing the same events, so the C equality operator should never be applied to event groups.

Some constraints apply to event groups. When these are created, they are added to the event groups they apply to. To visit all the constraints that apply to a given event group, call

```
int KheEventGroupConstraintCount(KHE_EVENT_GROUP eg);
KHE_CONSTRAINT KheEventGroupConstraint(KHE_EVENT_GROUP eg, int i);
```

There may be any number of avoid split assignments constraints, spread events constraints, and link events constraints, in any order.

Function

```
void KheEventGroupDebug(KHE_EVENT_GROUP eg, int verbosity,
  int indent, FILE *fp);
```

produces a debug print of `eg` onto `fp` with the given verbosity and indent, in the usual way.

3.6.2. Events

An event is created and added to an instance by calling

```
bool KheEventMake(KHE_INSTANCE ins, char *id, char *name, char *color,
  int duration, int workload, KHE_TIME preassigned_time, KHE_EVENT *e);
```

This works in the usual way, returning `false` only if `id` is non-`NULL` and is already used by an existing event of `ins`. Parameter `color` is an optional color to be used when printing the event in timetables. If non-`NULL`, its value must be a legal Web colour (`"#7CFC00"` for example, or a colour name). A duration and workload are compulsory (the XML specification states that a missing workload is taken to be equal to the duration), but the preassigned time may be `NULL`. The back pointer may be set and retrieved by

```
void KheEventSetBack(KHE_EVENT e, void *back);
void *KheEventBack(KHE_EVENT e);
```

as usual, and the other attributes may be retrieved by

```
KHE_INSTANCE KheEventInstance(KHE_EVENT e);
char *KheEventId(KHE_EVENT e);
char *KheEventName(KHE_EVENT e);
char *KheEventColor(KHE_EVENT e);
int KheEventDuration(KHE_EVENT e);
int KheEventWorkload(KHE_EVENT e);
KHE_TIME KheEventPreassignedTime(KHE_EVENT e);
```

There are two other useful query functions. First,

```
int KheEventIndex(KHE_EVENT e);
```

returns the index number of `e` (0 for the first event inserted, 1 for the next, etc.). This number has no timetabling significance; it is included merely for convenience. Second,

```
int KheEventDemand(KHE_EVENT e);
```

returns the *demand* of `e`, defined to be its duration multiplied by the number of its event resources (in matching terms, the number of demand tixels). This is included as a measure of the overall bulk of an event, useful for sorting events by estimated difficulty of timetabling.

Each event also contains any number of event resources. These are added to their events as they are created. To visit them, call

```
int KheEventResourceCount(KHE_EVENT e);
KHE_EVENT_RESOURCE KheEventResource(KHE_EVENT e, int i);
```

in the usual way. There is also

```
bool KheEventRetrieveEventResource(KHE_EVENT e, char *role,
  KHE_EVENT_RESOURCE *er);
```

which attempts to retrieve an event resource from `e` with the given `role`. If there is such an event resource, the function sets `*er` to that event resource and returns `true`. If not, `*er` is not changed and `false` is returned.

Each event also contains any number of event resource groups. These are added to their events as they are created. To visit them, call

```
int KheEventResourceGroupCount(KHE_EVENT e);
KHE_EVENT_RESOURCE_GROUP KheEventResourceGroup(KHE_EVENT e, int i);
```

as usual.

For convenience, an event group is created for each event, holding just that event. Call

```
KHE_EVENT_GROUP KheEventSingletonEventGroup(KHE_EVENT event);
```

to retrieve this event group. Other events may not be added to it.

Some constraints apply to events. When these constraints are created, they are added to the events they apply to. To visit all the constraints applicable to a given event, call

```
int KheEventConstraintCount(KHE_EVENT e);
KHE_CONSTRAINT KheEventConstraint(KHE_EVENT e, int i);
```

There may be any number of assign time constraints, prefer times constraints, split events constraints, and distribute split events constraints, in any order, except that an event with a preassigned time cannot have assign time constraints and prefer times constraints.

Following the general pattern given in Section 1.3, function

```
bool KheEventSimilar(KHE_EVENT e1, KHE_EVENT e2);
```

returns `true` if `e1` and `e2` are similar: if they have the same duration and similar event resources. The exact definition is as follows. An event is *admissible* if it has one or more admissible event resources. An event resource is admissible if its hard domain (reflecting its prefer resources constraints and any preassignment) is an admissible resource group, as defined in Section 3.5.5. An event is always similar to itself. Two distinct events are similar if they are admissible, have equal durations, and their admissible event resources (taken in any order) have equal hard domains.

There is also

```
bool KheEventMergeable(KHE_EVENT e1, KHE_EVENT e2, int slack);
```

which returns `true` if `e1` and `e2` could reasonably be considered to be split fragments of a single larger event: if their event resources correspond, ignoring differences in the order in which they appear in the two events. If `slack` is non-zero, `KheEventMergeable` returns `true` even if up to `slack` event resources in `e1` do not correspond with any event resource in `e2` and vice versa. Two event resources correspond when they have the same resource type, the same preassigned resource, equal hard domains as returned by `KheEventResourceHardDomain`, and equal hard-and-soft domains as returned by `KheEventResourceHardAndSoftDomain`. Like those two functions, `KheEventMergeable` can only be called after the instance is complete.

A reasonable way to decide whether two events must be disjoint in time is to call

```
bool KheEventSharePreassignedResource(KHE_EVENT e1, KHE_EVENT e2,
  KHE_RESOURCE *r);
```

If `e1` and `e2` share a preassigned resource which has a required avoid clashes constraint, this function returns `true` and sets `r` to one such resource; otherwise it returns `false` and sets `r` to `NULL`. It should only be called after the instance is complete.

Function

```
void KheEventDebug(KHE_EVENT e, int verbosity, int indent, FILE *fp);
```

produces a debug print of `e` onto `fp` with the given verbosity and indent, in the usual way.

3.6.3. Event resources

An event resource is created and added to an event by the call

```
bool KheEventResourceMake(KHE_EVENT event, KHE_RESOURCE_TYPE rt,
  KHE_RESOURCE preassigned_resource, char *role, int workload,
  KHE_EVENT_RESOURCE *er);
```

This returns `false` only when the optional `role` parameter (used only when writing XML) is non-`NULL` and there is already an event resource within `event` with this value for `role`. Parameter `preassigned_resource` is an optional resource preassignment and may be `NULL`.

To set and retrieve the back pointer of an event resource, call

```
void KheEventResourceSetBack(KHE_EVENT_RESOURCE er, void *back);
void *KheEventResourceBack(KHE_EVENT_RESOURCE er);
```

as usual. The other attributes may be retrieved by

```
KHE_INSTANCE KheEventResourceInstance(KHE_EVENT_RESOURCE er);
int KheEventResourceInstanceIndex(KHE_EVENT_RESOURCE er);
KHE_EVENT KheEventResourceEvent(KHE_EVENT_RESOURCE er);
int KheEventResourceEventIndex(KHE_EVENT_RESOURCE er);
KHE_RESOURCE_TYPE KheEventResourceResourceType(KHE_EVENT_RESOURCE er);
KHE_RESOURCE KheEventResourcePreassignedResource(KHE_EVENT_RESOURCE er);
char *KheEventResourceRole(KHE_EVENT_RESOURCE er);
int KheEventResourceWorkload(KHE_EVENT_RESOURCE er);
```

`KheEventResourceInstance` is the enclosing instance; `KheEventResourceInstanceIndex` is the index of `er` in that instance (the number `i` such that `KheInstanceEventResource(ins, i)` returns `er`). `KheEventResourceEvent` is the enclosing event; `KheEventResourceEventIndex` is the index of `er` in that event (the number `i` such that `KheEventResource(e, i)` returns `er`).

Some constraints apply to event resources. When these are created, they are added to the event resources they apply to. To visit the constraints that apply to a given event resource, call

```
int KheEventResourceConstraintCount(KHE_EVENT_RESOURCE er);
KHE_CONSTRAINT KheEventResourceConstraint(KHE_EVENT_RESOURCE er, int i);
```

There may be any number of assign resource constraints, prefer resources constraints, and avoid split assignments constraints, in any order, except that an event resource with a preassigned resource cannot have assign resource constraints and prefer resources constraints. If the `i`'th constraint is an avoid split assignments constraint, function

```
int KheEventResourceConstraintEventGroupIndex(KHE_EVENT_RESOURCE er, int i);
```

may be called to find the event group index within that constraint that contains `er`. (It returns `-1` if the `i`'th constraint is not an avoid split assignments constraint.)

After the instance is complete but not before, functions

```
KHE_RESOURCE_GROUP KheEventResourceHardDomain(KHE_EVENT_RESOURCE er);
KHE_RESOURCE_GROUP KheEventResourceHardAndSoftDomain(KHE_EVENT_RESOURCE er);
```

return domains suited to `er`. The resource group returned by `KheEventResourceHardDomain` is the intersection of the domains of the required prefer resources constraints, with weight greater than 0, of `er` and other event resources that share a required avoid split assignments constraint of weight greater than 0 with `er`, either directly or indirectly via any number of intermediate event resources. If any of these event resources is preassigned, then the singleton resource groups containing the preassigned resources are intersected along with the other groups. The same is true of `KheEventResourceHardAndSoftDomain`, except that both hard and soft prefer resources and avoid split assignments constraints are used, producing smaller domains in general.

These functions are not recommended for use when solving, since `KheTaskTreeMake` offers a more sophisticated way of initializing the domains of tasks. `KheEventResourceHardDomain` is used when deciding whether events are similar.

Function

```
void KheEventResourceDebug(KHE_EVENT_RESOURCE er, int verbosity,
  int indent, FILE *fp);
```

produces a debug print of `er` onto `fp` with the given verbosity and indent, in the usual way.

3.6.4. Event resource groups

An event resource group is created and added to an event by the call

```
KHE_EVENT_RESOURCE_GROUP KheEventResourceGroupMake(KHE_EVENT event,
  KHE_RESOURCE_GROUP rg);
```

Its attributes may be retrieved by calling

```
KHE_EVENT KheEventResourceGroupEvent(KHE_EVENT_RESOURCE_GROUP erg);
KHE_RESOURCE_GROUP KheEventResourceGroupResourceGroup(
  KHE_EVENT_RESOURCE_GROUP erg);
```

In addition to making a new event resource group object, `KheEventResourceGroupMake` calls `KheEventResourceMake` once for each resource of `rg`, with the resource for its `preassigned_resource` parameter and the obvious values for its other parameters. This satisfies the semantic requirement that adding a resource group should be just like adding its resources individually. These added event resources appear on the list of event resources of the event just like other event resources; they can be distinguished from them only by calling

```
KHE_EVENT_RESOURCE_GROUP KheEventResourceEventResourceGroup(
  KHE_EVENT_RESOURCE er);
```

which returns the event resource group that caused `er` to be created when there is one, and `NULL` when `er` was created directly. For example, when printing XML files, KHE calls this function once for each event resource, to decide whether it should be printed explicitly or omitted because it is part of an event resource group. Function

```
void KheEventResourceGroupDebug(KHE_EVENT_RESOURCE_GROUP erg,
  int verbosity, int indent, FILE *fp);
```

produces a debug print of erg onto fp with the given verbosity and indent, in the usual way.

3.7. Constraints

Some attributes of constraints are common to all kinds of constraints; others vary from one kind of constraint to another. Accordingly, KHE offers type KHE_CONSTRAINT, which is the abstract supertype of all kinds of constraints, and one subtype of this type for each kind of constraint.

To set and retrieve the back pointer of a constraint object, call

```
void KheConstraintSetBack(KHE_CONSTRAINT c, void *back);
void *KheConstraintBack(KHE_CONSTRAINT c);
```

as usual. To retrieve the other attributes common to all kinds of constraints, use functions

```
KHE_INSTANCE KheConstraintInstance(KHE_CONSTRAINT c);
char *KheConstraintId(KHE_CONSTRAINT c);
char *KheConstraintName(KHE_CONSTRAINT c);
bool KheConstraintRequired(KHE_CONSTRAINT c);
int KheConstraintWeight(KHE_CONSTRAINT c);
KHE_COST KheConstraintCombinedWeight(KHE_CONSTRAINT c);
KHE_COST_FUNCTION KheConstraintCostFunction(KHE_CONSTRAINT c);
int KheConstraintIndex(KHE_CONSTRAINT c);
KHE_CONSTRAINT_TAG KheConstraintTag(KHE_CONSTRAINT c);
```

KheConstraintInstance returns the instance; KheConstraintId and KheConstraintName return the constraint's Id and Name (as usual, these are optional in KHE, needed only when writing XML). KheConstraintRequired is true when the Required attribute is true.

KheConstraintWeight is the weight given to violations of the constraint. As explained in Section 6.1, KheConstraintCombinedWeight is similar, except that hard constraints are weighted more heavily; KHE_COST is also defined there. KheConstraintCostFunction is the cost function used when calculating the cost of deviations, of type

```
typedef enum {
  KHE_SUM_STEPS_COST_FUNCTION,
  KHE_STEP_SUM_COST_FUNCTION,
  KHE_SUM_COST_FUNCTION,
  KHE_SUM_SQUARES_COST_FUNCTION,
  KHE_SQUARE_SUM_COST_FUNCTION
} KHE_COST_FUNCTION;
```

KheConstraintIndex returns an automatically generated index number for c: 0 for the first constraint created, 1 for the second, and so on. KheConstraintTag is the type tag which determines which concrete kind of constraint this is, with type

```
typedef enum {
  KHE_ASSIGN_RESOURCE_CONSTRAINT_TAG,
  KHE_ASSIGN_TIME_CONSTRAINT_TAG,
  KHE_SPLIT_EVENTS_CONSTRAINT_TAG,
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT_TAG,
  KHE_PREFER_RESOURCES_CONSTRAINT_TAG,
  KHE_PREFER_TIMES_CONSTRAINT_TAG,
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT_TAG,
  KHE_SPREAD_EVENTS_CONSTRAINT_TAG,
  KHE_LINK_EVENTS_CONSTRAINT_TAG,
  KHE_ORDER_EVENTS_CONSTRAINT_TAG,
  KHE_AVOID_CLASHES_CONSTRAINT_TAG,
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT_TAG,
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT_TAG,
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT_TAG,
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT_TAG,
  KHE_LIMIT_WORKLOAD_CONSTRAINT_TAG,
  KHE_CONSTRAINT_TAG_COUNT
} KHE_CONSTRAINT_TAG;
```

The last value is not a valid tag; it counts the number of constraints, allowing code of the form

```
for( tag = 0;  tag < KHE_CONSTRAINT_TAG_COUNT;  tag++ )
  ...
```

to be written which visits every tag, now and in the future.

The number of points of application of a constraint is returned by

```
int KheConstraintAppliesToCount(KHE_CONSTRAINT c);
```

For an assign resource constraint this is the total number of event resources; for a split events constraint it is the total number of events plus the sizes of the event groups; and so on.

Given a tag, one can obtain a string representation of the constraint name by calling

```
char *KheConstraintTagShow(KHE_CONSTRAINT_TAG tag);
char *KheConstraintTagShowSpaced(KHE_CONSTRAINT_TAG tag);
```

The first returns an unspaced form (`"AssignResourceConstraint"` and so on), the second returns a spaced form (`"Assign Resource Constraint"` and so on). There is also

```
KHE_CONSTRAINT_TAG KheStringToConstraintTag(char *str);
```

which implements the inverse function, from unspaced constraint names to constraint tags, and

```
char *KheCostFunctionShow(KHE_COST_FUNCTION cf);
```

which returns a cost function's string representation, and

```
void KheConstraintDebug(KHE_CONSTRAINT c, int verbosity,
  int indent, FILE *fp);
```

which produces a debug print of `c` onto `fp` with the given verbosity and indent.

The names of the concrete subtypes themselves are

```
KHE_ASSIGN_RESOURCE_CONSTRAINT
KHE_ASSIGN_TIME_CONSTRAINT
KHE_SPLIT_EVENTS_CONSTRAINT
KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT
KHE_PREFER_RESOURCES_CONSTRAINT
KHE_PREFER_TIMES_CONSTRAINT
KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT
KHE_SPREAD_EVENTS_CONSTRAINT
KHE_LINK_EVENTS_CONSTRAINT
KHE_ORDER_EVENTS_CONSTRAINT
KHE_AVOID_CLASHES_CONSTRAINT
KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT
KHE_LIMIT_IDLE_TIMES_CONSTRAINT
KHE_CLUSTER_BUSY_TIMES_CONSTRAINT
KHE_LIMIT_BUSY_TIMES_CONSTRAINT
KHE_LIMIT_WORKLOAD_CONSTRAINT
```

Downcasting and upcasting between `KHE_CONSTRAINT` and each of these subtypes, using C casts, is a normal part of the use of KHE. Alternatively, since C casts can also be used for unsafe things, explicit functions are offered for upcasting:

```
KHE_CONSTRAINT KheFromAssignResourceConstraint(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c);
KHE_CONSTRAINT KheFromAssignTimeConstraint(
  KHE_ASSIGN_TIME_CONSTRAINT c);
KHE_CONSTRAINT KheFromSplitEventsConstraint(
  KHE_SPLIT_EVENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromDistributeSplitEventsConstraint(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromPreferResourcesConstraint(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_CONSTRAINT KheFromPreferTimesConstraint(
  KHE_PREFER_TIMES_CONSTRAINT c);
KHE_CONSTRAINT KheFromAvoidSplitAssignmentsConstraint(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromSpreadEventsConstraint(
  KHE_SPREAD_EVENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromLinkEventsConstraint(
  KHE_LINK_EVENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromOrderEventsConstraint(
  KHE_ORDER_EVENTS_CONSTRAINT c);
KHE_CONSTRAINT KheFromAvoidClashesConstraint(
  KHE_AVOID_CLASHES_CONSTRAINT c);
KHE_CONSTRAINT KheFromAvoidUnavailableTimesConstraint(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
KHE_CONSTRAINT KheFromLimitIdleTimesConstraint(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
KHE_CONSTRAINT KheFromClusterBusyTimesConstraint(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
KHE_CONSTRAINT KheFromLimitBusyTimesConstraint(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
KHE_CONSTRAINT KheFromLimitWorkloadConstraint(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c);
```

and for downcasting:

```
KHE_ASSIGN_RESOURCE_CONSTRAINT
  KheToAssignResourceConstraint(KHE_CONSTRAINT c);
KHE_ASSIGN_TIME_CONSTRAINT
  KheToAssignTimeConstraint(KHE_CONSTRAINT c);
KHE_SPLIT_EVENTS_CONSTRAINT
  KheToSplitEventsConstraint(KHE_CONSTRAINT c);
KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT
  KheToDistributeSplitEventsConstraint(KHE_CONSTRAINT c);
KHE_PREFER_RESOURCES_CONSTRAINT
  KheToPreferResourcesConstraint(KHE_CONSTRAINT c);
KHE_PREFER_TIMES_CONSTRAINT
  KheToPreferTimesConstraint(KHE_CONSTRAINT c);
KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT
  KheToAvoidSplitAssignmentsConstraint(KHE_CONSTRAINT c);
KHE_SPREAD_EVENTS_CONSTRAINT
  KheToSpreadEventsConstraint(KHE_CONSTRAINT c);
KHE_LINK_EVENTS_CONSTRAINT
  KheToLinkEventsConstraint(KHE_CONSTRAINT c);
KHE_ORDER_EVENTS_CONSTRAINT
  KheToOrderEventsConstraint(KHE_CONSTRAINT c);
KHE_AVOID_CLASHES_CONSTRAINT
  KheToAvoidClashesConstraint(KHE_CONSTRAINT c);
KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT
  KheToAvoidUnavailableTimesConstraint(KHE_CONSTRAINT c);
KHE_LIMIT_IDLE_TIMES_CONSTRAINT
  KheToLimitIdleTimesConstraint(KHE_CONSTRAINT c);
KHE_CLUSTER_BUSY_TIMES_CONSTRAINT
  KheToClusterBusyTimesConstraint(KHE_CONSTRAINT c);
KHE_LIMIT_BUSY_TIMES_CONSTRAINT
  KheToLimitBusyTimesConstraint(KHE_CONSTRAINT c);
KHE_LIMIT_WORKLOAD_CONSTRAINT
  KheToLimitWorkloadConstraint(KHE_CONSTRAINT c);
```

The downcasting functions check that their parameter is of the correct type, and abort if not.

3.7.1. Assign resource constraints

An assign resource constraint is created and added to an instance by

```
bool KheAssignResourceConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  char *role, KHE_ASSIGN_RESOURCE_CONSTRAINT *c);
```

This accepts the attributes common to all constraints, followed by an optional `role`, which is specific to this kind of constraint. As usual, if successful it returns `true`, setting `*c` to the new constraint; if not (which can only be because `id` is non-`NULL` and equal to the Id of an existing constraint of `ins`), then it returns `false`, setting `*c` to `NULL`.

The attributes common to all kinds of constraints may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operations on that type. The attribute specific to assign resources constraints may be retrieved by calling

```
char *KheAssignResourceConstraintRole(KHE_ASSIGN_RESOURCE_CONSTRAINT c);
```

Initially the constraint has no points of application. There are two ways to add them. The first is to give `NULL` for `role`, then add the event resources that this constraint applies to by calling

```
void KheAssignResourceConstraintAddEventResource(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, KHE_EVENT_RESOURCE er);
```

as often as necessary. It is an error to call this function when `er` contains a preassigned resource, since assign resource constraints do not apply to event resources with preassigned resources. To visit the event resources of `c`, call

```
int KheAssignResourceConstraintEventResourceCount(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c);
KHE_EVENT_RESOURCE KheAssignResourceConstraintEventResource(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, int i);
```

as usual.

The second way to add event resources, used when reading XML files, is to give a non-`NULL` value for `role`, then add events and event groups. To add events and visit them, the calls are

```
void KheAssignResourceConstraintAddEvent(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, KHE_EVENT e);
int KheAssignResourceConstraintEventCount(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c);
KHE_EVENT KheAssignResourceConstraintEvent(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, int i);
```

To add event groups and visit them, the calls are

```
void KheAssignResourceConstraintAddEventGroup(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, KHE_EVENT_GROUP eg);
int KheAssignResourceConstraintEventGroupCount(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c);
KHE_EVENT_GROUP KheAssignResourceConstraintEventGroup(
  KHE_ASSIGN_RESOURCE_CONSTRAINT c, int i);
```

When this is done, KHE stores the events and event groups in the constraint so that they can be written out again correctly later, but it also works out which event resources the constraint applies to and calls `KheAssignResourceConstraintAddEventResource` for each of them, taking due note of the XML rule that it does not apply when an event does not contain an event resource with the specified role, or when such an event resource has a preassigned resource.

The constraint density of the assign resources constraints of an instance (Section 3.3) is their number of their points of application divided by the number of event resources without preassigned resources.

3.7.2. Assign time constraints

An assign time constraint is created and added to an instance by

```
bool KheAssignTimeConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_ASSIGN_TIME_CONSTRAINT *c);
```

As usual, if successful it returns `true`, setting `*c` to the new constraint; if not (which can only be because `id` is non-`NULL` and equal to the Id of an existing constraint of `ins`), then it returns `false`, setting `*c` to `NULL`. The attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operations on that type.

The points of application of an assign time constraint are events, and the XML file allows them to be given individually and in groups. To add individual events and visit them, call

```
void KheAssignTimeConstraintAddEvent(KHE_ASSIGN_TIME_CONSTRAINT c,
  KHE_EVENT e);
int KheAssignTimeConstraintEventCount(KHE_ASSIGN_TIME_CONSTRAINT c);
KHE_EVENT KheAssignTimeConstraintEvent(KHE_ASSIGN_TIME_CONSTRAINT c,
  int i);
```

To add groups of events and visit them, call

```
void KheAssignTimeConstraintAddEventGroup(KHE_ASSIGN_TIME_CONSTRAINT c,
  KHE_EVENT_GROUP eg);
int KheAssignTimeConstraintEventGroupCount(
  KHE_ASSIGN_TIME_CONSTRAINT c);
KHE_EVENT_GROUP KheAssignTimeConstraintEventGroup(
  KHE_ASSIGN_TIME_CONSTRAINT c, int i);
```

The XML specification states that assign time constraints skip events with preassigned times, whether those events are mentioned or not. Accordingly, although such events are added to constraints by the calls just given, the reverse links, from the events to the constraint, are added only to events that do not have preassigned times.

The constraint density of the assign times constraints of an instance (Section 3.3) is their number of points of application divided by the number of events without preassigned times.

3.7.3. Split events constraints

A split events constraint is created and added to an instance by

```
bool KheSplitEventsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int min_duration, int max_duration, int min_amount,
  int max_amount, KHE_SPLIT_EVENTS_CONSTRAINT *c);
```

in the usual way. Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. The exceptions are

```
int KheSplitEventsConstraintMinDuration(KHE_SPLIT_EVENTS_CONSTRAINT c);
int KheSplitEventsConstraintMaxDuration(KHE_SPLIT_EVENTS_CONSTRAINT c);
int KheSplitEventsConstraintMinAmount(KHE_SPLIT_EVENTS_CONSTRAINT c);
int KheSplitEventsConstraintMaxAmount(KHE_SPLIT_EVENTS_CONSTRAINT c);
```

which return the various attributes specific to split events constraints.

The points of application are events, and, as for assign time constraints, these may be added and visited individually:

```
void KheSplitEventsConstraintAddEvent(KHE_SPLIT_EVENTS_CONSTRAINT c,
  KHE_EVENT e);
int KheSplitEventsConstraintEventCount(KHE_SPLIT_EVENTS_CONSTRAINT c);
KHE_EVENT KheSplitEventsConstraintEvent(KHE_SPLIT_EVENTS_CONSTRAINT c,
  int i);
```

and also in groups:

```
void KheSplitEventsConstraintAddEventGroup(
  KHE_SPLIT_EVENTS_CONSTRAINT c, KHE_EVENT_GROUP eg);
int KheSplitEventsConstraintEventGroupCount(
  KHE_SPLIT_EVENTS_CONSTRAINT c);
KHE_EVENT_GROUP KheSplitEventsConstraintEventGroup(
  KHE_SPLIT_EVENTS_CONSTRAINT c, int i);
```

All the events are linked to the constraint, unlike for assign time constraints.

The constraint density of the split events constraints of an instance (Section 3.3) is their number of points of application divided by the total number of events.

3.7.4. Distribute split events constraints

A distribute split events constraint is created and added to an instance by

```
bool KheDistributeSplitEventsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int duration, int minimum, int maximum,
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT *c);
```

in the usual way. Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. The exceptions are

```
int KheDistributeSplitEventsConstraintDuration(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
int KheDistributeSplitEventsConstraintMinimum(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
int KheDistributeSplitEventsConstraintMaximum(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
```

which return the various attributes specific to distribute split events constraints.

The points of application are events, and, as for split events constraints, these may be added and visited individually:

```
void KheDistributeSplitEventsConstraintAddEvent(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c, KHE_EVENT e);
int KheDistributeSplitEventsConstraintEventCount(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
KHE_EVENT KheDistributeSplitEventsConstraintEvent(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c, int i);
```

and also in groups:

```
void KheDistributeSplitEventsConstraintAddEventGroup(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c, KHE_EVENT_GROUP eg);
int KheDistributeSplitEventsConstraintEventGroupCount(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c);
KHE_EVENT_GROUP KheDistributeSplitEventsConstraintEventGroup(
  KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT c, int i);
```

All the events are linked to the constraint.

The constraint density of the distribute split events constraints of an instance (Section 3.3) is their number of points of application divided by the total number of events.

3.7.5. Prefer resources constraints

A prefer resources constraint is created and added to an instance by

```
bool KhePreferResourcesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  char *role, KHE_PREFER_RESOURCES_CONSTRAINT *c);
```

As usual, the only reason for returning `false` is that `id` is non-`NULL` and there is already a constraint in `ins` with this `id`. Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operations on that type; the exception is `role`, which is retrieved by calling

```
char *KhePreferResourcesConstraintRole(KHE_PREFER_RESOURCES_CONSTRAINT c);
```

since it is specific to this constraint type.

In the XML specification, the resources that make up the domain of the constraint may be added in groups or individually. To add them in groups, and to visit the groups, call

```
bool KhePreferResourcesConstraintAddResourceGroup(
  KHE_PREFER_RESOURCES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KhePreferResourcesConstraintResourceGroupCount(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_RESOURCE_GROUP KhePreferResourcesConstraintResourceGroup(
  KHE_PREFER_RESOURCES_CONSTRAINT c, int i);
```

The `bool` result type of `KhePreferResourcesConstraintAddResourceGroup` (and other functions below) is explained at the end of this section. To add and visit resources individually, call

```
bool KhePreferResourcesConstraintAddResource(
  KHE_PREFER_RESOURCES_CONSTRAINT c, KHE_RESOURCE r);
int KhePreferResourcesConstraintResourceCount(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_RESOURCE KhePreferResourcesConstraintResource(
  KHE_PREFER_RESOURCES_CONSTRAINT c, int i);
```

After the instance is complete, but not before, function

```
KHE_RESOURCE_GROUP KhePreferResourcesConstraintDomain(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
```

returns the domain of `c` as a single resource group. If exactly one resource group or one resource was added, this resource group will be that resource group or the automatically created singleton resource group for that resource; otherwise it will be created by taking the union of everything added. This resource group may be used like any other, except for a problem in one special case: when no resource groups or resources are added, the domain is not only an empty resource group but also has a `NULL` resource type.

The points of application of prefer resources constraints are event resources, and they are handled in the same way as for assign resource constraints. That is, one can load the event resources directly by having a `NULL` value for `role` and calling

```
bool KhePreferResourcesConstraintAddEventResource(
  KHE_PREFER_RESOURCES_CONSTRAINT c, KHE_EVENT_RESOURCE er);
int KhePreferResourcesConstraintEventResourceCount(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_EVENT_RESOURCE KhePreferResourcesConstraintEventResource(
  KHE_PREFER_RESOURCES_CONSTRAINT c, int i);
```

or load them indirectly by loading events:

```
bool KhePreferResourcesConstraintAddEvent(
  KHE_PREFER_RESOURCES_CONSTRAINT c, KHE_EVENT e);
int KhePreferResourcesConstraintEventCount(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_EVENT KhePreferResourcesConstraintEvent(
  KHE_PREFER_RESOURCES_CONSTRAINT c, int i);
```

and event groups:

```
bool KhePreferResourcesConstraintAddEventGroup(
  KHE_PREFER_RESOURCES_CONSTRAINT c, KHE_EVENT_GROUP eg,
  KHE_EVENT *problem_event);
int KhePreferResourcesConstraintEventGroupCount(
  KHE_PREFER_RESOURCES_CONSTRAINT c);
KHE_EVENT_GROUP KhePreferResourcesConstraintEventGroup(
  KHE_PREFER_RESOURCES_CONSTRAINT c, int i);
```

When `KhePreferResourcesConstraintAddEventGroup` returns `false`, `problem_event` is set to the first event that caused the problem. The rules for skipping inappropriate events are as for assign resource constraints.

The resources, resource groups, and event resources of a prefer resources constraint all have a resource type attribute. All these resources types must be equal. This is why the operations above for adding a resource, resource group, event resource, event, or event group all have a `bool` result type: they all return `false` and add nothing if the operation would add an entity with a different resource type from something added previously.

The constraint density of the prefer resources constraints of an instance (Section 3.3) is their number of points of application divided by the number of event resources without preassigned resources.

3.7.6. Prefer times constraints

A prefer times constraint is created and added to an instance by

```
bool KhePreferTimesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int duration, KHE_PREFER_TIMES_CONSTRAINT *c);
```

As usual, the only possible reason for returning `false` is that `id` is non-`NULL` and there is already a constraint in `ins` with this `id`. A duration is optional; to not give one (meaning that the constraint applies for all durations), use the special value `KHE_ANY_DURATION`, a synonym for 0.

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operations on that type; the exception is `duration`, which is retrieved by calling

```
int KhePreferTimesConstraintDuration(KHE_PREFER_TIMES_CONSTRAINT c);
```

since it is specific to this constraint type.

In the XML specification, the times that make up the domain of the constraint may be added in groups or individually. To add them in groups, and to visit the groups, call

```
void KhePreferTimesConstraintAddTimeGroup(
  KHE_PREFER_TIMES_CONSTRAINT c, KHE_TIME_GROUP tg);
int KhePreferTimesConstraintTimeGroupCount(
  KHE_PREFER_TIMES_CONSTRAINT c);
KHE_TIME_GROUP KhePreferTimesConstraintTimeGroup(
  KHE_PREFER_TIMES_CONSTRAINT c, int i);
```

To add and visit times individually, call

```
void KhePreferTimesConstraintAddTime(
  KHE_PREFER_TIMES_CONSTRAINT c, KHE_TIME t);
int KhePreferTimesConstraintTimeCount(
  KHE_PREFER_TIMES_CONSTRAINT c);
KHE_TIME KhePreferTimesConstraintTime(
  KHE_PREFER_TIMES_CONSTRAINT c, int i);
```

After the instance is complete, but not before, function

```
KHE_TIME_GROUP KhePreferTimesConstraintDomain(
  KHE_PREFER_TIMES_CONSTRAINT c);
```

returns the domain of c as a single time group. If exactly one time group or one time was added, this time group will be that time group or the automatically created singleton time group for that time; otherwise it will be created by taking the union of everything added. This time group may be used like any other.

The points of application of prefer times constraints are events, and they can be added and visited individually:

```
void KhePreferTimesConstraintAddEvent(
  KHE_PREFER_TIMES_CONSTRAINT c, KHE_EVENT e);
int KhePreferTimesConstraintEventCount(
  KHE_PREFER_TIMES_CONSTRAINT c);
KHE_EVENT KhePreferTimesConstraintEvent(
  KHE_PREFER_TIMES_CONSTRAINT c, int i);
```

or in groups:

```
void KhePreferTimesConstraintAddEventGroup(
  KHE_PREFER_TIMES_CONSTRAINT c, KHE_EVENT_GROUP eg);
int KhePreferTimesConstraintEventGroupCount(
  KHE_PREFER_TIMES_CONSTRAINT c);
KHE_EVENT_GROUP KhePreferTimesConstraintEventGroup(
  KHE_PREFER_TIMES_CONSTRAINT c, int i);
```

The XML specification states that prefer times constraints skip events with preassigned times, whether those events are mentioned or not. Accordingly, although such events are added to constraints by the calls just given, the reverse links, from the events to the constraint, are added only to events that do not have preassigned times.

The constraint density of the prefer times constraints of an instance (Section 3.3) is their number of points of application divided by the number of events without preassigned times.

3.7.7. Avoid split assignments constraints

An avoid split assignments constraint is created and added to an instance by

```
bool KheAvoidSplitAssignmentsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  char *role, KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT *c);
```

As usual, the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type, except that to retrieve the `role` attribute the call is

```
char *KheAvoidSplitAssignmentsConstraintRole(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c);
```

The `role` attribute may be `NULL`.

The handling of the points of application of an avoid split assignments constraint is somewhat complex, because one point of application is fundamentally a set of event resources (the XML file identifies each set by an event group and a role), so that the points of application overall form a set of sets of event resources. We will first explain how to add these points of application when reading an XML file, and then how to do it directly.

When reading an XML file, a non-`NULL` `role` is passed, and then each event group is added in the usual way. To add an event group and to visit the event groups, the calls are

```
bool KheAvoidSplitAssignmentsConstraintAddEventGroup(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c, KHE_EVENT_GROUP eg,
  KHE_EVENT *problem_event);
int KheAvoidSplitAssignmentsConstraintEventGroupCount(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c);
KHE_EVENT_GROUP KheAvoidSplitAssignmentsConstraintEventGroup(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c, int i);
```

Behind the scenes, the appropriate event resources are retrieved from the events of each event group and added automatically, so that nothing further needs to be done. A `false` result returned by `KheAvoidSplitAssignmentsConstraintAddEventGroup` indicates that one of the events of `eg` does not contain an event resource with the required non-`NULL` `role`. In this case, `*problem_event` will contain the first event of `eg` with this problem on return.

When the instance is not derived from an XML file it may be more convenient to add event resources directly. For the sake of this case, `role` may be `NULL`, and the `eg` parameter of `KheAvoidSplitAssignmentsConstraintAddEventGroup` may also be `NULL`. If either is `NULL`, event resources are not added automatically.

To add event resources manually, and to visit event resources (whether added automatically or manually), the calls are

```
void KheAvoidSplitAssignmentsConstraintAddEventResource(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c, int eg_index,
  KHE_EVENT_RESOURCE er);
int KheAvoidSplitAssignmentsConstraintEventResourceCount(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c, int eg_index);
KHE_EVENT_RESOURCE KheAvoidSplitAssignmentsConstraintEventResource(
  KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT c, int eg_index, int er_index);
```

These functions add an event resource to the `eg_index`'th point of application of `c`, return the

number of event resources at that point, and return the `er_index`'th event resource at that point. They define the required set of sets of event resources.

Usually, constraints are added to the instance and to the entities they apply to. For avoid split assignments constraints this would mean adding the constraint to the instance and the event groups. This is done, but, for convenience, each avoid split assignments constaint is also added to each of its event resources.

The constraint density of the avoid split assignments constraints of an instance (Section 3.3) is the number of event resources in all points of application divided by the number of event resources without preassigned resources.

3.7.8. Spread events constraints

A spread events constraint is created and added to an instance by

```
bool KheSpreadEventsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_TIME_SPREAD ts, KHE_SPREAD_EVENTS_CONSTRAINT *c);
```

where type `KHE_TIME_SPREAD` is explained below. Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. The exception is

```
KHE_TIME_SPREAD KheSpreadEventsConstraintTimeSpread(
  KHE_SPREAD_EVENTS_CONSTRAINT c);
```

which returns the time spread. Type `KHE_TIME_SPREAD` is an object which describes the time groups that the constraint requires the event group to spread through, and the limits on the number of events that may touch each time group. Time spread objects are immutable, and may be shared among any number of constraints. To create a time spread object, call

```
KHE_TIME_SPREAD KheTimeSpreadMake(KHE_INSTANCE ins);
```

Initially this has no time groups. To add them, call

```
void KheTimeSpreadAddLimitedTimeGroup(KHE_TIME_SPREAD ts,
  KHE_LIMITED_TIME_GROUP ltg);
```

repeatedly. To retrieve the limited time groups of a time spread, call

```
int KheTimeSpreadLimitedTimeGroupCount(KHE_TIME_SPREAD lts);
KHE_LIMITED_TIME_GROUP KheTimeSpreadLimitedTimeGroup(
  KHE_TIME_SPREAD lts, int i);
```

An object of type `KHE_LIMITED_TIME_GROUP` contains what one element of a time spread needs: a time group plus a minimum and maximum number of events. It may be created by calling

```
KHE_LIMITED_TIME_GROUP KheLimitedTimeGroupMake(KHE_TIME_GROUP tg,
  int minimum, int maximum);
```

and functions

```
KHE_TIME_GROUP KheLimitedTimeGroupTimeGroup(KHE_LIMITED_TIME_GROUP ltg);
int KheLimitedTimeGroupMinimum(KHE_LIMITED_TIME_GROUP ltg);
int KheLimitedTimeGroupMaximum(KHE_LIMITED_TIME_GROUP ltg);
```

retrieve its attributes.

Two other operations on time spreads, available only after the instance is complete, provide information that may be useful to solvers:

```
bool KheTimeSpreadTimeGroupsDisjoint(KHE_TIME_SPREAD ts);
bool KheTimeSpreadCoversWholeCycle(KHE_TIME_SPREAD ts);
```

`KheTimeSpreadTimeGroupsDisjoint` returns `true` when the time groups of `ts`'s limited time groups are pairwise disjoint. `KheTimeSpreadCoversWholeCycle` returns `true` when every time of the cycle appears in at least one of the time groups of `ts`'s limited time groups.

Spread events apply to event groups; the operations for adding and visiting them are

```
void KheSpreadEventsConstraintAddEventGroup(
  KHE_SPREAD_EVENTS_CONSTRAINT c, KHE_EVENT_GROUP eg);
int KheSpreadEventsConstraintEventGroupCount(
  KHE_SPREAD_EVENTS_CONSTRAINT c);
KHE_EVENT_GROUP KheSpreadEventsConstraintEventGroup(
  KHE_SPREAD_EVENTS_CONSTRAINT c, int i);
```

as usual.

The constraint density of the spread events constraints of an instance (Section 3.3) is the number of events in their points of application, divided by the number of events.

3.7.9. Link events constraints

A link events constraint is created and added to an instance by

```
bool KheLinkEventsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_LINK_EVENTS_CONSTRAINT *c);
```

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. One point of application of a link events constraint is an event group; one constraint may contain any number of these. The operations for adding them are

```
void KheLinkEventsConstraintAddEventGroup(KHE_LINK_EVENTS_CONSTRAINT c,
  KHE_EVENT_GROUP eg);
int KheLinkEventsConstraintEventGroupCount(KHE_LINK_EVENTS_CONSTRAINT c);
KHE_EVENT_GROUP KheLinkEventsConstraintEventGroup(
  KHE_LINK_EVENTS_CONSTRAINT c, int i);
```

as usual.

The constraint density of the link events constraints of an instance (Section 3.3) is the number of events in their points of application, divided by the number of events.

3.7.10. Order events constraints

An order events constraint is created and added to an instance by

```
bool KheOrderEventsConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_ORDER_EVENTS_CONSTRAINT *c);
```

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type.

One point of application of an order events constraint is a pair of instance events, together with integer minimum and maximum separations. To add one point of application, call

```
void KheOrderEventsConstraintAddEventPair(KHE_ORDER_EVENTS_CONSTRAINT c,
  KHE_EVENT first_event, KHE_EVENT second_event, int min_separation,
  int max_separation);
```

Both `min_separation` and `max_separation` must be non-negative. Infinity, the default value of `max_separation` in the XML format, is implemented by passing `INT_MAX`.

To retrieve the number of points of application and the attributes of each, call

```
int KheOrderEventsConstraintEventPairCount(
  KHE_ORDER_EVENTS_CONSTRAINT c);
KHE_EVENT KheOrderEventsConstraintFirstEvent(
  KHE_ORDER_EVENTS_CONSTRAINT c, int i);
KHE_EVENT KheOrderEventsConstraintSecondEvent(
  KHE_ORDER_EVENTS_CONSTRAINT c, int i);
int KheOrderEventsConstraintMinSeparation(
  KHE_ORDER_EVENTS_CONSTRAINT c, int i);
int KheOrderEventsConstraintMaxSeparation(
  KHE_ORDER_EVENTS_CONSTRAINT c, int i);
```

in the usual way. The value of `KheOrderEventsConstraintEventPairCount(c)` is the same as the value of `KheConstraintAppliesToCount((KHE_CONSTRAINT) c)`.

The constraint density of the order events constraints of an instance (Section 3.3) is their number of points of application divided by the number of events.

3.7.11. Avoid clashes constraints

An avoid clashes constraint is created and added to an instance by

```
bool KheAvoidClashesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_AVOID_CLASHES_CONSTRAINT *c);
```

as usual. The attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type.

Avoid clashes constraints apply to resource groups and resources. To add and visit resource

groups, the operations are

```
void KheAvoidClashesConstraintAddResourceGroup(
  KHE_AVOID_CLASHES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheAvoidClashesConstraintResourceGroupCount(
  KHE_AVOID_CLASHES_CONSTRAINT c);
KHE_RESOURCE_GROUP KheAvoidClashesConstraintResourceGroup(
  KHE_AVOID_CLASHES_CONSTRAINT c, int i);
```

while to add and visit resources the operations are

```
void KheAvoidClashesConstraintAddResource(
  KHE_AVOID_CLASHES_CONSTRAINT c, KHE_RESOURCE r);
int KheAvoidClashesConstraintResourceCount(
  KHE_AVOID_CLASHES_CONSTRAINT c);
KHE_RESOURCE KheAvoidClashesConstraintResource(
  KHE_AVOID_CLASHES_CONSTRAINT c, int i);
```

These all work in the usual way.

The constraint density of the avoid clashes constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

3.7.12. Avoid unavailable times constraints

An avoid unavailable times constraint is created and added to an instance by

```
bool KheAvoidUnavailableTimesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT *c);
```

in the usual way. To add the resource groups and resources defining the points of application, and to visit them, call

```
void KheAvoidUnavailableTimesConstraintAddResourceGroup(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheAvoidUnavailableTimesConstraintResourceGroupCount(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
KHE_RESOURCE_GROUP KheAvoidUnavailableTimesConstraintResourceGroup(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, int i);
```

for resource groups and

```
void KheAvoidUnavailableTimesConstraintAddResource(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, KHE_RESOURCE r);
int KheAvoidUnavailableTimesConstraintResourceCount(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
KHE_RESOURCE KheAvoidUnavailableTimesConstraintResource(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, int i);
```

for individual resources. The XML format allows the unavailable times themselves to be defined

by both time groups and times. To add time groups and visit them, call

```
void KheAvoidUnavailableTimesConstraintAddTimeGroup(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, KHE_TIME_GROUP tg);
int KheAvoidUnavailableTimesConstraintTimeGroupCount(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
KHE_TIME_GROUP KheAvoidUnavailableTimesConstraintTimeGroup(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, int i);
```

To add individual times and visit them, call

```
void KheAvoidUnavailableTimesConstraintAddTime(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, KHE_TIME t);
int KheAvoidUnavailableTimesConstraintTimeCount(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
KHE_TIME KheAvoidUnavailableTimesConstraintTime(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c, int i);
```

These functions all work in the usual way. Function

```
KHE_TIME_GROUP KheAvoidUnavailableTimesConstraintUnavailableTimes(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
```

returns a time group containing the union of the time groups and times of `c`, and

```
KHE_TIME_GROUP KheAvoidUnavailableTimesConstraintAvailableTimes(
  KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT c);
```

returns a time group containing the complement of those times—the available times. Both functions may be called only after construction of the instance is complete.

The constraint density of the avoid unavailable times constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

3.7.13. Limit idle times constraints

A limit idle times constraint is created and added to an instance by

```
bool KheLimitIdleTimesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int minimum, int maximum, KHE_LIMIT_IDLE_TIMES_CONSTRAINT *c);
```

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type; the exceptions are

```
int KheLimitIdleTimesConstraintMinimum(KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
int KheLimitIdleTimesConstraintMaximum(KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
```

which are specific to this kind of constraint.

A limit idle times constraint requires time groups, which are added and visited by calling

```
void KheLimitIdleTimesConstraintAddTimeGroup(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, KHE_TIME_GROUP tg);
int KheLimitIdleTimesConstraintTimeGroupCount(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
KHE_TIME_GROUP KheLimitIdleTimesConstraintTimeGroup(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, int i);
```

After the instance ends, the following queries are available:

```
bool KheLimitIdleTimesConstraintTimeGroupsDisjoint(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
bool KheLimitIdleTimesConstraintTimeGroupsCoverWholeCycle(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
```

They return `true` when the time groups of `c` are pairwise disjoint, and when their union covers the whole cycle.

A limit idle times constraint also requires the resource groups and resources which define its points of application. Resource groups are added and visited by calling

```
void KheLimitIdleTimesConstraintAddResourceGroup(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheLimitIdleTimesConstraintResourceGroupCount(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
KHE_RESOURCE_GROUP KheLimitIdleTimesConstraintResourceGroup(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, int i);
```

and individual resources are added and visited by calling

```
void KheLimitIdleTimesConstraintAddResource(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, KHE_RESOURCE r);
int KheLimitIdleTimesConstraintResourceCount(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c);
KHE_RESOURCE KheLimitIdleTimesConstraintResource(
  KHE_LIMIT_IDLE_TIMES_CONSTRAINT c, int i);
```

in the usual way.

The constraint density of the limit idle times constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

3.7.14. Cluster busy times constraints

A cluster busy times constraint is created and added to an instance by

```
bool KheClusterBusyTimesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int minimum, int maximum, KHE_CLUSTER_BUSY_TIMES_CONSTRAINT *c);
```

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type; the exceptions are

```
int KheClusterBusyTimesConstraintMinimum(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
int KheClusterBusyTimesConstraintMaximum(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
```

which are specific to this kind of constraint.

A cluster busy times constraint requires time groups, which are added and visited by

```
void KheClusterBusyTimesConstraintAddTimeGroup(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, KHE_TIME_GROUP tg);
int KheClusterBusyTimesConstraintTimeGroupCount(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
KHE_TIME_GROUP KheClusterBusyTimesConstraintTimeGroup(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, int i);
```

It also requires the resource groups and resources which define the points of application of the constraint. Resource groups are added and visited by calling

```
void KheClusterBusyTimesConstraintAddResourceGroup(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheClusterBusyTimesConstraintResourceGroupCount(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
KHE_RESOURCE_GROUP KheClusterBusyTimesConstraintResourceGroup(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, int i);
```

and individual resources are added and visited by calling

```
void KheClusterBusyTimesConstraintAddResource(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, KHE_RESOURCE r);
int KheClusterBusyTimesConstraintResourceCount(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c);
KHE_RESOURCE KheClusterBusyTimesConstraintResource(
  KHE_CLUSTER_BUSY_TIMES_CONSTRAINT c, int i);
```

in the usual way.

The constraint density of the cluster busy times constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

3.7.15. Limit busy times constraints

A limit busy times constraint is created and added to an instance by

```
bool KheLimitBusyTimesConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  KHE_RESOURCE applies_to, int minimum, int maximum,
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT *c);
```

Most of these attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. The exceptions are

```
int KheLimitBusyTimesConstraintMinimum(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
int KheLimitBusyTimesConstraintMaximum(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
```

which are specific to this kind of constraint.

A limit busy times constraint requires time groups, which are added and visited by

```
void KheLimitBusyTimesConstraintAddTimeGroup(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, KHE_TIME_GROUP tg);
int KheLimitBusyTimesConstraintTimeGroupCount(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
KHE_TIME_GROUP KheLimitBusyTimesConstraintTimeGroup(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, int i);
```

repeatedly. After the instance is complete, but not before, function

```
KHE_TIME_GROUP KheLimitBusyTimesConstraintDomain(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
```

returns the domain of `c` (that is, the set union of the times in its time groups) as a single time group. This time group may be used like any other.

A limit busy times constraint also requires the resource groups and resources which define the points of application of the constraint. Resource groups are added and visited by calling

```
void KheLimitBusyTimesConstraintAddResourceGroup(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheLimitBusyTimesConstraintResourceGroupCount(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
KHE_RESOURCE_GROUP KheLimitBusyTimesConstraintResourceGroup(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, int i);
```

and individual resources are added and visited by calling

```
void KheLimitBusyTimesConstraintAddResource(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, KHE_RESOURCE r);
int KheLimitBusyTimesConstraintResourceCount(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c);
KHE_RESOURCE KheLimitBusyTimesConstraintResource(
  KHE_LIMIT_BUSY_TIMES_CONSTRAINT c, int i);
```

in the usual way.

The constraint density of the limit busy times constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

3.7.16. Limit workload constraints

A limit workload constraint is created and added to an instance by

```
bool KheLimitWorkloadConstraintMake(KHE_INSTANCE ins, char *id,
  char *name, bool required, int weight, KHE_COST_FUNCTION cf,
  int minimum, int maximum, KHE_LIMIT_WORKLOAD_CONSTRAINT *c);
```

Most of the attributes may be retrieved by upcasting to `KHE_CONSTRAINT` and calling the relevant operation on that type. The exceptions are

```
int KheLimitWorkloadConstraintMinimum(KHE_LIMIT_WORKLOAD_CONSTRAINT c);
int KheLimitWorkloadConstraintMaximum(KHE_LIMIT_WORKLOAD_CONSTRAINT c);
```

which return the resource that `c` applies to, the minimum, and the maximum.

Limit workload constraints do not require time groups, because they always apply to the entire cycle. As usual, they require the resource groups and resources which define the points of application of the constraint. Resource groups are added and visited by calling

```
void KheLimitWorkloadConstraintAddResourceGroup(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c, KHE_RESOURCE_GROUP rg);
int KheLimitWorkloadConstraintResourceGroupCount(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c);
KHE_RESOURCE_GROUP KheLimitWorkloadConstraintResourceGroup(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c, int i);
```

and individual resources are added and visited by calling

```
void KheLimitWorkloadConstraintAddResource(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c, KHE_RESOURCE r);
int KheLimitWorkloadConstraintResourceCount(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c);
KHE_RESOURCE KheLimitWorkloadConstraintResource(
  KHE_LIMIT_WORKLOAD_CONSTRAINT c, int i);
```

in the usual way.

The constraint density of the limit workload constraints of an instance (Section 3.3) is the number of points of application divided by the number of resources.

Chapter 4. Solutions

4.1. Overview

A solution is represented by an object of type KHE_SOLN ('solution' is always abbreviated to 'soln' in the KHE interface). Any number of solutions may exist and be operated on simultaneously. Instances are immutable after creation, and operations that change instances only assemble them, they do not disassemble them. In contrast, each operation that changes a solution is paired with one that changes it back. This supports not just the assembly of a fixed solution, such as one read from a file, but also the changes and testing of alternatives needed when solving an instance.

Within each solution are KHE_MEET objects representing meets (also called split events or sub-events), each of which may be assigned a time, and KHE_TASK objects representing the resource elements of meets, each of which may be assigned a resource. Although most meets are derived from events and most tasks are derived from event resources, these derivations are optional. Only meets and tasks that are so derived are considered part of the solution to the original instance, but other meets and tasks may be present to help with solving. Several meets may be derived from one event; these are the split events or sub-events of that event in the solution.

At all times, the solution (however incomplete it may be) has a definite numerical *cost*, a 64-bit integer measuring the badness of the solution which is always available via function KheSolnCost (Chapter 6). It may be used to guide the search for good solutions.

A solution must obey a condition called the *solution invariant* throughout its lifetime; this is an unbreakable constraint. A precise statement of the solution invariant appears in Section 4.12. Every operation that changes a solution in a way that could violate the invariant is implemented with two functions, which look generically like this:

```
bool KheOperationCheck(...);
bool KheOperation(...);
```

The two functions accept the same inputs and return the same value in a given solution state. The first returns true if the change would not violate the invariant, but itself changes nothing. The second also returns true if the change would not violate the invariant, but in that case it also makes the change. It changes nothing if the change would violate the invariant.

The relationship between the solution invariant and the constraints of the original instance is rather subtle. Should a constraint be incorporated into the invariant, so that no solution (not even a partial solution) will ever violate it? KHE leaves this question to the user. Some operations do incorporate constraints into the solution invariant, but those operations are all optional.

Some aspects of solution entities that may be changed have operations of the form

```
void KheEntityAspectFix(ENTITY e);
void KheEntityAspectUnFix(ENTITY e);
bool KheEntityAspectIsFixed(ENTITY e);
```

The first fixes that aspect of the entity—prevents later operations from changing it; the second removes the fix; the third returns `true` when the fix is in place. Initially everything is unfixed. Fixing a fixed aspect, and unfixing an unfixed aspect, do nothing. When the current value of some aspect will remain unchanged for a long time, fixing that aspect may have a significant efficiency payoff. This is because fixing detaches attached monitors (Chapter 6) whose cost is 0 and cannot change while the current fixes are in place, which can save a lot of time. Unfixing attaches those unattached monitors which could have non-zero cost given the unfix.

There are three levels of operations. At the lowest level are *basic operations*, which carry out basic queries and changes to a solution, such as assigning or unassigning the time of a meet. Above them are *helper functions*, which implement commonly needed sequences of basic operations, such as swaps. Some helper functions utilize optimizations that make them significantly more efficient than the equivalent sequences of basic operations.

At the highest level are *solvers*, which make large-scale changes to solutions. A complete algorithm for solving an instance is a solver, but so are operations with more modest scope, such as assigning times to the meetings of one form, assigning rooms, and so on.

KHE supplies several solvers, documented in later chapters, and the user is free to write others. As a matter of good design, solvers should not have behind-the-scenes access to KHE's data structures; they should use only the operations described in this guide and made available by header file `khe.h`. The solvers supplied by KHE follow this rule.

4.2. Solution objects

To create a solution for a given instance, initially with no meets or tasks, call

```
KHE_SOLN KheSolnMake(KHE_INSTANCE ins);
```

`KheInstanceMakeEnd` must be complete before `KheSolnMake` is called. To delete `soln` and everything in it, and remove it from its solution groups, if any, call

```
void KheSolnDelete(KHE_SOLN soln);
```

The memory consumed by `soln` and everything in it will be freed.

A solution may lie in any number of solution groups. To add it to a solution group and delete it from a solution group, use functions `KheSolnGroupAddSoln` and `KheSolnGroupDeleteSoln` from Section 2.2. To visit the solution groups containing `soln`, call

```
int KheSolnSolnGroupCount(KHE_SOLN soln);
KHE_SOLN_GROUP KheSolnSolnGroup(KHE_SOLN soln, int i);
```

in the usual way.

A solution has an optional Description attribute which may contain arbitrary text saying what is distinctive about the solution. This attribute may be set and retrieved by calling

```
void KheSolnSetDescription(KHE_SOLN soln, char *description);
char *KheSolnDescription(KHE_SOLN soln);
```

The default value is `NULL`, meaning no description.

A solution also has an optional RunningTime attribute giving the wall clock time to produce the solution, in seconds. This attribute may be set and retrieved by calling

```
void KheSolnSetRunningTime(KHE_SOLN soln, float running_time);
float KheSolnRunningTime(KHE_SOLN soln);
```

The default value is `-1.0`, meaning that no running time is known. KHE makes no attempt to ensure that the value stored in this field is honest.

Solution objects and their components have back pointers in the usual way. These may be changed at any time. To set and retrieve the back pointer of a solution object, call

```
void KheSolnSetBack(KHE_SOLN soln, void *back);
void *KheSolnBack(KHE_SOLN soln);
```

as usual. Function

```
KHE_INSTANCE KheSolnInstance(KHE_SOLN soln);
```

returns the instance that the solution is for.

Another way to create a solution is

```
KHE_SOLN KheSolnCopy(KHE_SOLN soln);
```

which returns a copy of `soln`. The copy is exact except that it does not lie in any solution groups. Immutable elements, such as anything from the instance, and time, resource, and event groups created within the solution, are shared, as are back pointers.

Copying is useful when forking a solution process part-way through: the original solution may continue down one thread, and the copy, which is quite independent, may be given to the other thread. Care is needed in one respect, however: it is not safe to make two copies of one solution simultaneously, even though the original solution is unaffected by copying it. This is because the copy algorithm uses temporary forwarding pointers in the objects of the solution.

Even semantically unimportant things, such as the order of items in sets, are preserved by `KheSolnCopy`. If the same solution algorithm is run on the original and the copy, and it does not depend on anything peculiar such as elapsed time or the memory addresses of its objects, it should produce the same solution. The author has verified this for `KheGeneralSolve2014` (Section 8.1). Diversity can be obtained by changing the copy's diversifier (Section 4.5).

The specification of `qsort` states that when two elements compare equal, their order in the final result is undefined. So the author has tried to eliminate all such cases in the comparison functions packaged with KHE. Index numbers, returned by functions such as `KheMeetSolnIndex` and `KheTaskSolnIndex`, are useful for breaking ties consistently as a last resort.

To visit the meets of a solution, in an unspecified order, call

```
int KheSolnMeetCount(KHE_SOLN soln);
KHE_MEET KheSolnMeet(KHE_SOLN soln, int i);
```

The meets visited include the *cycle meets* described in Section 4.8.3. To visit the meets of a solution derived from a given event, call

```
int KheEventMeetCount(KHE_SOLN soln, KHE_EVENT e);
KHE_MEET KheEventMeet(KHE_SOLN soln, KHE_EVENT e, int i);
```

The first returns the number of meets derived from `e` (possibly 0), and the second returns the `i`'th of these meets, in an unspecified order.

To visit the tasks of a solution, in an unspecified order, call

```
int KheSolnTaskCount(KHE_SOLN soln);
KHE_TASK KheSolnTask(KHE_SOLN soln, int i);
```

To visit the tasks derived from a given event resource, call

```
int KheEventResourceTaskCount(KHE_SOLN soln, KHE_EVENT_RESOURCE er);
KHE_TASK KheEventResourceTask(KHE_SOLN soln, KHE_EVENT_RESOURCE er,
  int i);
```

There is one for each meet derived from the event containing `er`.

A solution may also contain *nodes* and *taskings*, as explained in Chapter 5. To visit the nodes in an unspecified order, call

```
int KheSolnNodeCount(KHE_SOLN soln);
KHE_NODE KheSolnNode(KHE_SOLN soln, int i);
```

To visit the taskings, call

```
int KheSolnTaskingCount(KHE_SOLN soln);
KHE_TASKING KheSolnTasking(KHE_SOLN soln, int i);
```

in the usual way.

As an aid to debugging, function

```
void KheSolnDebug(KHE_SOLN soln, int verbosity, int indent, FILE *fp);
```

prints information about the current solution onto file `fp` with the given verbosity and indent, as described for debug functions in general in Section 1.3. Verbosity 1 prints just the instance name and current cost, verbosity 2 adds a breakdown of the current cost by constraint type (only constraint types with non-zero cost are printed), verbosity 3 adds debug prints of the solution's defects (Section 6.2), and verbosity 4 prints further details.

4.3. Complete representation and preassignment conversion

A solution is a *complete representation* when it satisfies the following two conditions:

- For each event `e` of the solution's instance, the total duration of the meets derived from `e` is equal to the duration of `e`;
- For each event resource `er` of the solution's instance, each meet derived from the event containing `er` contains a task derived from `er`.

Complete representation does not rule out extra meets or tasks. It has nothing to do with being a complete solution, in the sense of assigning a time to every meet and a resource to every task.

KHE does not require a solution to be a complete representation, since that would be too restrictive when building and modifying solutions. However, the cost it reports for a solution is correct only when that solution is a complete representation. This is because, behind the scenes, KHE needs to be able to see a meet with no assigned time in order for it to realize that an assign time constraint is being violated, and similarly for the other constraints.

There is a standard procedure, part of the XML specification, for converting a solution into a complete representation:

1. For each event `e` of the solution's instance, if there are no meets derived from `e`, then insert one meet whose duration is the duration of `e`, and whose assigned time is the preassigned time of `e`, or is absent if `e` has no preassigned time. Initially, this meet contains no tasks, but that may be changed by the third rule.

2. If now there is an event `e` such that the total duration of the meets derived from `e` is not equal to the duration of `e`, then that is an error and the XML file is rejected.

3. For each event resource `er` of each event `e` of the instance, for each meet derived from `e`, if that meet does not contain a task derived from `er`, then add one. Its assigned resource is the preassigned resource of `er` if there is one, or is absent if `er` has no preassigned resource.

This procedure, minus the conversions from preassignments to assignments, is implemented by

```
bool KheSolnMakeCompleteRepresentation(KHE_SOLN soln,
  KHE_EVENT *problem_event);
```

For each event `e`, it finds the total duration of the meets derived from `e`. If that is greater than the duration of `e` it returns `false` with `*problem_event` set to `e`. If it is less, then one meet derived from `e` is added whose duration makes up the difference. The domain of this meet has the usual default value: the preassigned time of `e` if any, or else the largest legal domain, `KheSolnPackingTimeGroup(soln)` (Section 4.8.3). Then, within each meet derived from an event, just created or not, it adds a task for each event resource `er` not already represented. The domain of this task has the usual default value: the preassigned resource of `er` if any, or else the largest legal domain, `KheResourceTypeFullResourceGroup(rt)`, where `rt` is `er`'s resource type.

`KheSolnMakeCompleteRepresentation` has two uses. The first is in `KheArchiveRead` (Section 2.3), which applies it to each solution it reads, as the XML specification requires, and then calls these two public functions to convert preassignments into assignments:

```
void KheSolnAssignPreassignedTimes(KHE_SOLN soln);
void KheSolnAssignPreassignedResources(KHE_SOLN soln,
  KHE_RESOURCE_TYPE rt);
```

`KheSolnAssignPreassignedTimes` assigns the obvious time to each preassigned unassigned meet. `KheSolnAssignPreassignedResources` assigns the obvious resource to each preassigned unassigned task of type `rt` (any type if `rt` is `NULL`).

The second use for `KheSolnMakeCompleteRepresentation` is to build a solution from

scratch, ready for solving. The solution returned by `KheSolnMake` has no meets except for the initial cycle meet, and it has no tasks. `KheSolnMakeCompleteRepresentation` is a very convenient way to add both. When solving, it is usually called immediately after `KheSolnMake` and `KheSolnSplitCycleMeet` (Section 4.8.3). The solution changes as solving proceeds, but it remains a complete representation throughout, except perhaps during brief reconstructions. A call to `KheSolnAssignPreassignedResources` is also a good idea, since it does no harm and ensures that resource constraints involving preassigned resources will contribute to the cost of the solution as soon as the meets they are preassigned to are assigned times. On the other hand, it may be better not to assign preassigned times at this point; Section 10.4 has the alternatives.

4.4. Solution time, resource, and event groups

Groups are important in solving. A solver needs to be able to construct its own, since the ones declared in the instance might not be enough. (Conceivably, a solver could need its own times and resources as well, but that possibility is not currently supported.) Accordingly, the following functions are provided for constructing a time group while solving:

```
void KheSolnTimeGroupBegin(KHE_SOLN soln);
void KheSolnTimeGroupAddTime(KHE_SOLN soln, KHE_TIME t);
void KheSolnTimeGroupSubTime(KHE_SOLN soln, KHE_TIME t);
void KheSolnTimeGroupUnion(KHE_SOLN soln, KHE_TIME_GROUP tg2);
void KheSolnTimeGroupIntersect(KHE_SOLN soln, KHE_TIME_GROUP tg2);
void KheSolnTimeGroupDifference(KHE_SOLN soln, KHE_TIME_GROUP tg2);
KHE_TIME_GROUP KheSolnTimeGroupEnd(KHE_SOLN soln);
```

The first operation begins the process; the next five do what the corresponding operations for instance time groups do, and the last operation returns the finished time group. Its kind will be `KHE_TIME_GROUP_KIND_ORDINARY`, and its `id` and `name` attributes will be `NULL`.

A similar set of operations constructs a resource group:

```
void KheSolnResourceGroupBegin(KHE_SOLN soln, KHE_RESOURCE_TYPE rt);
void KheSolnResourceGroupAddResource(KHE_SOLN soln, KHE_RESOURCE r);
void KheSolnResourceGroupSubResource(KHE_SOLN soln, KHE_RESOURCE r);
void KheSolnResourceGroupUnion(KHE_SOLN soln, KHE_RESOURCE_GROUP rg2);
void KheSolnResourceGroupIntersect(KHE_SOLN soln, KHE_RESOURCE_GROUP rg2);
void KheSolnResourceGroupDifference(KHE_SOLN soln, KHE_RESOURCE_GROUP rg2);
KHE_RESOURCE_GROUP KheSolnResourceGroupEnd(KHE_SOLN soln);
```

and an event group:

```
void KheSolnEventGroupBegin(KHE_SOLN soln);
void KheSolnEventGroupAddEvent(KHE_SOLN soln, KHE_EVENT e);
void KheSolnEventGroupSubEvent(KHE_SOLN soln, KHE_EVENT e);
void KheSolnEventGroupUnion(KHE_SOLN soln, KHE_EVENT_GROUP eg2);
void KheSolnEventGroupIntersect(KHE_SOLN soln, KHE_EVENT_GROUP eg2);
void KheSolnEventGroupDifference(KHE_SOLN soln, KHE_EVENT_GROUP eg2);
KHE_EVENT_GROUP KheSolnEventGroupEnd(KHE_SOLN soln);
```

All the usual operations may be applied to these groups. The functions use `soln` as a factory object instead of the group itself, to ensure that groups are complete and immutable (apart from their back pointers) by the time they are given to the user. Groups are deleted when their solution is deleted. They know which instance they are for, but the instance, being immutable after creation, is not aware of their existence.

Within one solution, when calls to `KheSolnTimeGroupEnd` return groups containing the same elements, the objects returned are the same too. This is done using a hash table of time groups. It allows the user to experiment with many time groups, without worrying about their memory cost. This is not being done for resource and event groups yet; it should be.

4.5. Diversification

One strategy for finding good solutions is to find many solutions and choose the best. This only works when the solutions are diverse, creating a need to find ways to produce diversity.

Each solution contains a non-negative integer *diversifier*. Its initial value is 0, but it may be set and retrieved at any time by

```
void KheSolnSetDiversifier(KHE_SOLN soln, int val);
int KheSolnDiversifier(KHE_SOLN soln);
```

When solutions are created that need to be diverse, each is given a different diversifier. When an algorithm reaches a point where it could equally well follow any one of several paths, it consults the diversifier when making its choice.

Suppose the diversifier has value d and a point is reached where there are c alternatives, for some $c \geq 1$. A simple approach is to choose the `i`th alternative (counting from 0), where

```
i = d % c;
```

We call a function $D(d, c)$ which returns an integer i s.t. $0 \leq i < c$ a *diversification function.*

How should we choose diversifiers and diversification functions to ensure that we really do get diversity? One possibility is to start with a random integer and change it using a random number generator, passing the current value as seed, each time the diversifier is consulted. But there is no way to analyse the effect of this, so instead we are going to examine what happens when the diversifiers are fixed successive integers starting from 0.

What we want is a little hard to grasp. Suppose that, at some points in the algorithm, it is offered a choice between 1 alternative; at others, there are 2 alternatives, and so on, with a maximum of n alternatives. For a given diversifier, there are $n!$ different functions of the number of choices. Ideally we would want all of these functions to turn up as d varies over its range.

It is not obvious, but it is a fact that the modulus function above does turn up every function when n is 1, 2 or 3, but when n is 4 it produces 12 distinct functions, only half the possible 24 functions, as the following tables, obtained by running `khe -d4`, show:

```
   d |  1  2
 ----+------
   0 |  0  0
   1 |  0  1
 ----+------
```

```
   d |  1  2  3
 ----+---------
   0 |  0  0  0
   1 |  0  1  1
   2 |  0  0  2
   3 |  0  1  0
   4 |  0  0  1
   5 |  0  1  2
 ----+---------
```

```
   d |  1  2  3  4
 ----+------------
   0 |  0  0  0  0
   1 |  0  1  1  1
   2 |  0  0  2  2
   3 |  0  1  0  3
   4 |  0  0  1  0
   5 |  0  1  2  1
   6 |  0  0  0  2
   7 |  0  1  1  3
   8 |  0  0  2  0
   9 |  0  1  0  1
  10 |  0  0  1  2
  11 |  0  1  2  3
  12 |  0  0  0  0  (same as 0)
  13 |  0  1  1  1  (same as 1)
  14 |  0  0  2  2  (same as 2)
  15 |  0  1  0  3  (same as 3)
  16 |  0  0  1  0  (same as 4)
  17 |  0  1  2  1  (same as 5)
  18 |  0  0  0  2  (same as 6)
  19 |  0  1  1  3  (same as 7)
  20 |  0  0  2  0  (same as 8)
  21 |  0  1  0  1  (same as 9)
  22 |  0  0  1  2  (same as 10)
  23 |  0  1  2  3  (same as 11)
 ----+------------
```

Each row is one value of d, and each column is one value of c. What this means is that if, during the course of one run, no more than 4 choices are offered at any one point, then only 12 distinct solutions can emerge, no matter how many are begun.

The most natural diversification function which produces distinct outcomes is probably

```
(d / fact(c - 1)) % c
```

where `fact` is the factorial function. (To avoid overflow, in practice one stops multiplying as soon as the value exceeds `d`.) Each line is something like the binary representation of `d`, only in a factorial number system rather than binary:

```
  d |  1  2
----+------
  0 |  0  0
  1 |  0  1
----+------
```

```
  d |  1  2  3
----+---------
  0 |  0  0  0
  1 |  0  1  0
  2 |  0  0  1
  3 |  0  1  1
  4 |  0  0  2
  5 |  0  1  2
----+---------
```

```
  d |  1  2  3  4
----+------------
  0 |  0  0  0  0
  1 |  0  1  0  0
  2 |  0  0  1  0
  3 |  0  1  1  0
  4 |  0  0  2  0
  5 |  0  1  2  0
  6 |  0  0  0  1
  7 |  0  1  0  1
  8 |  0  0  1  1
  9 |  0  1  1  1
 10 |  0  0  2  1
 11 |  0  1  2  1
 12 |  0  0  0  2
 13 |  0  1  0  2
 14 |  0  0  1  2
 15 |  0  1  1  2
 16 |  0  0  2  2
 17 |  0  1  2  2
 18 |  0  0  0  3
 19 |  0  1  0  3
 20 |  0  0  1  3
 21 |  0  1  1  3
 22 |  0  0  2  3
 23 |  0  1  2  3
----+------------
```

But there is still a problem: if all alternatives have 4 choices, say, then the first 6 threads will produce the same result despite differing in `d`. The solution to this seems to be function

```
(d / fact(c - 1) + d % fact(c - 1)) % c
```

Delightfully, it produces

```
  d |  1  2
----+------
  0 |  0  0
  1 |  0  1
----+------
```

```
  d |  1  2  3
----+---------
  0 |  0  0  0
  1 |  0  1  1
  2 |  0  0  1
  3 |  0  1  2
  4 |  0  0  2
  5 |  0  1  0
----+---------
```

```
  d |  1  2  3  4
----+------------
  0 |  0  0  0  0
  1 |  0  1  1  1
  2 |  0  0  1  2
  3 |  0  1  2  3
  4 |  0  0  2  0
  5 |  0  1  0  1
  6 |  0  0  0  1
  7 |  0  1  1  2
  8 |  0  0  1  3
  9 |  0  1  2  0
 10 |  0  0  2  1
 11 |  0  1  0  2
 12 |  0  0  0  2
 13 |  0  1  1  3
 14 |  0  0  1  0
 15 |  0  1  2  1
 16 |  0  0  2  2
 17 |  0  1  0  3
 18 |  0  0  0  3
 19 |  0  1  1  0
 20 |  0  0  1  1
 21 |  0  1  2  2
 22 |  0  0  2  3
 23 |  0  1  0  0
----+------------
```

and is diverse up to $c = 8$ at least. Function

```
int KheSolnDiversifierChoose(KHE_SOLN soln, int c);
```

implements this function, returning a non-negative integer less than `c`.

It is quite reasonable for *every* algorithm faced with an arbitrary choice to diversify. It is easy to do, and it provides a continual prodding towards diversity that should drive solutions with different diversifiers further and further apart as solving continues, always provided that there are sufficiently many choices.

4.6. Visit numbers

Some algorithms, such as tabu search and ejection chains, need to know whether some part of the solution has changed recently. KHE supports this with a system of *visit numbers*.

A visit number is just an integer stored at some point in the solution. The KHE platform initializes visit numbers (to 0) and copies them, but does not otherwise use them. The user is free to set their values in any way at any time, using operations that look generically like this:

```
void KheSolnEntitySetVisitNum(KHE_SOLN_ENTITY e, int num);
int KheSolnEntityVisitNum(KHE_SOLN_ENTITY e);
```

But there is also a conventional way to use visit numbers, as follows.

The solution object contains a *global visit number* which is used differently from the others. The following operations are applicable to it:

```
void KheSolnSetGlobalVisitNum(KHE_SOLN soln, int num);
int KheSolnGlobalVisitNum(KHE_SOLN soln);
void KheSolnNewGlobalVisit(KHE_SOLN soln);
```

The first two operations are not usually used directly. The third increases the global visit number by one. This new value has not previously been assigned to any visit number.

The visit numbers of other solution entities should never exceed the global visit number. The operations for other solution entities look generically like this:

```
void KheSolnEntitySetVisitNum(KHE_SOLN_ENTITY e, int num);
int KheSolnEntityVisitNum(KHE_SOLN_ENTITY e);
bool KheSolnEntityVisited(KHE_SOLN_ENTITY e, int slack);
void KheSolnEntityVisit(KHE_SOLN_ENTITY e);
void KheSolnEntityUnVisit(KHE_SOLN_ENTITY e);
```

Type `SOLN_ENTITY` is fictitious and so are these functions; they just display the standard pattern. The first two are the standard ones. The third returns the value of the condition

```
KheSolnVisitNum(soln) - KheSolnEntityVisitNum(e) <= slack
```

where `soln` is the solution containing `e`. The fourth sets `e`'s visit number to its solution object's visit number, and the last sets it to one less than its solution's visit number.

These operations may be used to implement tabu search efficiently as follows. Suppose for

example that a change to the assignment of `meet` is to remain tabu until at least `tabu_len` other changes have been made. The code for this is

```
if( !KheMeetVisited(meet, tabu_len) )
{
  KheSolnNewVisit(KheMeetSoln(meet));
  KheMeetVisit(meet);
  ... change the assignment of meet ...
}
```

To ensure that everything is visitable initially, call

```
KheSolnSetVisitNum(soln, tabu_len);
```

It is easy to generalize this code to other operations.

One form of the ejection chains algorithm requires that once a meet (or other entity) has been changed during the current visit, it must remain tabu until a new visit is started in the outer loop of the algorithm. The code for this is

```
if( !KheMeetVisited(meet, 0) )
{
  KheMeetVisit(meet);
  ... change the assignment of meet ...
}
```

A variant of this idea makes `meet` tabu to recursive calls, but not tabu for the entire remainder of the current visit. The code for this is

```
if( !KheMeetVisited(meet, 0) )
{
  KheMeetVisit(meet);
  ... change the assignment of meet and recurse ...
  KheMeetUnVisit(meet);
}
```

Only meets in the direct line of the recursion are tabu.

4.7. Running times and time limits

Each solution contains a timer object of the kind defined in Section 8.5.1. It is initialized when the solution is created, and copied when it is copied. A call to

```
float KheSolnTimeNow(KHE_SOLN soln);
```

returns the number of seconds of wall clock time since the original creation, to a precision much better than one second. As explained in Section 8.5.1, if the binary was compiled with the `KHE_USE_TIMING` preprocessor flag set to `0`, `KheSolnTimeNow(soln)` will always return `-1.0`.

Each solution also contains a `float` value intended to hold the wall clock time in seconds taken to complete the solution. It is initialized to `-1.0`, meaning undefined, and is set and

retrieved by the `KheSolnSetRunningTime` and `KheSolnRunningTime` operations described in Section 4.2. The honest way to set the running time is to make the call

```
KheSolnSetRunningTime(soln, KheSolnTimeNow(soln));
```

at the end of the solve. Since wall clock time is measured, the stored value will be misleading if the solve was part of a thread that had to wait for processor time.

Also stored is an optional soft time limit, which may be set and retrieved like this:

```
void KheSolnSetTimeLimit(KHE_SOLN soln, float limit_in_secs);
float KheSolnTimeLimit(KHE_SOLN soln);
```

The default value of this limit is `-1.0`, a special value whose meaning is ‘no limit’. Setting a time limit does not prevent a solve from exceeding it. Instead, the user who wishes to enforce it must periodically call `KheSolnTimeNow` and compare its result with the time limit. We therefore describe it as a *soft time limit*. A convenient way to make this comparison is to call

```
bool KheSolnTimeLimitReached(KHE_SOLN soln);
```

which returns `true` when `KheSolnTimeLimit(soln)` is not `-1.0`, `KheSolnTimeNow(soln)` is not `-1.0`, and `KheSolnTimeNow(soln) >= KheSolnTimeLimit(soln)`.

4.8. Meets

A meet is created by calling

```
KHE_MEET KheMeetMake(KHE_SOLN soln, int duration, KHE_EVENT e);
```

This creates and adds to `soln` a new meet of the given duration, which must be at least 1. If `e` is non-`NULL`, it indicates that this meet is derived from event `e`. Initially the meet contains no tasks; they must be added separately. A meet may be deleted from its solution by calling

```
void KheMeetDelete(KHE_MEET meet);
```

Any tasks within `meet` are also deleted. If `meet` is assigned to another meet, or any other meets are assigned to it, all those assignments are removed. The meet is also deleted from any node (Section 5.2) it may lie in.

The back pointer of a meet may be set and retrieved by

```
void KheMeetSetBack(KHE_MEET meet, void *back);
void *KheMeetBack(KHE_MEET meet);
```

and the visit number by

```
void KheMeetSetVisitNum(KHE_MEET meet, int num);
int KheMeetVisitNum(KHE_MEET meet);
bool KheMeetVisited(KHE_MEET meet, int slack);
void KheMeetVisit(KHE_MEET meet);
void KheMeetUnVisit(KHE_MEET meet);
```

as usual. The other attributes of a meet are accessed by

```
KHE_SOLN KheMeetSoln(KHE_MEET meet);
int KheMeetSolnIndex(KHE_MEET meet);
int KheMeetDuration(KHE_MEET meet);
KHE_EVENT KheMeetEvent(KHE_MEET meet);
```

These return the enclosing solution, `meet`'s index in that solution (that is, the value of `i` for which `KheSolnMeet(soln, i)` returns `meet`), its duration, and the event that `meet` is derived from (possibly `NULL`). Index numbers change when meets are deleted (the hole left by the deletion of a meet, if not last, is plugged by the last meet), so care is needed. There is also

```
bool KheMeetIsPreassigned(KHE_MEET meet, TIME *time);
```

which returns `true` when `KheMeetEvent(meet) != NULL` and that event has a preassigned time; `meet` is called a *preassigned meet* in that case. If `time != NULL`, then `*time` is set to the event's preassigned time if `meet` is preassigned, and to `NULL` otherwise.

When deciding what order to assign meets in, it is handy to have some measure of how difficult they are to timetable. Functions

```
int KheMeetAssignedDuration(KHE_MEET meet);
int KheMeetDemand(KHE_MEET meet);
```

attempt to provide this. `KheMeetAssignedDuration` is the duration of `meet` if it is assigned, or 0 otherwise. `KheMeetDemand(meet)` is the sum, over `meet` and all meets assigned to `meet`, directly or indirectly, of the product of the duration of the meet and the number of tasks it contains. This value is stored in the meet and kept up to date as solutions change, so a call on `KheMeetDemand` costs almost nothing.

A task is added to its meet when it is created, and removed from its meet when it is deleted. To visit the tasks of a meet, call

```
int KheMeetTaskCount(KHE_MEET meet);
KHE_TASK KheMeetTask(KHE_MEET meet, int i);
bool KheMeetRetrieveTask(KHE_MEET meet, char *role, KHE_TASK *task);
bool KheMeetFindTask(KHE_MEET meet, KHE_EVENT_RESOURCE er,
  KHE_TASK *task);
```

The first two traverse the tasks. The order of tasks within meets is not significant, and it may change as tasks are created and deleted. `KheMeetRetrieveTask` retrieves a task which is derived from an event resource with the given `role`, if present. `KheMeetFindTask` is similar, but it looks for a task derived from event resource `er`, rather than for a role. There are also

```
bool KheMeetContainsResourcePreassignment(KHE_MEET meet,
  KHE_RESOURCE r, KHE_TASK *task);
bool KheMeetContainsResourceAssignment(KHE_MEET meet,
  KHE_RESOURCE r, KHE_TASK *task);
```

which return `true` if `meet` contains a task preassigned or assigned `r`, setting `*task` to one if so. Here a task is considered to be preassigned if it is derived from a preassigned event resource.

A meet contains an optional *assignment*, which assigns the meet to a particular offset in another meet, thereby fixing its time relative to the starting time of the other meet, and a *time domain* which restricts the times it may start at to an arbitrary subset of the times of the cycle. These attributes are described in detail in later sections.

A meet may optionally be contained in one node (Chapter 5). Functions

```
KHE_NODE KheMeetNode(KHE_MEET meet);
int KheMeetNodeIndex(KHE_MEET meet);
```

return the node containing `meet`, and the index of `meet` in that node, or `NULL` and `-1` if none.

As an aid to debugging, function

```
void KheMeetDebug(KHE_MEET meet, int verbosity, int indent, FILE *fp);
```

prints `meet` onto `fp` with the given verbosity and indent (for which see Section 1.3). Verbosity 1 prints just an identifying name; verbosity 2 adds the chain of assignments leading out of `meet`.

The name is usually the name of `meet`'s event, between quotes. If there is more than one meet corresponding to that event, this will be followed by a colon and the number `i` for which `KheEventMeet(soln, e, i)` equals `meet`. Alternatively, if `meet` is a cycle meet (Section 4.8.3), the name is its starting time (a time name or else an index) between slashes.

4.8.1. Splitting and merging

A meet may be split into two meets whose durations sum to the duration of the original meet:

```
bool KheMeetSplitCheck(KHE_MEET meet, int duration1, bool recursive);
bool KheMeetSplit(KHE_MEET meet, int duration1, bool recursive,
  KHE_MEET *meet1, KHE_MEET *meet2);
```

These functions follow the pattern described earlier for operations that might violate the solution invariant, in that both return `true` if the split is permitted. The second actually carries out the split, setting `*meet1` and `*meet2` to the new meets if the split is permitted, and leaving them unchanged if not. The original meet, `meet`, is undefined after a successful split, unless `meet1` or `meet2` is set to `&meet` (this may seem dangerous, but it does what is wanted whether the split succeeds or not). The split meet may be a cycle meet, in which case so are the two fragments.

The first new meet, `*meet1`, has duration `duration1`, and the second, `*meet2`, has the remaining duration. Parameter `duration1` must be such that both meets have duration at least 1, otherwise both functions abort. Their back pointers are set to the back pointer of `meet`. If `meet` is assigned, `*meet1` has the same target meet and offset as `meet`, while `*meet2` has the same target meet, but its offset is `duration1` larger, making the two meets adjacent in time.

If `recursive` is `true`, any meets assigned to `meet` that span the split point will also be split, into one meet for the part overlapping `*meet1` and one for the part overlapping `*meet2`. This process proceeds recursively as deeply as required.

The two split functions return `true` if these two conditions hold:

- Either `recursive` is `true`, or else no meets assigned to `meet` span the split point.

- The meets resulting from each split have copies of the meet bounds (Section 4.8.4) of the meets they are fragments of. Nevertheless their domains usually change, owing to meet bounds with specific `duration` attributes. This must cause no incompatibilities with the domains of other meets connected to them by assignments, allowing for offsets. When a cycle meet (Section 4.8.3) splits, the two fragments have the appropriate singleton domains. Domain incompatibilities cannot occur in that case.

If these conditions hold, `meet` is said to be *splittable* at `duration1`.

When a meet splits, its tasks split too. This produces what is typically required when assigning rooms: the fragments are free to be assigned different resources. The other possibility, where the fragments are required to be assigned the same resource, can be obtained by assigning the fragmentary tasks to each other. This must be done separately.

The next two functions are concerned with merging two meets into one:

```
bool KheMeetMergeCheck(KHE_MEET meet1, KHE_MEET meet2);
bool KheMeetMerge(KHE_MEET meet1, KHE_MEET meet2, bool recursive,
  KHE_MEET *meet);
```

Parameters `meet1` and `meet2` become undefined after a successful merge, unless `meet` is set to `&meet1` or `&meet2`.

If `recursive` is `true`, after merging `meet1` and `meet2`, `KheMeetMerge` searches for pairs of meets, one formerly assigned to the end of `meet1`, the other formerly assigned to the beginning of `meet2`, which are mergeable according to `KheMeetMergeCheck`, and merges each such pair. This process proceeds recursively as deeply as required. `KheMeetMergeCheck` has no `recursive` parameter because its result does not depend on whether the merge is recursive.

The functions return `true` if all these conditions hold:

- The two meets are distinct.
- The two meets have the same value of `KheMeetIsCycleMeet` (Section 4.8.3).
- The two meets have the same value of `KheMeetEvent`, possibly `NULL`.
- The two meets have the same value of `KheMeetNode`, possibly `NULL`.
- The two meets are both either assigned to the same meet, or not assigned. If assigned, the offset of one (it may be either) must equal the offset plus duration of the other, ensuring they are adjacent in time. Cycle meets, although never assigned, must also be adjacent in time.
- The two meets have the same number of tasks, and the order of their tasks may be permuted so that corresponding tasks are compatible. Two tasks are compatible when they have the same taskings, domains, event resources, and assignments.

- The result meet takes over the meet bounds (Section 4.8.4) of one of the meets being merged. Nevertheless its domain usually changes, owing to meet bounds with non-zero `duration` attributes. This must cause no incompatibilities with the domains of other meets connected to it by assignments, allowing for offsets. When cycle meets (Section 4.8.3) merge, the result meet has the singleton domain of the chronologically first meet. Domain incompatibilities cannot occur in that case.

If all these conditions hold, `meet1` and `meet2` are said to be *mergeable*. These conditions usually hold trivially when merging the results of a previous split. The merged meet's attributes (including its meet bounds and the order of its tasks) may come from either `meet1` or `meet2`; the choice is deliberately left unspecified, and the user must not depend on it.

It is now clear why `KheMeetMergeCheck` does not need a `recursive` parameter: because none of the conditions just given depend on whether the merge is recursive. Recursive merges are only attempted when `KheMergeCheck` says they will succeed. So instead of preventing the top-level merge, an unacceptable recursive merge simply does not happen.

4.8.2. Assignment

KHE's basic operations do not include assigning a time to a meet. A meet is either unassigned or else assigned to another meet at a given offset, fixing the starting times of the two meets relative to each other, but not assigning a specific time to either. For example, if `m1` is assigned to `m2` at offset 2, then whatever time `m2` eventually starts at, `m1` will start two times later. Of course, ultimately meets need to be assigned times. This is done by assigning them to special meets called *cycle meets* (Section 4.8.3).

Assigning one meet to another supports *hierarchical timetabling*, in which several meets are timetabled relative to each other, then the whole group is timetabled into a larger context, and so on. One simple application is in handling link events constraints. Assigning all the linked events except one to that exception guarantees that the linked events will be simultaneous; the time eventually assigned to the exception becomes the time assigned to all.

The fundamental meet assignment operations are

```
bool KheMeetMoveCheck(KHE_MEET meet, KHE_MEET target_meet, int offset);
bool KheMeetMove(KHE_MEET meet, KHE_MEET target_meet, int offset);
```

`KheMeetMove` changes the assignment of `meet` from whatever it is now to `target_meet` at `offset`. If `target_meet` is `NULL`, the move is an unassignment and `offset` is ignored.

These functions follow the usual pattern, returning `true` if the move can be carried out, with `KheMeetMove` actually doing it if so. They return `true` if all of the following conditions hold:

- `KheMeetAssignIsFixed` (see below) returns `false`.

- The `meet` parameter is not a cycle meet.

- The move actually changes the assignment: either `target_meet` is `NULL` and `meet`'s current assignment is non-`NULL`, or `target_meet` is non-`NULL` and `meet`'s current assignment is not to `target_meet` at `offset`.

- The `offset` parameter is in range: if `target_meet` is non-`NULL`, then `offset >= 0` and `offset <= KheMeetDuration(target_meet) - KheMeetDuration(meet)`;
- If `target_meet` is non-`NULL`, then the time domain (Section 4.8.4) of `target_meet` is a subset of the time domain of `meet`.
- The node rule (Section 4.12) would not be violated if the move was carried out.

If all these conditions hold, then `meet` is said to be *moveable* to `target_meet` at `offset`. Returning `false` when the move changes nothing reflects the practical reality that no solver wants to waste time on such moves.

KHE offers several convenience functions based on `KheMeetMoveCheck` and `KheMeetMove`. For assigning a meet there is

```
bool KheMeetAssignCheck(KHE_MEET meet, KHE_MEET target_meet, int offset);
bool KheMeetAssign(KHE_MEET meet, KHE_MEET target_meet, int offset);
```

Assigning is the same as moving except that `meet` is expected to be unassigned to begin with, and `KheMeetAssignCheck` and `KheMeetAssign` return `false` if not. For unassigning there is

```
bool KheMeetUnAssignCheck(KHE_MEET meet);
bool KheMeetUnAssign(KHE_MEET meet);
```

Unassigning is the same as moving to `NULL`. For swapping there is

```
bool KheMeetSwapCheck(KHE_MEET meet1, KHE_MEET meet2);
bool KheMeetSwap(KHE_MEET meet1, KHE_MEET meet2);
```

A swap is two moves, one of `meet1` to whatever `meet2` is assigned to, and the other of `meet2` to whatever `meet1` is assigned to. It succeeds whenever those two moves succeed.

`KheMeetSwap` has two useful properties. First, exchanging the order of its parameters never affects what it does. Second, the code fragment

```
if( KheMeetSwap(meet1, meet2) )
  KheMeetSwap(meet1, meet2);
```

leaves the solution in its original state whether the swap occurs or not.

A variant of the swapping idea called *block swapping* is offered:

```
bool KheMeetBlockSwapCheck(KHE_MEET meet1, KHE_MEET meet2);
bool KheMeetBlockSwap(KHE_MEET meet1, KHE_MEET meet2);
```

Block swapping is the same as ordinary swapping except that it treats one very special case in a different way: the case when both meets are initially assigned to the same meet, at different offsets which cause them to be adjacent, but not overlapping, in time. In this case, both meets remain assigned to the same meet afterwards, and the later meet is assigned the offset of the earlier one, but the earlier one is not necessarily assigned the offset of the later one. Instead, it is assigned that offset which places it adjacent to the other meet.

For example, when swapping a meet of duration 1 assigned to the first time on Monday with

a meet of duration 2 assigned to the second time on Monday, `KheMeetBlockSwap` would move the first meet to the third time on Monday, not the second time. This is much more likely to work well when the two meets have preassigned resources in common. It is the same as an ordinary swap when the meets have the same duration, but it is different when their durations differ. The two useful properties of ordinary swaps also hold for block swaps.

A meet's assignment may be retrieved by calling

```
KHE_MEET KheMeetAsst(KHE_MEET meet);
int KheMeetAsstOffset(KHE_MEET meet);
```

These return the meet that `meet` is assigned to, and the offset into that meet. If there is no assignment, the values returned are `NULL` and `-1`.

Although a meet may only be assigned to one meet, any number of meets may be assigned to a meet, each with its own offset. Functions

```
int KheMeetAssignedToCount(KHE_MEET target_meet);
KHE_MEET KheMeetAssignedTo(KHE_MEET target_meet, int i);
```

visit all the meets that are assigned to a given meet, in an unspecified order which could change when a meet is assigned to or unassigned from `target_meet`. (What actually happens is that an assignment is added to the end, and the hole created by the unassignment of any element other than the last is plugged with the last element.)

Given that a meet can be assigned to another meet at some offset, it follows that a chain of assignments can be built up, from one meet to another and another and so on. Function

```
KHE_MEET KheMeetRoot(KHE_MEET meet, int *offset_in_root);
```

returns the *root* of `meet`: the last meet on the chain of assignments leading out of `meet`. It also sets `*offset_in_root` to the offset of `meet` in its root meet, which is just the sum of the offsets along the assignment path. One function which uses `KheMeetRoot` is

```
bool KheMeetOverlap(KHE_MEET meet1, KHE_MEET meet2);
```

This returns `true` if `meet1` and `meet2` can be proved to overlap in time, because they have the same root meet, and their offsets in that root meet and durations make them overlap. Also,

```
bool KheMeetAdjacent(KHE_MEET meet1, KHE_MEET meet2, bool *swap);
```

returns `true` if `meet1` and `meet2` can be proved to be immediately adjacent in time (but not overlapping), because they have the same root meet, and their offsets in that root meet and durations make them adjacent. If so, it also sets `*swap` to `true` if `meet2` precedes `meet1`, and to `false` otherwise. Again, the meets are required to have the same root meet. This implies that a meet assigned to the end of one cycle meet (Section 4.8.3) is not reported to be adjacent to a meet assigned to the start of the next cycle meet. This is usually what is wanted in practice.

Meet assignments may be fixed and unfixed, by calling

```
void KheMeetAssignFix(KHE_MEET meet);
void KheMeetAssignUnFix(KHE_MEET meet);
bool KheMeetAssignIsFixed(KHE_MEET meet);
```

Any attempt to change the assignment of meet will fail while the fix is in place. When several events are linked by a link events constraint, assigning the meets of all but one of them to the meets of that one and fixing those assignments, or assigning the meets of all of them to some other set of meets and fixing those assignments, has a significant efficiency payoff.

A call to KheMeetMoveCheck(meet, target_meet, offset) returns false irrespective of target_meet and offset when meet is a cycle meet or its assignment is fixed. Function

```
bool KheMeetIsMovable(KHE_MEET meet);
```

returns true when neither of these conditions holds, so that KheMeetMoveCheck can be expected to return true for at least some target meets and offsets.

Two similar functions follow chains of fixed assignments:

```
KHE_MEET KheMeetFirstMovable(KHE_MEET meet, int *offset_in_result);
KHE_MEET KheMeetLastFixed(KHE_MEET meet, int *offset_in_result);
```

KheMeetFirstMovable returns the first meet m on the chain of assignments out of meet such that KheMeetIsMovable(m) holds. If there is no such meet it returns NULL. It is used when changing the time assigned to meet: this can be done only by changing the assignment of KheMeetFirstMovable(meet), or of a movable meet further along the chain, and this is only possible when the result is non-NULL. KheMeetLastFixed returns the last meet on the chain of fixed assignments out of meet; that is, it follows the chain of assignments out of meet until it reaches a meet whose target meet is NULL or whose assignment is not fixed, and returns that meet. Its result is always non-NULL, and could be a cycle meet. It is used to decide whether two meets are fixed to the same meet, directly or indirectly. In both functions, the result could be meet itself, and *offset_in_result is set to the offset of meet in the result, if non-NULL.

4.8.3. Cycle meets and time assignment

Even if most meets are assigned to other meets, there must be a way to associate a particular starting time with a meet eventually. Rather than having two kinds of assignment, one to a meet and one to a time, which might conflict, KHE has a special kind of meet called a *cycle meet*. A cycle meet has type KHE_MEET as usual, and it has many of the properties of ordinary meets. But it is also associated with a particular starting time (and its domain is fixed to just that time and cannot be changed), and so by assigning a meet to a cycle meet one also assigns a time.

A cycle meet cannot be assigned to another meet; its assignment is fixed to NULL and cannot be changed. Cycle meets may be split (their offspring are also cycle meets) and merged. They may even be deleted, but that is not likely to ever be a good idea.

The user cannot create cycle meets directly. Instead, one cycle meet is created automatically whenever a solution is created. The starting time of this *initial cycle meet* is the first time of the cycle, and its duration is the number of times of the cycle. When solving, it is usual to split the initial cycle meet into one meet for each block of times not separated by a meal break or the end of a day, to prevent other meets from being assigned times which cause them to span these breaks. A function for this appears below. When evaluating a fixed solution, it is usual to not split the initial cycle meet, since the other meets already have unchangeable starting times and durations, and splitting the initial cycle meet might prevent them from being assigned to cycle meets.

To find out whether a given meet is a cycle meet, call

```
bool KheMeetIsCycleMeet(KHE_MEET meet);
```

Cycle meets appear on the list of all meets contained in a solution. They are not stored separately anywhere. So the way to find them all is

```
for( i = 0;  i < KheSolnMeetCount(soln);  i++ )
{
  meet = KheSolnMeet(soln, i);
  if( KheMeetIsCycleMeet(meet) )
    visit_cycle_meet(meet);
}
```

However, cycle meets are usually near the front of the list, so this can be optimized as follows:

```
time_count = KheInstanceTimeCount(KheSolnInstance(soln));
durn = 0;
for( i = 0;  i < KheSolnMeetCount(soln) && durn < time_count;  i++ )
{
  meet = KheSolnMeet(soln, i);
  if( KheMeetIsCycleMeet(meet) )
  {
    visit_cycle_meet(meet);
    durn += KheMeetDuration(meet);
  }
}
```

The loop terminates as soon as the total duration of the cycle meets visited reaches the number of times in the instance.

Solutions offer several functions whose results depend on cycle meets. They notice when cycle meets are split, and adjust their results accordingly. Functions

```
KHE_MEET KheSolnTimeCycleMeet(KHE_SOLN soln, KHE_TIME t);
int KheSolnTimeCycleMeetOffset(KHE_SOLN soln, KHE_TIME t);
```

return the unique cycle meet running at time `t`, and the offset of `t` within that meet. Function

```
KHE_TIME_GROUP KheSolnPackingTimeGroup(KHE_SOLN soln, int duration);
```

returns a time group containing the times at which a meet of the given duration may begin. For example, if the initial cycle meet has not been split, `KheSolnPackingTimeGroup(soln, 2)` will contain every time except the last in the cycle; if the initial cycle meet has been split into one meet for each day, it will contain every time except the last in each day; and so on.

As mentioned earlier, when solving it is usual to split the initial cycle meet into one fragment for each maximal block of times not spanning a meal break or end of day. The XML format does not record this information, but solver

```
void KheSolnSplitCycleMeet(KHE_SOLN soln);
```

is able to infer it, as follows. Say that two events of `soln`'s instance are related if they share a required link events constraint with non-zero weight. Find the equivalence classes of the reflexive transitive closure of this relation. For each class, examine the required split events constraints with non-zero weight of the events of the class to determine what durations the meets derived from the events of this class may have. Also determine whether the starting time of the class is preassigned, because one of its events has a preassigned time.

For each permitted duration, consult the required prefer times constraints of non-zero weight of the events of the class to see when its meets of that duration could begin. If a meet `m` with duration 2 can begin at time `t`, there cannot be a break after time `t`; if a meet `m` with duration 3 can begin at time `t`, there cannot be a break after time `t` or after the time following `t`, if any; and so on. Accumulating all this information for all classes determines the set of times which cannot be followed by a break. All other times can be followed by a break, and the initial cycle event is split at these times, and also at times where a break is explicitly allowed by function `KheTimeBreakAfter` from Section 3.4.2.

These functions move a meet to a time, following the familiar pattern:

```
bool KheMeetMoveTimeCheck(KHE_MEET meet, KHE_TIME t);
bool KheMeetMoveTime(KHE_MEET meet, KHE_TIME t);
```

They work by converting `t` into a cycle meet and offset, via functions `KheSolnTimeCycleMeet` and `KheSolnTimeCycleMeetOffset` above, and calling `KheMeetMoveCheck` and `KheMeetMove`. Meets may also be assigned to cycle meets directly, using `KheMeetMove` and the rest. The direct route is more convenient in general solving, since time assignment is then not a special case.

The following functions are also offered:

```
bool KheMeetAssignTimeCheck(KHE_MEET meet, KHE_TIME t);
bool KheMeetAssignTime(KHE_MEET meet, KHE_TIME t);
bool KheMeetUnAssignTimeCheck(KHE_MEET meet);
bool KheMeetUnAssignTime(KHE_MEET meet);
KHE_TIME KheMeetAsstTime(KHE_MEET meet);
```

The first four are wrappers for `KheMeetAssignCheck`, `KheMeetAssign`, `KheMeetUnAssignCheck`, and `KheMeetUnAssign`. `KheMeetAsstTime` follows the assignments of `meet` as far as possible, and if it arrives in a cycle meet, it returns the starting time of `meet`; otherwise it returns `NULL`.

4.8.4. Meet domains and bounds

Each meet contains a time group called its *domain*, retrievable by calling

```
KHE_TIME_GROUP KheMeetDomain(KHE_MEET meet);
```

When a meet is assigned a time, that time must be an element of its domain.

More precisely, the solution invariant says that `meet`'s domain must be a superset of the domain of the meet it is assigned to, if any, adjusted for offsets. So, given a chain of assignments beginning at `meet` and ending at a cycle meet, the domain of `meet` must be a superset of the domain of the cycle meet, adjusted for offsets. Since the domain of a cycle meet is a singleton set defining a time, the time assigned to `meet` by this chain of assignments lies in `meet`'s domain.

Meet domains cannot be set directly. Instead, *meet bound* objects influence them. This may seem unnecessarily complicated, but meet bounds have several major advantages over setting domains directly, including allowing restrictions on domains to be added and removed independently, and doing the right thing when meets split and merge.

When meets split and merge, their durations change, and this usually requires a change of domain. For example, a meet of duration 2 cannot be assigned the last time on any day, but if it is split, the fragments may be. Accordingly, a meet bound object stores a whole set of time groups, one for each possible duration. Only one time group influences a meet's domain at any moment: the one corresponding to the meet's current duration. But the others remain in reserve for when the meet's duration is changed by a split or merge.

To create a meet bound object, call

```
KHE_MEET_BOUND KheMeetBoundMake(KHE_SOLN soln,
  bool occupancy, KHE_TIME_GROUP dft_tg);
```

See below for the `occupancy` and `dft_tg` parameters. To delete a meet bound object, call

```
bool KheMeetBoundDeleteCheck(KHE_MEET_BOUND mb);
bool KheMeetBoundDelete(KHE_MEET_BOUND mb);
```

This includes deleting `mb` from each meet it is added to, and is permitted when all of those deletions are permitted, according to `KheMeetDeleteMeetBoundCheck`, defined below.

To retrieve the attributes defined when a meet bound is created, call

```
KHE_SOLN KheMeetBoundSoln(KHE_MEET_BOUND mb);
int KheMeetBoundSolnIndex(KHE_MEET_BOUND mb);
bool KheMeetBoundOccupancy(KHE_MEET_BOUND mb);
KHE_TIME_GROUP KheMeetBoundDefaultTimeGroup(KHE_MEET_BOUND mb);
```

These are rarely accessed in practice.

As mentioned above, a meet bound is supposed to define a time group for each possible duration. These time groups can be set manually by making any number of calls to

```
void KheMeetBoundAddTimeGroup(KHE_MEET_BOUND mb,
  int duration, KHE_TIME_GROUP tg);
```

Each declares that when `mb` is applied to a meet of the given `duration`, it restricts its domain to be a subset of `tg`. They may be retrieved by

```
KHE_TIME_GROUP KheMeetBoundTimeGroup(KHE_MEET_BOUND mb, int duration);
```

In both functions, `duration` may be any positive integer, provided it is not unreasonably large. Two calls to `KheMeetBoundAddTimeGroup` with the same `duration` are pointless, but if they occur, the second takes effect. There is no need to specify a time group for every possible duration: durations other than those covered by calls to `KheMeetBoundAddTimeGroup` are assigned time groups using the `occupancy` and `dft_tg` arguments of `KheMeetBoundMake`. To explain them we need to delve deeper.

There are really two kinds of domains. Those we have dealt with so far may be called

starting-time domains, because they restrict the starting times of meets. They are appropriate, for example, when expressing prefer times and spread events constraints (which constrain starting times) structurally. The others may be called *occupancy domains*, because they restrict the whole set of times a meet occupies, not just its starting time. For example, a meet of duration 2 should not start immediately before a time when one of its resources is unavailable: the complement of a resource's set of unavailable times is an occupancy domain, not a starting-time domain.

KHE works directly only with starting-time domains, not occupancy domains, so what is needed is a function to convert an occupancy domain into a starting-time domain:

```
KHE_TIME_GROUP KheSolnStartingTimeGroup(KHE_SOLN soln, int duration,
  KHE_TIME_GROUP tg);
```

This returns the set of times that a meet of the given duration could start without any part of it lying outside `tg`. In other words, it accepts occupancy domain `tg` and returns the equivalent starting-time domain for a meet of the given duration. When `duration` is 1, the result is just `tg`. As `duration` increases the result shrinks, eventually becoming empty.

To return to meet bounds. When `occupancy` is `false`, the time group used by the meet bound for durations not set explicitly is `dft_tg`. It may be best to set all durations explicitly in this case. When `occupancy` is `true`, the value used for any unspecified duration is

```
KheSolnStartingTimeGroup(soln, duration, dft_tg);
```

These values could be passed explicitly, but this way they can be (and are) created only when needed, and there is no need to know the maximum duration. For example, let `available_tg` be the set of times that some resource is available. Then the meet bound created by

```
KheMeetBoundMake(soln, true, available_tg);
```

ensures that a meet lies entirely within this set of times, whatever duration it has.

A meet `m` may have any number of meet bounds. Its domain is the intersection, over all its meet bounds `mb`, of `KheMeetBoundTimeGroup(mb, KheMeetDuration(m))`, or the full cycle if none. A meet bound may be added to any number of meets. To add a meet bound, call

```
bool KheMeetAddMeetBoundCheck(KHE_MEET meet, KHE_MEET_BOUND mb);
bool KheMeetAddMeetBound(KHE_MEET meet, KHE_MEET_BOUND mb);
```

These follow the usual form, returning `true` when the addition is permitted (when the change in `meet`'s domain it causes does not violate the solution invariant), with `KheMeetAddMeetBound` actually carrying out the addition in that case. To delete a meet bound from a meet, call

```
bool KheMeetDeleteMeetBoundCheck(KHE_MEET meet, KHE_MEET_BOUND mb);
bool KheMeetDeleteMeetBound(KHE_MEET meet, KHE_MEET_BOUND mb);
```

This too is not always permitted, because it may increase `meet`'s domain, which may violate the solution invariant with respect to the domains of meets assigned to `meet`.

While a meet bound is added to at least one meet, it is not permitted to change its time groups (that is, calls to `KheMeetBoundAddTimeGroup` are prohibited).

To visit the meet bounds added to a given meet, call

```
int KheMeetMeetBoundCount(KHE_MEET meet);
KHE_MEET_BOUND KheMeetMeetBound(KHE_MEET meet, int i);
```

as usual. To visit the meets to which a given meet bound has been added, call

```
int KheMeetBoundMeetCount(KHE_MEET_BOUND mb);
KHE_MEET KheMeetBoundMeet(KHE_MEET_BOUND mb, int i);
```

The relationship between meets and meet bounds is a many-to-many one.

When a meet is split, its meet bounds are added to both fragments; and when two meets are merged, one (either) of the two sets of meet bounds is used for the merged meet. Although the meet bounds are the same, the durations change, so the domains may change too. Splits and merges are only permitted when the new domains do not violate the solution invariant.

Adding a meet bound to a meet has some cost in run time, but is fast enough to use within solvers. KHE intersects the bound's time group's bit set with the current domain's bit set, looks up the result in a hash table of all time groups known to the solution, and either uses an existing time group returned by the lookup, or makes and uses a new one, which is then added to the table. Deleting a meet bound is much the same, except that the bit sets of the remaining bounds' time groups are intersected to obtain the new domain. Time groups are immutable during solving and may be shared. Meet bound objects are obtained from free lists held in the solution object.

When `KheMeetMake` makes a meet derived from an event with a preassigned time, it adds to the meet a meet bound whose default time group is the singleton time group containing that time. No other special arrangements are made for meets derived from preassigned events.

4.8.5. Automatic domains

Cycle meets have fixed singleton domains, and meets derived from events can also be assigned fixed domains, based on their durations and the constraints that apply to them.

When solving hierarchically there may be other meets, lying at intermediate levels, for which there is no obvious fixed domain. Instead, the domain of such a meet needs to be the largest domain consistent with the domains of the meets assigned to it: the intersection of those domains, allowing for offsets, or the full set of times if no meets are assigned to it.

As meets are assigned to and unassigned from such a meet, its domain changes automatically. At any moment it does have a domain, however, defined by the rule just given, and this domain must satisfy the solution invariant as usual.

A newly created meet has a fixed domain. To convert it to the automatic form, call

```
bool KheMeetSetAutoDomainCheck(KHE_MEET meet, bool automatic);
bool KheMeetSetAutoDomain(KHE_MEET meet, bool automatic);
```

Assigning `true` to `automatic` gives the meet an automatic domain. This will return `false` if `meet` is a cycle meet, or if `meet` is derived from an event or contains tasks, as discussed below. Assigning `false` returns the meet to a fixed domain. Meet bounds are not affected by automatic domains; what is affected is whether they are used to construct the domain or not.

`KheMeetDomain` returns `NULL` when the meet has an automatic domain. It is important not to mistake this for 'having no domain,' a concept not defined by KHE. Function

```
KHE_TIME_GROUP KheMeetDescendantsDomain(KHE_MEET meet);
```

returns the intersection of the domains of the descendants of `meet`, including `meet` itself, adjusted for offsets, or the full time group if there are no such meets or they all have automatic domains. It may thus be used to find the true domain of a meet when `KheMeetDomain` returns `NULL`. It is relatively slow and not intended for use during solving.

When a meet with an automatic domain is split, its two fragments have automatic domains. When two meets are joined, they must both either have automatic domains or not; and if both do, then the joined meet has an automatic domain.

A meet with an automatic domain may not be derived from an event, and it may not have tasks. These two conditions are naturally satisfied by the kinds of meets that need automatic domains. They are necessary, since otherwise KHE would be forced to maintain explicit domains as meets are assigned and unassigned, which would not be efficient. As it is, automatic domains are implemented by having the domain test bypass meets whose domains are automatic, as though each such meet was replaced by the collection of meets assigned to it.

4.9. Tasks

A task is a demand for one resource. It is created by calling

```
KHE_TASK KheTaskMake(KHE_SOLN soln, KHE_RESOURCE_TYPE rt,
  KHE_MEET meet, KHE_EVENT_RESOURCE er);
```

The task lies in `soln` and has resource type `rt`. When parameter `meet` is non-`NULL`, the task lies within `meet`, representing a demand for one resource, of type `rt`, at the times when `meet` is running. When `meet` is `NULL`, the task still demands a resource, but at no times, making it useful only as a target for the assignment of other tasks, as explained below.

Parameter `er` may be non-`NULL` only when `meet` is non-`NULL` and derived from some event `e`. In that case, `er` must be one of `e`'s event resources. Its presence causes the task to consider itself to be derived from event resource `er`.

When first created, a meet has no tasks. They must be created separately by calls to `KheTaskMake`. Function `KheSolnMakeCompleteRepresentation` (Section 4.3) does this. When a task's enclosing meet splits, the task splits too. And when two meets merge, their tasks must be compatible and are merged pairwise, inversely to the split.

A task contains an optional *assignment* to another task, and a *resource domain* which restricts the resources it may be assigned to an arbitrary subset of the resources of its type. These attributes are described in detail in later sections.

A task may be deleted by calling

```
void KheTaskDelete(KHE_TASK task);
```

This removes the task from its meet, if any, and unassigns any assignments involving the task.

The back pointer of a task may be set and retrieved by

```
void KheTaskSetBack(KHE_TASK task, void *back);
void *KheTaskBack(KHE_TASK task);
```

as usual, and the usual visit number operations are available:

```
void KheTaskSetVisitNum(KHE_TASK task, int num);
int KheTaskVisitNum(KHE_TASK task);
bool KheTaskVisited(KHE_TASK task, int slack);
void KheTaskVisit(KHE_TASK task);
void KheTaskUnVisit(KHE_TASK task);
```

The attributes of a task related to its meet may be retrieved by

```
KHE_MEET KheTaskMeet(KHE_TASK task);
int KheTaskMeetIndex(KHE_TASK task);
int KheTaskDuration(KHE_TASK task);
float KheTaskWorkload(KHE_TASK task);
```

If there is no meet, `KheTaskMeet` returns `NULL` and `KheTaskDuration` and `KheTaskWorkload` return 0. If there is a meet and event resource, `KheTaskWorkload` returns the workload of the task, defined in accord with the XML format's definition to be

$$w(task) = \frac{d(meet)w(er)}{d(e)}$$

where $d(meet)$ is the duration of `task`'s meet, $w(er)$ is the workload of the task's event resource, and $d(e)$ is the duration of the task's meet's event. See below for the similar and more generally useful `KheTaskTotalDuration` and `KheTaskTotalWorkload` operations. Other attributes of a task may be accessed by

```
KHE_SOLN KheTaskSoln(KHE_TASK task);
int KheTaskSolnIndex(KHE_TASK task);
KHE_RESOURCE_TYPE KheTaskResourceType(KHE_TASK task);
KHE_EVENT_RESOURCE KheTaskEventResource(KHE_TASK task);
```

These return the solution containing `task`, the index of `task` in its solution (the value of `i` for which `KheSolnTask(soln, i)` returns `task`), the task's resource type, and its event resource (if any). Index numbers may change when tasks are deleted (what actually happens is that the hole left by the deletion of a task, if not last, is plugged by the last task), so care is needed. Also,

```
bool KheTaskIsPreassigned(KHE_TASK task, KHE_RESOURCE *r);
```

returns `true` when `KheTaskEventResource(task) != NULL` and that event resource has a preassigned resource; `task` is called a *preassigned task* in that case. If `r != NULL`, then `*r` is set to the event resource's preassigned resource if `task` is preassigned, and to `NULL` otherwise.

A task may lie in a *tasking*, which is an arbitrary set of tasks (Section 5.5). Functions

```
KHE_TASKING KheTaskTasking(KHE_TASK task);
int KheTaskTaskingIndex(KHE_TASK task);
```

return the tasking containing `task` and the index of `task` in that tasking, or `NULL` and `-1` if the task does not lie in a tasking. Finally,

```
void KheTaskDebug(KHE_TASK task, int verbosity, int indent, FILE *fp);
```

produces the usual debug print of `task` onto `fp` with the given verbosity and indent.

4.9.1. Assignment

Just as KHE assigns one meet to another meet, not to a time, so it assigns one task to another task, not to a resource. Accordingly, the assignment operations for tasks parallel those for meets, the main difference being that there is no offset.

The fundamental task assignment operations are

```
bool KheTaskMoveCheck(KHE_TASK task, KHE_TASK target_task);
bool KheTaskMove(KHE_TASK task, KHE_TASK target_task);
```

`KheTaskMove` changes the assignment of `task` to `target_task`. If `target_task` is `NULL`, the move is an unassignment. These operations follow the usual pattern, returning `false` and changing nothing if they cannot be carried out. Here is the full list of reasons why this could happen:

- `task`'s assignment is fixed;
- `task` is a cycle task (Section 4.9.2);
- the move changes nothing: `target_task` is the same as `task`'s current assignment;
- `target_task` is non-`NULL` and the resource domain (Section 4.9.3) of `target_task` is not a subset of the resource domain of `task`.

As for meet moves, returning `false` when the move changes nothing reflects the practical reality that no solver wants to waste time on such moves.

KHE offers several convenience functions based on `KheTaskMoveCheck` and `KheTaskMove`. For assigning a task there is

```
bool KheTaskAssignCheck(KHE_TASK task, KHE_TASK target_task);
bool KheTaskAssign(KHE_TASK task, KHE_TASK target_task);
```

Assigning is the same as moving except that `task` is expected to be unassigned to begin with, and `KheTaskAssignCheck` and `KheTaskAssign` return `false` if not. For unassigning there is

```
bool KheTaskUnAssignCheck(KHE_TASK task);
bool KheTaskUnAssign(KHE_TASK task);
```

Unassigning is the same as moving to `NULL`. For swapping there is

```
bool KheTaskSwapCheck(KHE_TASK task1, KHE_TASK task2);
bool KheTaskSwap(KHE_TASK task1, KHE_TASK task2);
```

A swap is two moves, one of `task1` to whatever `task2` is assigned to, and the other of `task2` to whatever `task1` is assigned to. It succeeds whenever those two moves succeed. As for meet swaps, exchanging the parameters changes nothing, and code fragment

```
if( KheTaskSwap(task1, task2) )
  KheTaskSwap(task1, task2);
```

leaves the solution in its original state whether the swap occurs or not.

A task's assignment may be retrieved by calling

```
KHE_TASK KheTaskAsst(KHE_TASK task);
```

If there is no assignment, `NULL` is returned. Although a task may only be assigned to one task, any number of tasks may be assigned to a task. Functions

```
int KheTaskAssignedToCount(KHE_TASK target_task);
KHE_TASK KheTaskAssignedTo(KHE_TASK target_task, int i);
```

visit all the tasks that are assigned to `target_task`, in an unspecified order which could change when a task is assigned or unassigned from `target_task`. (What actually happens is that an assignment is added to the end, and the hole created by the unassignment of any element other than the last is plugged with the last element.) Functions

```
int KheTaskTotalDuration(KHE_TASK task);
float KheTaskTotalWorkload(KHE_TASK task);
```

return the total duration and workload of `task` and the tasks assigned to it, directly or indirectly. These functions are usually more appropriate than `KheTaskDuration` and `KheTaskWorkload`.

Given that a task can be assigned to another task, a chain of assignments can be built up, from one task to another and so on. Function

```
KHE_TASK KheTaskRoot(KHE_TASK task);
```

returns the *root* of `task`: the last task on the chain of assignments leading out of `task`, possibly `task` itself. The result is never `NULL`, but it could be a cycle task (Section 4.9.2).

Task assignments may be fixed and unfixed as usual, by calling

```
void KheTaskAssignFix(KHE_TASK task);
void KheTaskAssignUnFix(KHE_TASK task);
bool KheTaskAssignIsFixed(KHE_TASK task);
```

The assignment of `task` cannot be changed while the fix is in place. When several tasks are linked by an avoid split assignments constraint, assigning all but one of them to that one and fixing those assignments, or assigning all of them to some other task and fixing those assignments, has a significant efficiency payoff. Function

```
KHE_TASK KheTaskFirstUnFixed(KHE_TASK task);
```

returns the first task on the chain of assignments out of `task` whose assignment is not fixed (possibly `task`), or `NULL` if none. A solver can change the resource assigned to `task` only by changing the assignment of `KheTaskFirstUnFixed(task)`, or of a task further along the chain.

4.9.2. Cycle tasks and resource assignment

Just as meets are assigned times by assigning them, directly or indirectly, to cycle meets, so tasks are assigned resources by assigning them, directly or indirectly, to *cycle tasks*. A cycle task has type KHE_TASK as usual, and it has many of the properties of ordinary tasks. But it is also associated with a particular resource (and its domain is fixed to just that resource and cannot be changed), and so by assigning a task to a cycle task one also assigns a resource.

The user cannot create cycle tasks directly. Instead, one cycle task is created automatically for each resource whenever a solution is created. The first KheInstanceResourceCount tasks of a solution are its cycle tasks, in the order the resources appear in the instance. Function

```
bool KheTaskIsCycleTask(KHE_TASK task);
```

returns true when task is a cycle task. Function

```
KHE_TASK KheSolnResourceCycleTask(KHE_SOLN soln, KHE_RESOURCE r);
```

returns the cycle task representing r in soln.

These functions move a task to a resource, following the familiar pattern:

```
bool KheTaskMoveResourceCheck(KHE_TASK task, KHE_RESOURCE r);
bool KheTaskMoveResource(KHE_TASK task, KHE_RESOURCE r);
```

They work by converting r into a cycle task, via function KheSolnResourceCycleTask above, and calling KheTaskMoveCheck and KheTaskMove. Tasks may also be assigned to cycle tasks directly, using KheTaskMove and the rest.

The following functions are also offered:

```
bool KheTaskAssignResourceCheck(KHE_TASK task, KHE_RESOURCE r);
bool KheTaskAssignResource(KHE_TASK task, KHE_RESOURCE r);
bool KheTaskUnAssignResourceCheck(KHE_TASK task);
bool KheTaskUnAssignResource(KHE_TASK task);
KHE_RESOURCE KheTaskAsstResource(KHE_TASK task);
```

The first four are wrappers for KheTaskAssignCheck, KheTaskAssign, KheTaskUnAssignCheck, and KheTaskUnAssign. KheTaskAsstResource follows the assignments of task as far as possible. If it arrives at a cycle task, it returns the resource represented by that task, else it returns NULL.

To find the tasks assigned a given resource, either directly or indirectly via other tasks, call

```
int KheResourceAssignedTaskCount(KHE_SOLN soln, KHE_RESOURCE r);
KHE_TASK KheResourceAssignedTask(KHE_SOLN soln, KHE_RESOURCE r, int i);
```

When a resource r is assigned to a task, the task and all tasks assigned to it, directly or indirectly, go on the end of r's sequence. When r is unassigned from a task, the task and all tasks assigned to it, directly or indirectly, are removed, and the gaps are plugged by tasks taken from the end. The sequence does not include r's cycle task.

In practice, tasks are of three kinds: *cycle tasks*, which represent resources; *unfixed tasks*, which require assignment to cycle tasks; and *fixed tasks*, whose assignments are fixed to unfixed

tasks, relinquishing responsibility for assigning a resource to those tasks. Resource assignment algorithms are concerned with assigning or reassigning unfixed tasks.

4.9.3. Task domains and bounds

Each task contains a resource group called its *domain*, retrievable by calling

```
KHE_RESOURCE_GROUP KheTaskDomain(KHE_TASK task);
```

When a task is assigned a resource, that resource must be an element of its domain.

More precisely, the solution invariant says that `task`'s domain must be a superset of the domain of the task it is assigned to, if any. So, given a chain of assignments beginning at `task` and ending at a cycle task, the domain of `task` must be a superset of the domain of the cycle task. Since the domain of a cycle task is a singleton set defining a resource, the resource assigned to `task` by this chain of assignments lies in `task`'s domain.

Task domains cannot be set directly. Instead, *task bound* objects influence them. Task bounds work in the same way as meet bounds, except that the complications introduced by meet splitting are absent.

To create a task bound object, call

```
KHE_TASK_BOUND KheTaskBoundMake(KHE_SOLN soln, KHE_RESOURCE_GROUP rg);
```

To delete a task bound object, call

```
bool KheTaskBoundDeleteCheck(KHE_TASK_BOUND tb);
bool KheTaskBoundDelete(KHE_TASK_BOUND tb);
```

This includes deleting `tb` from each task it is added to, and is permitted when all of those deletions are permitted, according to `KheTaskDeleteTaskBoundCheck`, defined below.

To retrieve the attributes defined when a task bound is created, call

```
KHE_SOLN KheTaskBoundSoln(KHE_TASK_BOUND tb);
int KheTaskBoundSolnIndex(KHE_TASK_BOUND tb);
KHE_RESOURCE_GROUP KheTaskBoundResourceGroup(KHE_TASK_BOUND tb);
```

These are rarely accessed in practice.

A task may have any number of task bounds. Its domain is the intersection, over all its task bounds `tb`, of `KheTaskBoundResourceGroup(tb)`, or the full set of resources of its type if none. A task bound may be added to any number of tasks. To add a task bound, call

```
bool KheTaskAddTaskBoundCheck(KHE_TASK task, KHE_TASK_BOUND tb);
bool KheTaskAddTaskBound(KHE_TASK task, KHE_TASK_BOUND tb);
```

These follow the usual form, returning `true` when the addition is permitted (when the change in `task`'s domain it causes does not violate the solution invariant), with `KheTaskAddTaskBound` actually carrying out the addition in that case. To delete a task bound from a task, call

```
bool KheTaskDeleteTaskBoundCheck(KHE_TASK task, KHE_TASK_BOUND tb);
bool KheTaskDeleteTaskBound(KHE_TASK task, KHE_TASK_BOUND tb);
```

This too is not always permitted, because it may increase `task`'s domain, which may violate the solution invariant with respect to the domains of tasks assigned to `task`.

To visit the task bounds added to a given task, call

```
int KheTaskTaskBoundCount(KHE_TASK task);
KHE_TASK_BOUND KheTaskTaskBound(KHE_TASK task, int i);
```

as usual. To visit the tasks to which a given task bound has been added, call

```
int KheTaskBoundTaskCount(KHE_TASK_BOUND tb);
KHE_TASK KheTaskBoundTask(KHE_TASK_BOUND tb, int i);
```

The relationship between tasks and task bounds is a many-to-many one.

Adding a task bound to a task has some cost in run time, but is fast enough to use within solvers. The implementation parallels the one described previously for meet bounds.

When `KheTaskMake` makes a task derived from an event resource which has a preassigned resource, it adds to the task a task bound whose resource group is the singleton resource group containing that resource. No other special arrangements are made for tasks derived from preassigned event resources.

4.10. Marks and paths

Suppose you want to make the best time assignment for a meet. You try each assignment in turn, remembering the best so far and its solution cost, then finish off by re-doing the best one.

Now suppose the alternative operations are more complicated. For example, they might be Kempe meet moves (Section 10.2.2), each consisting of an unpredictable number of time assignments. The same program structure works, but undoing one alternative is much more complicated. Marks and paths solve these kinds of problems.

A *mark* is like a waymark on a journey: it marks a particular point, or state, that a solution has reached. It is created and deleted by

```
KHE_MARK KheMarkBegin(KHE_SOLN soln);
void KheMarkEnd(KHE_MARK mark, bool undo);
```

These operations must be called in matching pairs: for each call to `KheMarkBegin` there must be one later call to `KheMarkEnd` with the same mark object. Between these two calls there may be other calls to `KheMarkBegin` and `KheMarkEnd`, and those calls must occur in matching pairs.

`KheMarkEnd` deletes the mark created by the corresponding `KheMarkBegin`. If its `undo` parameter is `true`, it also undoes all operations on `soln` since the corresponding `KheMarkBegin`, returning the solution to its state when that call was made. Another way to undo is

```
void KheMarkUndo(KHE_MARK mark);
```

It undoes all operations on `soln` since the call to `KheMarkBegin` which returned `mark`, only without removing `mark`. It can only be called when it would be legal to call `KheMarkEnd` with the same value of `mark`: when `mark` is the mark returned most recently by a call to `KheMarkBegin`, apart from marks already completed by `KheMarkEnd`.

When undoing by either method, the resulting value of the solution may differ from the original in its naturally nondeterministic aspects, such as the set of unmatched demand monitors (but not their number), and the order of elements in arrays representing sets (of meets, etc.). But as a solution it will be the same as the original. KHE objects deleted while doing and re-created while undoing are re-created with the same memory addresses as the originals.

At any time between `KheMarkBegin` and its corresponding `KheMarkEnd`, functions

```
KHE_SOLN KheMarkSoln(KHE_MARK mark);
KHE_COST KheMarkSolnCost(KHE_MARK mark);
```

may be called to obtain `mark`'s solution and the solution cost at the time `KheMarkBegin` was called. Exploring the result of `KheMarkSoln` will reveal the solution as it is now, not as it was when `KheMarkBegin` was called.

All mark objects share access to one sequence, stored in the solution object, of records of the operations performed on the solution since the first call to `KheMarkBegin` whose corresponding `KheMarkEnd` has not occurred yet. When undoing, these operations are undone in reverse order and removed from the sequence. All changes to solutions, including changes to back pointers, are recorded, except changes to visit numbers, since undoing them would be inappropriate. A mark object holds a pointer to the solution object, its cost when `KheMarkBegin` was called, an index into the sequence saying where to stop undoing, and a sequence of paths, described next.

A *path* is like the route between two waymarks. A path is created by calling

```
KHE_PATH KheMarkAddPath(KHE_MARK mark);
```

and represents the route from the state of `mark`'s solution represented by `mark` to the state of that solution at the moment `KheMarkAddPath` is called. Concretely, a path holds a copy of the shared sequence of operations, taken at the moment `KheMarkAddPath` is called, from its mark's index to the end. As well as being returned, a path is stored in its mark and deleted by that mark's `KheMarkEnd`, if it has not been deleted before then. A path is meaningless after its mark ends.

In practice, this helper function may be more useful than `KheMarkAddPath`:

```
KHE_PATH KheMarkAddBestPath(KHE_MARK mark, int k);
```

It is written using the more basic functions given below. Its behaviour is equivalent to calling `KheMarkAddPath(mark)`, then sorting `mark`'s paths into increasing cost order, then deleting paths from the end as required to ensure that not more than `k` paths are kept. But rather than following this description literally, it uses an optimized method that only calls `KheMarkAddPath(mark)` when the resulting path would be one of those kept; it returns the new path in that case, and `NULL` otherwise. For example, `KheMarkAddBestPath(mark, 1)` saves only the best path, and only creates a path when it would be a new best.

Any number of paths may be stored in a mark, and they may be visited using

```
int KheMarkPathCount(KHE_MARK mark);
KHE_PATH KheMarkPath(KHE_MARK mark, int i);
```

as usual, and sorted by calling

```
void KheMarkPathSort(KHE_MARK mark,
  int(*compar)(const void *, const void *));
```

where `compar` is a function suited to passing to `qsort` when sorting an array of `KHE_PATH` objects. One such function, `KhePathIncreasingSolnCostCmp`, is provided, such that after calling

```
KheMarkPathSort(mark, &KhePathIncreasingSolnCostCmp);
```

the paths will be sorted into increasing solution cost order, so that the path with the smallest solution cost comes first. The following operations on paths are also available:

```
KHE_SOLN KhePathSoln(KHE_PATH path);
KHE_COST KhePathSolnCost(KHE_PATH path);
KHE_MARK KhePathMark(KHE_PATH path);
void KhePathDelete(KHE_PATH path);
void KhePathRedo(KHE_PATH path);
```

`KhePathSoln` returns `path`'s solution, and `KhePathSolnCost` returns the solution cost at the moment the path was created by `KheMarkAddPath`. `KhePathMark` returns `path`'s mark. `KhePathDelete` deletes `path`, including removing it from its mark. `KheMarkEnd` calls `KhePathDelete` for each of its paths; once a mark is deleted, its paths have no meaning.

When `KhePathRedo(path)` is called, the solution must be in the state it was in when `path`'s mark was created. It redoes `path`, without deleting or otherwise disturbing its mark, so that the state after it returns is the state at the end of `path`. This is the only way to redo a path, and because it checks that it starts from the same state that the path started from originally, it guarantees that the operations executed while redoing the path cannot fail. KHE objects created along the path and deleted during the undo (which must have occurred in order to return the solution to its original state) are re-created during the redo with the same memory addresses as the originals.

One application of marks and paths is the conversion of a sequence of operations into an *atomic sequence*, one which is either carried out completely or not at all:

```
mark = KheMarkBegin(soln);
success = SomeSequenceOfOperations(...);
KheMarkEnd(mark, !success);
```

If the sequence of operations is successful, it remains in place; otherwise the unsuccessful sequence, or whatever part if it was completed before failure occurred, is undone. Similarly,

```
mark = KheMarkBegin(soln);
SomeSequenceOfOperations(...);
KheMarkEnd(mark, KheSolnCost(soln) >= KheMarkSolnCost(mark));
```

keeps the sequence of operations if it reduces the cost of the solution, but not otherwise.

Another application is the coordination of complex searches, such as tree searches, which try many alternatives and keep the best. Before the search begins, create a mark, and pass it to the search function, so that whenever it finds a worthwhile state it can record it in the mark by calling `KheMarkAddPath` or `KheMarkAddBestPath`. (If the initial state is a valid solution, one that the rest of the search is trying to improve on, call `KheMarkAddPath` immediately after

`KheMarkBegin`.) Within the search function, create other marks as required so that subtrees can be undone by calling `KheMarkEnd(sub_mark, true)`. At the end, all worthwhile states are paths in the original mark, where they can be examined, sorted, or whatever—like this, perhaps:

```
if( KheMarkPathCount(mark) > 0 )
  KhePathRedo(KheMarkPath(mark, 0));
KheMarkEnd(mark, false);
```

when only the best path is kept. If it is safe to redo that path, there can be nothing to undo.

Marks and paths have been implemented carefully, and their running time is small. Indeed, it is usually faster to use marks and undoing to return a solution to a previous state, than to use operations opposite to the originals. This is because `KheMarkBegin` and `KheMarkEnd` call `KheSolnMatchingMarkBegin` and `KheSolnMatchingMarkEnd` (Section 7.2), and because there is no need to check that an undo is safe, as there is when carrying out an opposite operation.

4.11. Placeholder and invalid solutions

A solution can be converted to a *placeholder solution* by calling

```
void KheSolnReduceToPlaceholder(KHE_SOLN soln);
```

This deletes everything below `soln`: all its meets, all its tasks, and so on. It cannot be undone. It reclaims a great deal of memory, which is the point of it, but it makes `soln` unusable except that the following functions remain available and return their previous values:

```
char *KheSolnDescription(KHE_SOLN soln);
void *KheSolnBack(KHE_SOLN soln);
KHE_INSTANCE KheSolnInstance(KHE_SOLN soln);
KHE_SOLN_GROUP KheSolnSolnGroup(KHE_SOLN soln);
void *KheSolnImpl(KHE_SOLN soln);
int KheSolnDiversifier(KHE_SOLN soln);
int KheSolnVisitNum(KHE_SOLN soln);
float KheSolnTimeNow(KHE_SOLN soln);
void KheSolnSetTimeLimit(KHE_SOLN soln, float limit_in_secs);
float KheSolnTimeLimit(KHE_SOLN soln);
bool KheSolnTimeLimitReached(KHE_SOLN soln);
KHE_COST KheSolnCost(KHE_SOLN soln);
```

The functions defined below within this section also remain available. For example, placeholder solutions may be used to build a table of solutions showing their costs; but they cannot be used to find cost breakdowns by constraint type, or to print timetables, and so on.

To find out whether a solution is a placeholder, function

```
bool KheSolnIsPlaceholder(KHE_SOLN soln);
```

may be called. In practice this will usually be clear anyway from the algorithmic context.

A placeholder solution can also be an *invalid solution*, meaning that it was converted to a placeholder because it was invalid. In practice, this would only happen when reading a solution

from an archive (Section 2.3). Function

```
bool KheSolnIsInvalid(KHE_SOLN soln);
```

returns `true` if `soln` is invalid, and function

```
KML_ERROR KheSolnInvalidError(KHE_SOLN soln);
```

returns the first error that rendered `soln` invalid, or `NULL` if `soln` is not invalid. For type `KML_ERROR`, see Appendix A.4.2.

Function

```
void KheSolnReduceToInvalid(KHE_SOLN soln, KML_ERROR ke);
```

may be called to convert an ordinary solution, or a non-invalid placeholder solution, into an invalid solution whose error is `ke`. This function is offered only for completeness: there seems to be no reason for the user to ever call it.

4.12. The solution invariant

Here is the condition, called the solution invariant, that every solution always satisfies. The last three rules relate to data types introduced in Chapter 5.

1. The *meet rule*: if `meet` is assigned to `target_meet` at offset `offset`, then:
   (a) The value of `offset` is at least 0 and at most the duration of `target_meet` minus the duration of `meet`;
   (b) The time domain of `target_meet`, shifted right `offset` places, is a subset of the time domain of `meet`;
2. The *task rule*: if `task` is assigned to `target_task`, then the resource domain of `target_task` is a subset of the resource domain of `task`.
3. The *cycle rule*: the parent links of nodes may not form a cycle.
4. The *node rule*: if meet `meet` is assigned to meet `target_meet` and lies in node `n`, then `n` has a parent node and `target_meet` lies in that parent node.
5. The *layer rule*: every node of a layer has the same parent node as the layer.

No sequence of operations can bring a solution to a state that violates this invariant.

Chapter 5. Extra Types for Solving

This chapter introduces four types of objects that help with solving: *nodes*, *layers*, *zones*, and *taskings*. They are an integral part of a solution, being copied when it is copied and deleted when it is deleted. But they are not part of the XML model, so their use is optional. Nodes and layers together define the *layer tree*, a data structure invented by the author [7] for use in time assignment. Zones help to make time assignments regular, and taskings are used in resource assignment.

5.1. Layer trees

The layer tree is a data structure for organizing solutions during time assignment. It supports *hierarchical timetabling*, in which meets are timetabled together into small timetables called *tiles*, the tiles are timetabled together, and so on until a complete timetable is produced. Layer trees are recommended when solving general instances, since they gracefully handle awkward cases, such as linked events whose durations differ.

Layer trees are made of *nodes*, which form a tree (actually, a forest). Each node has an optional *parent node*. The nodes with a given parent are its *children*.

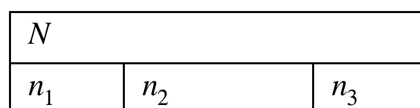
Within each node lie any number of meets. The *node rule*, part of the solution invariant (Section 4.12), imposes a structure on how the meets of nodes may be assigned: if `meet` is assigned to `target_meet` and lies in node `n`, then `n` has a parent node and `target_meet` lies in that parent node. A layer tree usually has a single root node containing the cycle meets, called the *cycle node*. If there is a cycle node, the node rule guarantees that if every non-cycle meet lying in a node is assigned to some meet, then every such meet is assigned a time.

A meet may lie in at most one node. When using layer trees, it is conventional for every meet to lie in a node except when it has received an assignment that is considered to be final. Omitting these finalized meets from nodes hides them from time assignment algorithms, which typically access meets via nodes.

When a meet splits, it is replaced in its node (if any) by the two fragments. When two meets merge, they must lie in the same node (or none), and they are replaced by the merged meet.

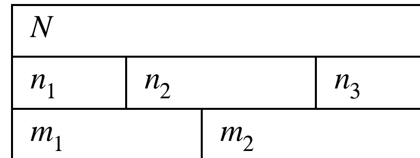
A *layer* is a subset of the children of some node with the property that none of the meets in the nodes of the layer may overlap in time. This could be for any reason, but it is usually because their meets all share a preassigned resource which possesses a required avoid clashes constraint. The property is not enforced by KHE; it is merely a convention.

Here are some examples of layer trees. The first has four nodes, N, n_1, n_2, and n_3. The n_i share a layer and are children of N, so their meets must be assigned to meets of N and should not overlap in time:



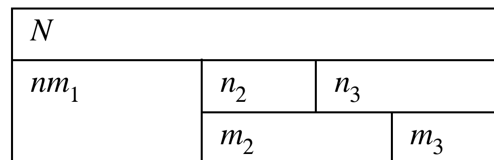
The nodes are shown as rectangles. The horizontal direction represents time. That the n_i share a layer is indicated by placing them alongside each other, and that they are children of N is indicated by placing them vertically below N.

In the next example, N has five children, lying in two layers, $\{n_1, n_2, n_3\}$ and $\{m_1, m_2\}$:



This could arise when one group of students attends the n_i while another group attends the m_i.

Finally, here is an example where a node lies in two layers (but still has only one parent):



The two layers $\{nm_1, n_2, n_3\}$ and $\{nm_1, m_2, m_3\}$ both contain node nm_1. This case arises naturally when an event (or a set of linked events) is attended by two groups of students, so that their timetables coincide at that event but may differ elsewhere.

The key operation in hierarchical timetabling is the assignment of the meets of all the children of a node to the meets of the node, so that meets that share a layer do not overlap. One way to construct a timetable is to build a single layer tree containing every meet, whose root node contains the cycle meets, and then apply this operation at each node in turn, visiting the nodes in postorder (that is, from the bottom up).

5.2. Nodes

To create a layer tree node, initially with no meets, no parent, and no children, call

```
KHE_NODE KheNodeMake(KHE_SOLN soln);
```

Its back pointer may be accessed by

```
void KheNodeSetBack(KHE_NODE node, void *back);
void *KheNodeBack(KHE_NODE node);
```

and its visit number by

```
void KheNodeSetVisitNum(KHE_NODE n, int num);
int KheNodeVisitNum(KHE_NODE n);
bool KheNodeVisited(KHE_NODE n, int slack);
void KheNodeVisit(KHE_NODE n);
void KheNodeUnVisit(KHE_NODE n);
```

as usual, and its other attributes may be retrieved by calling

```
KHE_SOLN KheNodeSoln(KHE_NODE node);
int KheNodeSolnIndex(KHE_NODE node);
```

Function KheNodeSolnIndex returns the *index number* of node, that is, the value of i for which KheSolnNode(soln, i), defined in Section 4.2, returns node. The index number may change when nodes are deleted (what actually happens is that the hole left by the deletion of a node, if not last, is plugged by the last node) so care is needed if node index numbers are stored. To visit the nodes of a solution in increasing index number order, use functions KheSolnNodeCount and KheSolnNode from Section 4.2. To delete a node, call

```
bool KheNodeDeleteCheck(KHE_NODE node);
bool KheNodeDelete(KHE_NODE node);
```

This deletes all parent-child links involving node, and deletes all meets from node (but does not delete them). It is permitted only when no meets assigned to node's meets lie in a node.

To make one node the parent of another, call

```
bool KheNodeAddParentCheck(KHE_NODE child_node, KHE_NODE parent_node);
bool KheNodeAddParent(KHE_NODE child_node, KHE_NODE parent_node);
```

These abort if child_node already has a parent; they return false and do nothing when adding the link would cause a cycle. To delete a parent-child link, call

```
bool KheNodeDeleteParentCheck(KHE_NODE child_node);
bool KheNodeDeleteParent(KHE_NODE child_node);
```

Deletion is permitted only when none of the meets of child_node is assigned. The gap created in the list of child nodes of the parent node by the deletion of child_node is filled by shuffling the following nodes down one place. To retrieve the parent of a node, call

```
KHE_NODE KheNodeParent(KHE_NODE node);
```

This returns NULL when node has no parent. Children are added and deleted, obviously, by adding and deleting parents. Functions

```
int KheNodeChildCount(KHE_NODE node);
KHE_NODE KheNodeChild(KHE_NODE node, int i);
```

visit a node's children in the usual way. There are also

```
bool KheNodeIsDescendant(KHE_NODE node, KHE_NODE ancestor_node);
bool KheNodeIsProperDescendant(KHE_NODE node, KHE_NODE ancestor_node);
```

KheNodeIsDescendant returns true when node is a descendant of ancestor_node, possibly ancestor_node itself; KheNodeIsProperDescendant returns true when node is a proper descendant of ancestor_node, that is, a descendant other than ancestor_node itself. They work in the obvious way, searching upwards from node for ancestor_node.

Several helper functions for rearranging nodes appear in Section 9.5. They are often more useful than KheNodeAddParent and KheNodeDeleteParent. Some of them call

```
void KheNodeSwapChildNodesAndLayers(KHE_NODE node1, KHE_NODE node2);
```

This function makes all the child nodes and child layers of `node1` into child nodes and child layers of `node2` and vice versa. The child nodes and layers are the exact same objects as before, stored in the same order as before; only their parent node is changed. Any assigned meets lying in child nodes of either node are unassigned (otherwise the node rule would be violated).

A meet may lie in at most one node, and function `KheMeetNode` (Section 4.8) returns the node containing a given meet, if any. To add a meet to a node and delete it, the operations are

```
bool KheNodeAddMeetCheck(KHE_NODE node, KHE_MEET meet);
bool KheNodeAddMeet(KHE_NODE node, KHE_MEET meet);
bool KheNodeDeleteMeetCheck(KHE_NODE node, KHE_MEET meet);
bool KheNodeDeleteMeet(KHE_NODE node, KHE_MEET meet);
```

`KheNodeAddMeetCheck` and `KheNodeAddMeet` abort if `meet` already lies in a node, and return `false` if it is already assigned to a meet not in the parent of `node`. `KheNodeDeleteMeetCheck` and `KheNodeDeleteMeet` abort if `meet` does not lie in `node`, and return `false` if a meet from a child of `node` is assigned to `meet`. Functions

```
int KheNodeMeetCount(KHE_NODE node);
KHE_MEET KheNodeMeet(KHE_NODE node, int i);
```

visit the meets of a node in the usual way. The order that meets are stored in nodes and returned by these functions is arbitrary, and the user can change it by calling

```
void KheNodeMeetSort(KHE_NODE node,
  int(*compar)(const void *, const void *))
```

where `compar` is a comparison function suitable for passing to `qsort`. Two such comparison functions are supplied. One sorts the meets into decreasing duration order:

```
int KheMeetDecreasingDurationCmp(const void *p1, const void *p2);
```

Here is the implementation:

```
int KheMeetDecreasingDurationCmp(const void *p1, const void *p2)
{
  KHE_MEET meet1 = * (KHE_MEET *) p1;
  KHE_MEET meet2 = * (KHE_MEET *) p2;
  if( KheMeetDuration(meet1) != KheMeetDuration(meet2) )
    return KheMeetDuration(meet2) - KheMeetDuration(meet1);
  else
    return KheMeetIndex(meet1) - KheMeetIndex(meet2);
}
```

Ties are broken by referring to the meet index. The other sorts meets by increasing value of the index of the target meet, breaking ties by increasing value of the target offset:

```
int KheMeetIncreasingAsstCmp(const void *p1, const void *p2)
```

This brings together meets whose assignments place them adjacent in time. Unassigned meets appear after assigned ones, but are not themselves sorted into any particular order.

Unlike cycle meets, which are different behind the scenes from other meets, cycle nodes are just ordinary nodes whose meets happen to be cycle meets. Accordingly, function

```
bool KheNodeIsCycleNode(KHE_NODE node);
```

merely returns `true` if `node` contains at least one meet, and its first meet is a cycle meet.

The total duration, assigned duration, and demand of the meets of `node` are returned by

```
int KheNodeDuration(KHE_NODE node);
int KheNodeAssignedDuration(KHE_NODE node);
int KheNodeDemand(KHE_NODE node);
```

The duration is kept up to date and stored in the node, so `KheNodeDuration` costs almost nothing. The other two have to sum values stored in the meets, which is slower but still fast.

Following the pattern laid down in Section 1.3, function

```
bool KheNodeSimilar(KHE_NODE node1, KHE_NODE node2);
```

returns `true` when `node1` and `node2` are similar: when they contain similar events. The exact rule is as follows. If `node1` and `node2` are the same node, they are similar. A node is *admissible* if all of its meets are derived from events, and for each event found among those meets, all of the meets of that event lie in the node. Thus, an admissible node can be considered as a set of events. Two distinct nodes are similar if they are admissible and each event in one can be matched up with a similar event in the other. The definition of similarity for events is as in Section 3.6.2.

A similar property is *regularity* (Section 5.4). Two nodes are regular when they are the same node or contain meets of equal durations and equal time domains. Function

```
bool KheNodeRegular(KHE_NODE node1, KHE_NODE node2, int *regular_count);
```

returns `true` when `node1` and `node2` are regular, and `false` otherwise. Either way, it reorders the meets of both nodes so that corresponding meets have equal durations and equal time domains, as far as possible; `*regular_count` is the number of such pairs. (So `true` is returned when `*regular_count` equals the number of meets in both nodes.)

Another function useful to solvers is

```
int KheNodeResourceDuration(KHE_NODE node, KHE_RESOURCE r);
```

This returns the total duration of meets in `node` and its descendants that contain a preassignment of `r`. If a meet contains two such preassignments, its duration is only counted once.

To make a debug print of `node` onto file `fp` with a given verbosity and indent, call

```
void KheNodeDebug(KHE_NODE node, int verbosity, int indent, FILE *fp);
```

Verbosity 1 prints just the node index number, verbosity 2 adds the duration and meets, verbosity 3 adds the node's children, and verbosity 4 adds its segments. There is also

```
void KheNodePrintTimetable(KHE_NODE node, int cell_width,
  int indent, FILE *fp);
```

which prints a timetable showing the meets of `node` across the top, and the assigned meets lying in child nodes of `node` on subsequent lines, one line per child layer. If `node` has child layers when it is called, those layers are used; otherwise `KheNodeChildLayersMake` and `KheNodeChildLayersDelete` are called to create layers at the start and delete them at the end. Parameter `cell_width` is the width of each cell, in characters.

5.3. Layers

A *layer* (not to be confused with the resource layer of Section 3.5.4) is a subset of the child nodes of some node. The intention is that the meets of a layer's nodes should not overlap in time, although this condition is not enforced.

For a given node there are two sets of layers of interest: the node's *parent layers*, which are the layers it lies in (it may lie in several), and its *child layers*, which are subsets of its child nodes. A node is a member of all of its parent layers and none of its child layers.

To create a layer of children of a given parent node, initially with no nodes, call

```
KHE_LAYER KheLayerMake(KHE_NODE parent_node);
```

It has a back pointer and a visit number, accessed by

```
void KheLayerSetBack(KHE_LAYER layer, void *back);
void *KheLayerBack(KHE_LAYER layer);

void KheLayerSetVisitNum(KHE_LAYER layer, int num);
int KheLayerVisitNum(KHE_LAYER layer);
bool KheLayerVisited(KHE_LAYER layer, int slack);
void KheLayerVisit(KHE_LAYER layer);
void KheLayerUnVisit(KHE_LAYER layer);
```

as usual. Functions

```
KHE_NODE KheLayerParentNode(KHE_LAYER layer);
int KheLayerParentNodeIndex(KHE_LAYER layer);
```

return the parent node of `layer` and the value of `i` for which `KheNodeChildLayer(KheLayerParentNode(layer), i)` returns `layer`. For convenience the solution containing it can be found by

```
KHE_SOLN KheLayerSoln(KHE_LAYER layer);
```

To delete the layer (but not its nodes), call

```
void KheLayerDelete(KHE_LAYER layer);
```

To add and delete a child node of `parent_node` from a layer, call

```
void KheLayerAddChildNode(KHE_LAYER layer, KHE_NODE node);
void KheLayerDeleteChildNode(KHE_LAYER layer, KHE_NODE node);
```

`KheLayerAddChildNode` aborts if `node`'s parent node and `layer`'s parent node are different, and `KheLayerDeleteChildNode` aborts if `node` does not lie in `layer`; otherwise, both succeed. When a child node is deleted from a layer, all later nodes are shuffled up one place to fill the gap. To visit the child nodes of a layer, call

```
int KheLayerChildNodeCount(KHE_LAYER layer);
KHE_NODE KheLayerChildNode(KHE_LAYER layer, int i);
```

To sort the child nodes of a layer, call

```
void KheLayerChildNodesSort(KHE_LAYER layer,
  int(*compar)(const void *, const void *));
```

where `compar` is a function suited to passing to `qsort` when it sorts an array of nodes.

Although much about layers is taken on trust, the *layer rule* is enforced: the parent node of each node of a layer equals the parent node of the layer. When the parent of a node is changed, the node is deleted from all the layers it lies in.

The usual reason why nodes are placed into a layer together is because their meets have one or more preassigned resources in common, and the resources have hard avoid clashes constraints, preventing the meets from overlapping in time. To document this reason when it applies, a layer contains a set of resources. To add and delete a resource from this set, the functions are

```
void KheLayerAddResource(KHE_LAYER layer, KHE_RESOURCE r);
void KheLayerDeleteResource(KHE_LAYER layer, KHE_RESOURCE r);
```

To visit this set of resources, the functions are

```
int KheLayerResourceCount(KHE_LAYER layer);
KHE_RESOURCE KheLayerResource(KHE_LAYER layer, int i);
```

There is no check that these resources are actually preassigned to the layer's meets.

When `KheLayerMake(parent_node)` is called, the resulting layer becomes a *child layer* of `parent_node`. To visit the child layers of a given node, call

```
int KheNodeChildLayerCount(KHE_NODE parent_node);
KHE_LAYER KheNodeChildLayer(KHE_NODE parent_node, int i);
```

Also,

```
void KheNodeChildLayersSort(KHE_NODE parent_node,
  int(*compar)(const void *, const void *));
```

sorts the child layers of `parent_node`, using `compar` (a function suited to passing to `qsort`) as the comparison function, and

```
void KheNodeChildLayersDelete(KHE_NODE parent_node);
```

deletes all the child layers of `parent_node`, without deleting any nodes.

When `KheLayerAddChildNode(layer, node)` is called, `layer` becomes a *parent layer* of `node`. To visit a node's parent layers, call

```
int KheNodeParentLayerCount(KHE_NODE child_node);
KHE_LAYER KheNodeParentLayer(KHE_NODE child_node, int i);
```

It is important to allow multiple parent layers in this way. For example, suppose there is one layer for the meets attended by Year 12 students and another for the meets attended by Year 11 students. If one of the Year 11 events is linked to one of the Year 12 events by a link events constraint, then there will usually be a single node whose subtree contains the meets of both events, and this node will appear in both layers. Function

```
bool KheNodeSameParentLayers(KHE_NODE node1, KHE_NODE node2);
```

returns `true` when `node1` and `node2` have the same parent layers.

Functions

```
int KheLayerDuration(KHE_LAYER layer);
int KheLayerMeetCount(KHE_LAYER layer);
```

return the total duration of `layer`'s child nodes and the number of meets in them. These values are stored in the layer and kept up to date as it changes, in the expectation that they will be used when sorting layers. Similarly,

```
int KheLayerAssignedDuration(KHE_LAYER layer);
int KheLayerDemand(KHE_LAYER layer);
```

return the total duration of the assigned meets of `layer`'s child nodes, and their total demand. These values are calculated on demand, not stored, so the functions are a bit slower. There are also set operations, implemented efficiently using bit vectors of node indexes:

```
bool KheLayerEqual(KHE_LAYER layer1, KHE_LAYER layer2);
bool KheLayerSubset(KHE_LAYER layer1, KHE_LAYER layer2);
bool KheLayerDisjoint(KHE_LAYER layer1, KHE_LAYER layer2);
bool KheLayerContains(KHE_LAYER layer, KHE_NODE node);
```

These return `true` if `layer1` and `layer2` contain the same nodes, if every node of `layer1` is a node of `layer2`, if `layer1` and `layer2` contain no nodes in common, and if `node` lies in `layer`.

Three functions offer more complex comparisons between layers:

```
bool KheLayerSame(KHE_LAYER layer1, KHE_LAYER layer2, int *same_count);
bool KheLayerSimilar(KHE_LAYER layer1, KHE_LAYER layer2,
  int *similar_count);
bool KheLayerRegular(KHE_LAYER layer1, KHE_LAYER layer2,
  int *regular_count);
```

These work in the same general way: they reorder the nodes in the two layers so that the first `*same_count` (etc.) nodes in `layer1` are equivalent in some way to the corresponding nodes in `layer2`, returning `true` if this accounts for all the nodes in both layers. `KheLayerSame` aligns nodes that are the identical same node; `KheLayerSimilar` aligns nodes that are similar, according to `KheNodeSimilar` from Section 5.2; and `KheLayerRegular` aligns nodes that are regular, according to `KheNodeRegular` from Section 5.2. If `layer1` and `layer2` are the same layer, all

three functions return `true` and set their count variable to the number of nodes in the layer. If some nodes are shared between the two layers, these are always considered equivalent and they always appear first after the layers are ordered.

These functions are implemented by calls to a more general function:

```
bool KheLayerAlign(KHE_LAYER layer1, KHE_LAYER layer2,
  bool (*node_equiv)(KHE_NODE node1, KHE_NODE node2), int *count);
```

which does the same kind of alignment, first bringing identical nodes to the front of both layers, then ordering the other nodes, calling `node_equiv` to decide whether two nodes are equivalent.

Two layers that share a common parent node may be merged:

```
void KheLayerMerge(KHE_LAYER layer1, KHE_LAYER layer2, KHE_LAYER *res);
```

The layers are deleted and replaced by layer `*res`, containing the nodes and resources of `layer1` and `layer2`. It makes sense to merge, for example, when one layer is a subset of the other.

As an aid to debugging, KHE offers function

```
void KheLayerDebug(KHE_LAYER layer, int verbosity, int indent, FILE *fp);
```

It sends a debug print of `layer` to `fp` in the usual way.

5.4. Zones

A *regular* timetable is one which has a pattern that makes it easy to understand. For example, if a train comes every 15 minutes, then that is a regular train timetable.

In high school timetabling, two forms of regularity are important. *Meet regularity* is achieved when meets which overlap in time have the same starting times and durations. It is automatic when all meets have duration 1, but not otherwise. For example, if there are two meets of duration 2, and one starts at the first time on Mondays while the second starts at the second time, that is not regular. Most instances seem to have meets of durations 1 and 2, with just a few meets of higher durations, and under those circumstances meet regularity is easy to achieve.

Node regularity is achieved when the meets of two nodes which overlap in time have the same starting times and durations. Node regularity makes a timetable easy to understand, and simplifies resource assignment by reducing the number of pairs of events whose meets overlap in time, by ensuring that they generally either overlap completely or not at all.

There seems to be little value in measuring regularity formally; the important thing is to encourage it. This is what zones are for.

For any node n, consider the set of all pairs of the form (m, o), where m is a meet lying in n, and o is a legal offset of m: if m has duration 1, o may only be 0; if m has duration 2, o may be 0 or 1; and so on. Such a pair is called a *meet-offset of n*. For example, if n contains the cycle meets, then there is a meet-offset of n for each time of the cycle.

A *zone* of node n is a subset of the meet-offsets of n. A zone may be created by calling

```
KHE_ZONE KheZoneMake(KHE_NODE node);
```

Initially it contains no meet-offsets. Functions

```
KHE_NODE KheZoneNode(KHE_ZONE zone);
int KheZoneNodeIndex(KHE_ZONE zone);
```

return `zone`'s node, which never changes, and the value of `i` for which `KheNodeZone(node, i)` returns `zone`. When a zone is deleted, the indexes of other zones in its node may change. (As usual, the gap left by the deletion of the zone is plugged by moving the last zone into it, unless the deleted zone was the last zone.) For convenience there is also

```
KHE_SOLN KheZoneSoln(KHE_ZONE zone);
```

which returns the solution containing `zone`'s node.

A zone has has the usual back pointer and visit number:

```
void KheZoneSetBack(KHE_ZONE zone, void *back);
void *KheZoneBack(KHE_ZONE zone);

void KheZoneSetVisitNum(KHE_ZONE zone, int num);
int KheZoneVisitNum(KHE_ZONE zone);
bool KheZoneVisited(KHE_ZONE zone, int slack);
void KheZoneVisit(KHE_ZONE zone);
void KheZoneUnVisit(KHE_ZONE zone);
```

A zone may be deleted by calling

```
void KheZoneDelete(KHE_ZONE zone);
```

and all the zones of a node may be deleted by calling

```
void KheNodeDeleteZones(KHE_NODE node);
```

Each meet-offset may lie in at most one zone. To add a meet-offset to a zone, and to delete a meet-offset from a zone, the operations are

```
void KheZoneAddMeetOffset(KHE_ZONE zone, KHE_MEET meet, int offset);
void KheZoneDeleteMeetOffset(KHE_ZONE zone, KHE_MEET meet, int offset);
```

To retrieve the zone of a meet-offset, call

```
KHE_ZONE KheMeetOffsetZone(KHE_MEET meet, int offset);
```

All these functions abort if `offset` is not a legal offset of `meet`. `KheZoneAddMeetOffset` also aborts if the meet-offset already lies in a zone, or `zone` is `NULL`, or `meet` does not lie in a node, or `zone` is not a zone of the node containing `meet`. `KheMeetOffsetZone` returns `NULL` if the meet-offset does not lie in any zone, as is the case by default.

The zones of a node may be accessed from the node in the usual way:

```
int KheNodeZoneCount(KHE_NODE node);
KHE_ZONE KheNodeZone(KHE_NODE node, int i);
```

They are returned in an arbitrary order. The meet-offsets of a zone may be accessed by calling

```
int KheZoneMeetOffsetCount(KHE_ZONE zone);
void KheZoneMeetOffset(KHE_ZONE zone, int i, KHE_MEET *meet, int *offset);
```

They are returned in an arbitrary order. Function

```
void KheZoneDebug(KHE_ZONE zone, int verbosity, int indent, FILE *fp);
```

produces a debug print of `zone` onto `fp` in the usual way.

When a meet is deleted from a node or deleted altogether, all the meet-offsets involving that meet are removed from their zones. When a meet is split or merged, the meet-offsets mutate in the appropriate way, but preserve their zones. For example, when a meet *m* of duration 3 is split into a meet m_1 of duration 1 and a meet m_2 of duration 2, the meet-offsets mutate as follows:

$$(m,0),(m,1),(m,2) \rightarrow (m_1,0),(m_2,0),(m_2,1)$$

Nothing constrains a zone to hold any particular meet-offsets, and indeed nothing requires zones to be created at all. The basic operations of KHE are not restricted in any way by zones. By convention only, some solvers use zones to encourage meet and node regularity. See Section 9.6 for solvers that install zones.

A useful helper function when using zones is

```
bool KheMeetMovePreservesZones(KHE_MEET meet1, int offset1,
  KHE_MEET meet2, int offset2, int durn);
```

Assuming that a meet of duration `durn` may be assigned to `meet1` at `offset1` and to `meet2` at `offset2`, this function returns `true` if that meet would be assigned to the same zones either way. It treats the `NULL` value returned at times by `KheMeetOffsetZone` as though it was a zone.

Another useful function is

```
int KheNodeIrregularity(KHE_NODE node);
```

It returns the *irregularity* of `node`: 0 if none of its meets is assigned, else the number of distinct zones of `n`'s parent node that the assigned meets of `n` are assigned to (counting `NULL` as a zone), minus one. For example, when `n`'s parent node has no zones, or all of the meets of `n` are assigned to the same zone, `n`'s irregularity is 0. One reasonable way to preserve existing regularity is to measure the irregularity of the nodes affected by an operation beforehand, measure it again afterwards, and undo the operation if irregularity has increased.

5.5. Taskings

A *tasking* is an object of type `KHE_TASKING` representing a set of tasks. A task may lie in at most one tasking at any one time. Taskings make useful parameters to resource solvers: the solver's job can be to assign resources to the tasks of the tasking—any subset of the tasks of a solution. For a deeper analysis of the role of taskings, see Section 11.3.2.

To create a tasking, initially with no tasks, call

```
KHE_TASKING KheTaskingMake(KHE_SOLN soln, KHE_RESOURCE_TYPE rt);
```

When `rt` is non-`NULL`, it signifies that all the tasks of the tasking have that type; but it may also be `NULL`, in which case there is no restriction. To retrieve the two attributes, call

```
KHE_SOLN KheTaskingSoln(KHE_TASKING tasking);
KHE_RESOURCE_TYPE KheTaskingResourceType(KHE_TASKING tasking);
```

To visit the taskings of a solution, call functions `KheSolnTaskingCount` and `KheSolnTasking` from Section 4.2. To delete a tasking, without deleting its tasks, call

```
void KheTaskingDelete(KHE_TASKING tasking);
```

To add a task to a tasking, and to delete it from a tasking, call

```
void KheTaskingAddTask(KHE_TASKING tasking, KHE_TASK task);
void KheTaskingDeleteTask(KHE_TASKING tasking, KHE_TASK task);
```

`KheTaskingAddTask` aborts if `task` already lies in a tasking, or if the resource type of `tasking` is non-`NULL` and `task` does not have that resource type. `KheTaskingDeleteTask` aborts if `task` does not lie in `tasking`. Functions

```
int KheTaskingTaskCount(KHE_TASKING tasking);
KHE_TASK KheTaskingTask(KHE_TASKING tasking, int i);
```

visit the tasks of a tasking in the usual way, and

```
void KheTaskingDebug(KHE_TASKING tasking, int verbosity,
  int indent, FILE *fp);
```

produces a debug print of `tasking`.

Chapter 6. Solution Monitoring

As a solution changes, it is continuously *monitored* by a hand-tuned constraint network.

6.1. Measuring cost

KHE measures the badness of a solution as a single integral value called the *cost*, or sometimes the *combined cost* because it includes the cost of both hard and soft constraint deviations. Storing costs in this way is convenient, because it allows costs to be assigned using `=`, added using `+`, and compared using `<` and so on in the usual way. The hard cost is shifted left by 32 bits, to ensure that it is more significant than any reasonable total soft cost, then added to the soft cost.

The type of a combined cost is `KHE_COST`, a synonym for the standard C 64-bit integer type `int64_t` (a fact best forgotten). To find the current combined cost of a solution, call

```
KHE_COST KheSolnCost(KHE_SOLN soln);
```

This value is stored explicitly in `soln`, so this function takes virtually no time to execute. Call

```
KHE_COST KheCost(int hard_cost, int soft_cost);
```

to create a combined cost. The two components of a combined cost may be accessed by

```
int KheHardCost(KHE_COST combined_cost);
int KheSoftCost(KHE_COST combined_cost);
```

There is also the constant `KheCostMax`, which returns the maximum value storable in a variable of type `KHE_COST` (a synonym for `INT64_MAX`) and the function

```
int KheCostCmp(KHE_COST cost1, KHE_COST cost2);
```

which returns an `int` which is less than, equal to, or greater than zero if the first argument is respectively less than, equal to, or greater than the second, as needed when sorting items by cost. The implementation does not make the mistake of merely subtracting `cost2` from `cost1`; the result then would be a `KHE_COST` which will usually overflow the `int` result.

The suggested way to display a combined cost is as a decimal number with the hard cost before the decimal point and the soft cost after. Five decimal places are displayed, allowing for soft costs up to 99999. Larger soft costs are displayed as 99999. To assist with this, function

```
double KheCostShow(KHE_COST combined_cost);
```

returns a value which, when printed with `printf` format `"%.5f"`, prints the cost in this format.

These functions assume that both components of the cost are non-negative. There is no problem with negative combined costs in themselves, but when a hard and soft cost are combined together, if either is negative they may be different if they are separated again.

6.2. Monitors

A *monitor* is an object, of type `KHE_MONITOR`, that monitors one part of a solution: typically, one point of application of one constraint. It contains the usual back pointer and visit number:

```
void KheMonitorSetBack(KHE_MONITOR m, void *back);
void *KheMonitorBack(KHE_MONITOR m);
void KheMonitorSetVisitNum(KHE_MONITOR m, int num);
int KheMonitorVisitNum(KHE_MONITOR m);
bool KheMonitorVisited(KHE_MONITOR m, int slack);
void KheMonitorVisit(KHE_MONITOR m);
void KheMonitorUnVisit(KHE_MONITOR m);
```

Operations

```
KHE_SOLN KheMonitorSoln(KHE_MONITOR m);
int KheMonitorSolnIndex(KHE_MONITOR m);
KHE_COST KheMonitorCost(KHE_MONITOR m);
KHE_COST KheMonitorLowerBound(KHE_MONITOR m);
```

return the enclosing solution, the index of `m` in that solution, the cost of what `m` is monitoring (kept up to date by KHE as the solution changes), and a constant lower bound on `KheMonitorCost`, which is usually 0 but will be non-zero when KHE can prove the lower bound easily.

Type `KHE_MONITOR` is the abstract supertype of many concrete subtypes, with these tags:

```
typedef enum {
  KHE_ASSIGN_RESOURCE_MONITOR_TAG,
  KHE_ASSIGN_TIME_MONITOR_TAG,
  KHE_SPLIT_EVENTS_MONITOR_TAG,
  KHE_DISTRIBUTE_SPLIT_EVENTS_MONITOR_TAG,
  KHE_PREFER_RESOURCES_MONITOR_TAG,
  KHE_PREFER_TIMES_MONITOR_TAG,
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR_TAG,
  KHE_SPREAD_EVENTS_MONITOR_TAG,
  KHE_LINK_EVENTS_MONITOR_TAG,
  KHE_ORDER_EVENTS_MONITOR_TAG,
  KHE_AVOID_CLASHES_MONITOR_TAG,
  KHE_AVOID_UNAVAILABLE_TIMES_MONITOR_TAG,
  KHE_LIMIT_IDLE_TIMES_MONITOR_TAG,
  KHE_CLUSTER_BUSY_TIMES_MONITOR_TAG,
  KHE_LIMIT_BUSY_TIMES_MONITOR_TAG,
  KHE_LIMIT_WORKLOAD_MONITOR_TAG,
  KHE_TIMETABLE_MONITOR_TAG,
  KHE_TIME_GROUP_MONITOR_TAG,
  KHE_ORDINARY_DEMAND_MONITOR_TAG,
  KHE_WORKLOAD_DEMAND_MONITOR_TAG,
  KHE_EVENNESS_MONITOR_TAG,
  KHE_GROUP_MONITOR_TAG,
  KHE_MONITOR_TAG_COUNT
} KHE_MONITOR_TAG;
```

Each monitor object contains a tag identifying its subtype, returned by

```
KHE_MONITOR_TAG KheMonitorTag(KHE_MONITOR m);
```

Monitors of the first sixteen types monitor one point of application of one constraint; their cost is the total cost of deviations at that point. They are described in detail in later sections of this chapter. Monitors of the last six types (from `KHE_TIMETABLE_MONITOR_TAG` onwards) do not monitor constraints. Timetable monitors hold the timetables of resources and events (Section 6.7); time group monitors (Section 6.8) are used within them. Ordinary and workload demand monitors monitor matchings, and evenness monitors monitor evenness (Chapter 7). Group monitors group together other monitors (Section 6.9). The last value is not a tag; it is a count of the number of monitor types, allowing code of the form

```
for( tag = 0;  tag < KHE_MONITOR_TAG_COUNT;  tag++ )
  ... do something for monitors of type tag ...
```

For those monitors that monitor a point of application of a constraint, functions

```
KHE_CONSTRAINT KheMonitorConstraint(KHE_MONITOR m);
char *KheMonitorAppliesToName(KHE_MONITOR m);
```

return the constraint and the name of the point of application (if this point is an event resource, the name of the enclosing event is returned). For other monitors they return `NULL`. Each constraint monitor also has functions which return the specific constraint and point of application.

The cost of a monitor is a function of its *deviation*, which is a non-negative integer. This value can be obtained by calling

```
int KheMonitorDeviation(KHE_MONITOR m);
char *KheMonitorDeviationDescription(KHE_MONITOR m);
```

These functions are intended for reporting, not solving. `KheMonitorDeviation` returns the deviation, and `KheMonitorDeviationDescription` returns a description of it: an expression, augmented with brief text, showing how it is calculated. The result string is stored in heap memory and may be freed by passing it to `MFree` (Appendix A.1) after use.

To visit the full set of monitors monitoring `soln`, call

```
int KheSolnMonitorCount(KHE_SOLN soln);
KHE_MONITOR KheSolnMonitor(KHE_SOLN soln, int i);
```

Although KHE does not fully specify the order in which these monitors appear, it does guarantee that the monitors which monitor constraints will appear together in the list in the order that their constraints appear in the input. It is best to select these monitors by testing whether the result of `KheMonitorConstraint` above is non-`NULL`.

To debug a monitor `m` with a given verbosity and indent, calll

```
void KheMonitorDebug(KHE_MONITOR m, int verbosity, int indent, FILE *fp);
```

The output starts with a `G`, `A` or `D` indicating whether the monitor is a group monitor, an attached non-group monitor, or a detached non-group monitor. This is followed by the number of paths

up from the monitor to the solution (Section 6.9), usually 0 or 1. Then comes the monitor's tag and cost, then other information depending on the monitor type and verbosity. There is also

```
char *KheMonitorTagShow(KHE_MONITOR_TAG tag);
```

which returns a string representation of `tag`. In practice a more useful function is

```
char *KheMonitorLabel(KHE_MONITOR m);
```

This returns `KheMonitorTagShow(KheMonitorTag(m))` if `m` is not a group monitor, and `m`'s subtag label if `m` is a group monitor.

6.3. Attaching, detaching, and provably zero fixed cost

For a monitor to be updated when the solution changes, there must be linkages from the appropriate points within the solution to the monitor. When these linkages are present, the monitor is said to be *attached to the solution*, or just *attached*. Monitors are attached to begin with, but they can be detached at any time, and even reattached later, by calling

```
void KheMonitorDetachFromSoln(KHE_MONITOR m);
void KheMonitorAttachToSoln(KHE_MONITOR m);
```

Even when detached, a monitor remembers which parts of the solution it is supposed to monitor, so the attach operation does not have to tell the monitor where to attach itself. To find out whether a monitor is currently attached or detached, call

```
bool KheMonitorAttachedToSoln(KHE_MONITOR m);
```

These three operations apply to all kinds of monitors except the group monitors of Section 6.9, to which the concept of attachment to the solution does not apply. Another function, highly recommended for calling at the end of a solve, is

```
void KheSolnEnsureOfficialCost(KHE_SOLN soln);
```

This ensures that all constraint monitors are both attached to the solution and reporting their cost to the solution, directly or indirectly via group monitors, and that all demand and evenness monitors are detached from the solution, guaranteeing that the solution cost is the official cost.

While a monitor is detached, it receives no information about changes to the solution, and, by definition, its cost is 0. Detaching a monitor may therefore change its cost. If there is a change in cost, it is reported to the monitor's parents (if it has any) as usual. Conversely, attaching a monitor brings it up to date with the current state of the solution, which again may change its cost; and again, if there is a change in cost it is reported to its parents (if it has any).

There are two main reasons for detaching a monitor. First, the user might make a deliberate choice to ignore some constraints. For example, a solver that works in two phases, first finding a solution that satisfies the hard constraints, and then attacking the soft ones, might detach the monitors for the soft constraints during its first phase. An example of this kind of deliberate choice is KHE's matching feature (Chapter 7), which is implemented with monitors. Unlike other monitors, matching monitors are detached initially. KHE makes this choice deliberately, on the grounds that the cost of the matching is not officially part of the cost function.

The second reason for detaching a monitor is that it may be clear that its cost will be zero for a long time. In that case, detaching it means that no time is spent keeping it up to date, yet it still reports the correct cost. For example, if the meets of one point of application of a link events constraint are assigned to each other and those assignments will not be removed, then it is safe to save time by detaching the corresponding monitor.

This reasoning was formerly embodied in a function called `KheMonitorAttachCheck`, which assumed that certain elements of the solution were unlikely to change, and detached monitors accordingly. `KheMonitorAttachCheck` has been withdrawn; the equivalent functionality is now obtained, more reliably, by calling the `Fix` and `UnFix` functions, as follows.

A monitor has *provably zero fixed cost* if enough of the solution is currently fixed (by calls to `KheMeetAssignFix` and `KheTaskAssignFix`) to allow KHE to prove that the monitor must have cost 0 while those fixes remain. For each kind of monitor, either a specific definition of when that kind of monitor has provably zero fixed cost is given below, or else that kind never has provably zero fixed cost.

When one of the fixing operations just listed is called, after doing the actual fixing KHE ensures that all monitors which did not have provably zero fixed cost before but now do are detached. When one of the corresponding unfix operations is called, after doing the actual unfixing it ensures that all monitors which had provably zero fixed cost before but now do not are attached. So there is no risk that detaching these monitors could lead to cost errors; as soon as unfixes make a non-zero cost possible, they are attached again.

6.4. Event monitors

An *event monitor* monitors one or more events. The set of monitors (attached or unattached) which monitor a given event may be found by calling

```
int KheSolnEventMonitorCount(KHE_SOLN soln, KHE_EVENT e);
KHE_MONITOR KheSolnEventMonitor(KHE_SOLN soln, KHE_EVENT e, int i);
```

These return the number of monitors that monitor `e` in `soln`, and the `i`th of these, as usual. The timetable monitor for event `e` (Section 6.7) is not visited by these functions; it may be retrieved by calling `KheEventTimetableMonitor`.

The total cost of these monitors measures how well `e` is timetabled. Functions

```
KHE_COST KheSolnEventCost(KHE_SOLN soln, KHE_EVENT e);
KHE_COST KheSolnEventMonitorCost(KHE_SOLN soln, KHE_EVENT e,
  KHE_MONITOR_TAG tag);
```

return the total cost of all the monitors monitoring `e`, and the total cost of all monitors monitoring `e` of a specific type, defined by `tag`. `KheSolnEventMonitorCost` returns 0 when `tag` does not specify one of the monitor types in the following subsections.

Each point of application of a spread events constraint or link events constraint is one event group, and a monitor of these kinds appears on the list of monitors of each of the events in its event group. Similarly, an order events monitor appears on the list of monitors of both of the events it monitors. If `KheSolnEventCost(soln, e)` is summed over all events, the cost of such monitors is counted repeatedly, and the total may exceed the total cost of all event monitors.

The following subsections list the various kinds of event monitors and the details specific to each of them. Their types (KHE_SPLIT_EVENTS_MONITOR and so on) may be obtained by downcasting from KHE_MONITOR after checking the type tag.

6.4.1. Split events monitors

A split events monitor has tag KHE_SPLIT_EVENTS_MONITOR_TAG and monitors an event which is one point of application of one split events constraint. Functions

```
KHE_SPLIT_EVENTS_CONSTRAINT KheSplitEventsMonitorConstraint(
  KHE_SPLIT_EVENTS_MONITOR m);
KHE_EVENT KheSplitEventsMonitorEvent(KHE_SPLIT_EVENTS_MONITOR m);
```

return the split events constraint and event being monitored, and

```
void KheSplitEventsMonitorLimits(KHE_SPLIT_EVENTS_MONITOR m,
  int *min_duration, int *max_duration, int *min_amount, int *max_amount);
```

sets the four last variables to the corresponding attributes of the monitor's constraint.

6.4.2. Distribute split events monitors

A distribute split events monitor has tag KHE_DISTRIBUTE_SPLIT_EVENTS_MONITOR_TAG and monitors one point of application of a distribute split events constraint (one event). Functions

```
KHE_DISTRIBUTE_SPLIT_EVENTS_CONSTRAINT
  KheDistributeSplitEventsMonitorConstraint(
  KHE_DISTRIBUTE_SPLIT_EVENTS_MONITOR m);
KHE_EVENT KheDistributeSplitEventsMonitorEvent(
  KHE_DISTRIBUTE_SPLIT_EVENTS_MONITOR m);
```

return the constraint and event being monitored, and

```
void KheDistributeEventsMonitorLimits(
  KHE_DISTRIBUTE_SPLIT_EVENTS_MONITOR m,
  int *duration, int *minimum, int *maximum, int *meet_count);
```

sets *duration, *minimum, and *maximum to the corresponding attributes of the monitor's constraint, and *meet_count to the number of meets derived from the monitored event whose duration is *duration (or to the total number of meets if *duration is KHE_ANY_DURATION).

6.4.3. Assign time monitors

An assign time monitor has tag KHE_ASSIGN_TIME_MONITOR_TAG and monitors an event which is one point of application of one assign time constraint. Functions

```
KHE_ASSIGN_TIME_CONSTRAINT KheAssignTimeMonitorConstraint(
  KHE_ASSIGN_TIME_MONITOR m);
KHE_EVENT KheAssignTimeMonitorEvent(KHE_ASSIGN_TIME_MONITOR m);
```

return the assign time constraint and event being monitored.

An assign time monitor does not have provably zero fixed cost when KheMeetAssignFix has been called for each of the meets derived from the event it monitors and the monitor has cost 0 when attached, because the assignments may be to other meets whose assignments are not fixed. The full assignment paths leading out of the monitored meets would need to be fixed; but that would be awkward to implement and give no efficiency payoff, because then the monitor would never be updated anyway. So an assign time monitor never has provably zero cost.

6.4.4. Prefer times monitors

A prefer times monitor has tag KHE_PREFER_TIMES_MONITOR_TAG and monitors an event which is one point of application of one prefer times constraint. Functions

```
KHE_PREFER_TIMES_CONSTRAINT KhePreferTimesMonitorConstraint(
  KHE_PREFER_TIMES_MONITOR m);
KHE_EVENT KhePreferTimesMonitorEvent(KHE_PREFER_TIMES_MONITOR m);
```

return the prefer times constraint and event being monitored.

6.4.5. Spread events monitors

A spread events monitor has tag KHE_SPREAD_EVENTS_MONITOR_TAG and monitors an event group which is one point of application of a spread events constraint. It appears in the list of monitors of all the events in its event group. Functions

```
KHE_SPREAD_EVENTS_CONSTRAINT KheSpreadEventsMonitorConstraint(
  KHE_SPREAD_EVENTS_MONITOR m);
KHE_EVENT_GROUP KheSpreadEventsMonitorEventGroup(
  KHE_SPREAD_EVENTS_MONITOR m);
```

return the spread events constraint and event group being monitored. There are also

```
int KheSpreadEventsMonitorTimeGroupCount(KHE_SPREAD_EVENTS_MONITOR m);
void KheSpreadEventsMonitorTimeGroup(KHE_SPREAD_EVENTS_MONITOR m, int i,
  KHE_TIME_GROUP *time_group, int *minimum, int *maximum, int *incidences);
```

The first returns the number of time groups (as in the corresponding constraint). The second returns the i'th time group and the minimum and maximum number of meets wanted there (again, as in the constraint), plus the current number of meets incident on that time group. If *incidences is less than *minimum or more than *maximum, a cost is incurred.

6.4.6. Link events monitors

A link events monitor has tag KHE_LINK_EVENTS_MONITOR_TAG and monitors an event group which is one point of application of a link events constraint. It appears in the list of monitors of all the events in its event group. Functions

```
KHE_LINK_EVENTS_CONSTRAINT KheLinkEventsMonitorConstraint(
  KHE_LINK_EVENTS_MONITOR m);
KHE_EVENT_GROUP KheLinkEventsMonitorEventGroup(
  KHE_LINK_EVENTS_MONITOR m);
```

return the link events constraint and event group being monitored.

A link events monitor has provably zero fixed cost when following to the end the chains of fixed assignments out of the meets of the events it monitors produces the same result for each event: the same offsets and durations within the same final meets. `KheMeetAssignFix` and `KheMeetAssignUnFix` may detach and attach link events monitors.

Detaching link events monitors is the most important service provided by fixing. Keeping these monitors up to date is slow, despite the author's best efforts to optimize. When the times of a set of linked events change together, an attached link events monitor receives the changes one by one, forcing it through a tedious sequence of cost changes beginning and ending with 0.

6.4.7. Order events monitors

An order events monitor has tag `KHE_ORDER_EVENTS_MONITOR_TAG` and monitors two events which together constitute one point of application of an order events constraint. It appears in the list of monitors of both events. Functions

```
KHE_ORDER_EVENTS_CONSTRAINT KheOrderEventsMonitorConstraint(
  KHE_ORDER_EVENTS_MONITOR m);
KHE_EVENT KheOrderEventsMonitorFirstEvent(KHE_ORDER_EVENTS_MONITOR m);
KHE_EVENT KheOrderEventsMonitorSecondEvent(KHE_ORDER_EVENTS_MONITOR m);
int KheOrderEventsMonitorMinSeparation(KHE_ORDER_EVENTS_MONITOR m);
int KheOrderEventsMonitorMaxSeparation(KHE_ORDER_EVENTS_MONITOR m);
```

return the constraint being monitored and the four attributes of the monitor: the two events monitored, and the minimum and maximum separations.

An order events monitor has provably zero fixed cost when both of its events are broken into a single meet, following to the end the chains of fixed assignments out of those two meets leads to the same final meet, and their separation (the offset into the final meet of the second meet, minus the duration plus offset into the final meet of the first meet) is in the legal range. `KheMeetAssignFix` and `KheMeetAssignUnFix` may detach and attach order events monitors.

6.5. Event resource monitors

An *event resource monitor* monitors one or more event resources. The monitors (attached or unattached) which monitor a given event resource may be visited by

```
int KheSolnEventResourceMonitorCount(KHE_SOLN soln, KHE_EVENT_RESOURCE er);
KHE_MONITOR KheSolnEventResourceMonitor(KHE_SOLN soln,
  KHE_EVENT_RESOURCE er, int i);
```

The total cost of these monitors measures how well `er` is timetabled. Functions

```
KHE_COST KheSolnEventResourceCost(KHE_SOLN soln, KHE_EVENT_RESOURCE er);
KHE_COST KheSolnEventResourceMonitorCost(KHE_SOLN soln,
  KHE_EVENT_RESOURCE er, KHE_MONITOR_TAG tag);
```

return the total cost of all the monitors monitoring `er`, and the total cost of all monitors monitoring `er` of a specific type, defined by `tag`. `KheSolnEventResourceMonitorCost` returns 0 when `tag` does not specify one of the monitor types in the following subsections.

Each point of application of an avoid split assignments constraint is a whole set of event resources, and a monitor of this kind is attached to each of the event resources in its set. If `KheSolnEventResourceCost(soln, er)` is summed over all event resources, such a monitor is counted repeatedly, so the total may exceed the total cost of all event resource monitors.

The following subsections list the various kinds of event resource monitors and the details specific to each of them. Their types (`KHE_ASSIGN_RESOURCE_MONITOR` and so on) may be obtained by downcasting from `KHE_MONITOR` after checking the type tag.

6.5.1. Assign resource monitors

An assign resource monitor has tag `KHE_ASSIGN_RESOURCE_MONITOR_TAG` and monitors an event resource which is one point of application of one assign resource constraint. Functions

```
KHE_ASSIGN_RESOURCE_CONSTRAINT KheAssignResourceMonitorConstraint(
  KHE_ASSIGN_RESOURCE_MONITOR m);
KHE_EVENT_RESOURCE KheAssignResourceMonitorEventResource(
  KHE_ASSIGN_RESOURCE_MONITOR m)
```

return the assign resource constraint and event resource being monitored. Like assign time monitors, assign resource monitors are never considered to have provably zero fixed cost.

6.5.2. Prefer resources monitors

A prefer resources monitor has tag `KHE_PREFER_RESOURCES_MONITOR_TAG` and monitors an event resource which is one point of application of one prefer resources constraint. Functions

```
KHE_PREFER_RESOURCES_CONSTRAINT KhePreferResourcesMonitorConstraint(
  KHE_PREFER_RESOURCES_MONITOR m);
KHE_EVENT_RESOURCE KhePreferResourcesMonitorEventResource(
  KHE_PREFER_RESOURCES_MONITOR m);
```

return the prefer resources constraint and event resource being monitored.

6.5.3. Avoid split assignments monitors

The operations for building avoid split assignments constraints accept a role and event groups, as required when reading XML. However, they also accept a set of event resources, and these are what are actually used. Accordingly, one avoid split assignments monitor monitors a set of event resources, and appears in the list of monitors of each of those event resources. Functions

```
KHE_AVOID_SPLIT_ASSIGNMENTS_CONSTRAINT
  KheAvoidSplitAssignmentsMonitorConstraint(
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR m)
int KheAvoidSplitAssignmentsMonitorEventGroupIndex(
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR m)
```

return the constraint and the index of the set of event resources being monitored, suitable for passing to functions `KheAvoidSplitAssignmentsConstraintEventResourceCount` and `KheAvoidSplitAssignmentsConstraintEventResource` (Section 3.7.7). There are also

```
int KheAvoidSplitAssignmentsMonitorResourceCount(
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR m);
KHE_RESOURCE KheAvoidSplitAssignmentsMonitorResource(
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR m, int i);
int KheAvoidSplitAssignmentsMonitorResourceMultiplicity(
  KHE_AVOID_SPLIT_ASSIGNMENTS_MONITOR m, int i);
```

The first returns the number of distinct resources currently assigned to tasks monitored by `m`. If `m` is a defect this number will be at least 2. The second and third return the `i`th of these distinct resources (in an arbitrary order) and the number of tasks monitored by `m` to which that resource is currently assigned. The monitor does not record which tasks those are.

An avoid split assignments monitor has provably zero fixed cost when the paths of fixed assignments leading out of the tasks it monitors have the same endpoint. `KheTaskAssignFix` and `KheTaskAssignUnFix` may detach and attach avoid split assignments monitors. Similarly to link events monitors, the efficiency payoff is significant.

6.6. Resource monitors

A *resource monitor* monitors a resource. The set of monitors (attached or unattached) which monitor a given resource may be visited by calling

```
int KheSolnResourceMonitorCount(KHE_SOLN soln, KHE_RESOURCE r);
KHE_MONITOR KheSolnResourceMonitor(KHE_SOLN soln, KHE_RESOURCE r, int i);
```

The total cost of these monitors measures how well `r` is timetabled. Functions

```
KHE_COST KheSolnResourceCost(KHE_SOLN soln, KHE_RESOURCE r);
KHE_COST KheSolnResourceMonitorCost(KHE_SOLN soln, KHE_RESOURCE r,
  KHE_MONITOR_TAG tag);
```

return the total cost of all the monitors monitoring `r`, and the total cost of all monitors monitoring `r` of a specific type, defined by `tag`. `KheSolnResourceMonitorCost` returns 0 when `tag` does not specify one of the monitor types in the following subsections.

The following subsections list the kinds of resource monitors and their features. Their types (`KHE_AVOID_CLASHES_MONITOR` etc.) may be obtained by downcasting from `KHE_MONITOR` after checking the type tag. Monitors of type `KHE_WORKLOAD_DEMAND_MONITOR`, defined in Section 7.4, are also visited by `KheSolnResourceMonitorCount` and `KheSolnResourceMonitor`. However, the timetable monitor for a resource is not visited by these functions; as explained in

Section 6.7, it is retrieved by calling KheResourceTimetableMonitor.

6.6.1. Avoid clashes monitors

An avoid clashes monitor has tag KHE_AVOID_CLASHES_MONITOR_TAG and monitors a resource which is one point of application of one avoid clashes constraint. Functions

```
KHE_AVOID_CLASHES_CONSTRAINT KheAvoidClashesMonitorConstraint(
  KHE_AVOID_CLASHES_MONITOR m);
KHE_RESOURCE KheAvoidClashesMonitorResource(
  KHE_AVOID_CLASHES_MONITOR m);
```

return the avoid clashes constraint and resource being monitored.

An avoid clashes monitor m may have non-zero KheMonitorLowerBound(m). Let t be the total duration of the events to which m's resource is preassigned which either have preassigned times or are subject to an assign time constraint of weight greater than m's weight. Then if t exceeds the number of times in the cycle, the excess is a lower bound on the number of defects that m must have in any reasonable solution (one in which violations of m are preferred to violations of the more expensive assign time constraints). Converting this number of defects into a cost using m's cost function in the usual way gives the lower bound.

6.6.2. Avoid unavailable times monitors

This monitor has tag KHE_AVOID_UNAVAILABLE_TIMES_MONITOR_TAG and monitors a resource which is one point of application of one avoid unavailable times constraint. Functions

```
KHE_AVOID_UNAVAILABLE_TIMES_CONSTRAINT
  KheAvoidUnavailableTimesMonitorConstraint(
  KHE_AVOID_UNAVAILABLE_TIMES_MONITOR m);
KHE_RESOURCE KheAvoidUnavailableTimesMonitorResource(
  KHE_AVOID_UNAVAILABLE_TIMES_MONITOR m);
```

return the avoid unavailable times constraint and resource being monitored.

An avoid unavailable times monitor m may have non-zero KheMonitorLowerBound(m). Suppose m's resource is subject to an avoid clashes constraint of weight greater than m's weight. Let t_1 be the total duration of the events to which m's resource is preassigned which either have preassigned times or are subject to an assign time constraint of weight greater than m's weight. Let t_2 be the number of times to be avoided according to m. Then if $t_1 + t_2$ exceeds the number of times in the cycle, the excess is a lower bound on the number of defects that m must have in any reasonable solution (one in which every meet is assigned a time, and violations of m are preferred to violations of the more expensive assign time and avoid clashes constraints). Converting this number of defects into a cost using m's cost function in the usual way gives the lower bound.

6.6.3. Limit idle times monitors

A limit idle times monitor has tag KHE_LIMIT_IDLE_TIMES_MONITOR_TAG and monitors a resource which is one point of application of one limit idle times constraint. Functions

```
KHE_LIMIT_IDLE_TIMES_CONSTRAINT KheLimitIdleTimesMonitorConstraint(
  KHE_LIMIT_IDLE_TIMES_MONITOR m);
KHE_RESOURCE KheLimitIdleTimesMonitorResource(
  KHE_LIMIT_IDLE_TIMES_MONITOR m);
```

return the limit idle times constraint and resource being monitored, and

```
int KheLimitIdleTimesMonitorTimeGroupMonitorCount(
  KHE_LIMIT_IDLE_TIMES_MONITOR m);
KHE_TIME_GROUP_MONITOR KheLimitIdleTimesMonitorTimeGroupMonitor(
  KHE_LIMIT_IDLE_TIMES_MONITOR m, int i);
```

visit the time group monitors (Section 6.8) that `m` monitors, one for each time group in the limit idle times constraint. These can be used to find out which time groups contain idle times.

6.6.4. Cluster busy times monitors

A cluster busy times monitor has tag `KHE_CLUSTER_BUSY_TIMES_MONITOR_TAG` and monitors a resource which is one point of application of one cluster busy times constraint. Functions

```
KHE_CLUSTER_BUSY_TIMES_CONSTRAINT KheClusterBusyTimesMonitorConstraint(
  KHE_CLUSTER_BUSY_TIMES_MONITOR m);
KHE_RESOURCE KheClusterBusyTimesMonitorResource(
  KHE_CLUSTER_BUSY_TIMES_MONITOR m);
```

return the cluster busy times constraint and resource being monitored. Function

```
void KheClusterBusyTimesMonitorBusyGroupCount(
  KHE_CLUSTER_BUSY_TIMES_MONITOR m,
  int *busy_group_count, int *minimum, int *maximum);
```

sets `*busy_group_count` to the number of busy time groups, and `*minimum` and `*maximum` to the `Minimum` and `Maximum` attributes of the cluster busy times constraint. If `m` has non-zero cost, then `*busy_group_count < *minimum` or `*busy_group_count > *maximum`. Functions

```
int KheClusterBusyTimesMonitorTimeGroupMonitorCount(
  KHE_CLUSTER_BUSY_TIMES_MONITOR m);
KHE_TIME_GROUP_MONITOR KheClusterBusyTimesMonitorTimeGroupMonitor(
  KHE_CLUSTER_BUSY_TIMES_MONITOR m, int i);
```

visit the time group monitors (Section 6.8) that `m` monitors, one for each time group in the cluster busy times constraint. These can be used to find out which time groups are busy.

6.6.5. Limit busy times monitors

A limit busy times monitor has tag `KHE_LIMIT_BUSY_TIMES_MONITOR` and monitors a resource which is one point of application of one limit busy times constraint. Functions

```
KHE_LIMIT_BUSY_TIMES_CONSTRAINT KheLimitBusyTimesMonitorConstraint(
  KHE_LIMIT_BUSY_TIMES_MONITOR m);
KHE_RESOURCE KheLimitBusyTimesMonitorResource(
  KHE_LIMIT_BUSY_TIMES_MONITOR m);
```

return the limit busy times constraint and resource being monitored. Functions

```
int KheLimitBusyTimesMonitorDefectiveTimeGroupCount(
  KHE_LIMIT_BUSY_TIMES_MONITOR m);
void KheLimitBusyTimesMonitorDefectiveTimeGroup(
  KHE_LIMIT_BUSY_TIMES_MONITOR m, int i, KHE_TIME_GROUP *tg,
  int *busy_count, int *minimum, int *maximum);
```

visit the time groups monitored by `m` that are currently defective, in unspecified order. For each `i`, `*tg` is set to one defective time group, `*busy_count` is set to the number of times `m`'s resource is busy during `*tg`, and `*minimum` and `*maximum` are set to the minimum and maximum values from the constraint; so either the resource is underloaded during `*tg` and `*busy_count < *minimum`, or the resource is overloaded during `*tg` and `*busy_count > *maximum`.

Limit busy times monitors contain a `ceiling` attribute, set and retrieved by

```
void KheLimitBusyTimesMonitorSetCeiling(KHE_LIMIT_BUSY_TIMES_MONITOR m,
  int ceiling);
int KheLimitBusyTimesMonitorCeiling(KHE_LIMIT_BUSY_TIMES_MONITOR m);
```

When `busy_count > ceiling`, the usual formula is overridden: the deviation is 0. For why this might be useful, consult Section 12.7.2. The default value of `ceiling` is `INT_MAX`, which effectively turns it off. If `m` is attached when `KheLimitBusyTimesMonitorSetCeiling` is called, it will be detached and reattached by the call.

A limit busy times monitor `m` may have non-zero `KheMonitorLowerBound(m)`. Suppose `m`'s resource is subject to an avoid clashes constraint of weight greater than `m`'s weight. Let t_1 be the total duration of the events to which `m`'s resource is preassigned which either have preassigned times or are subject to an assign times constraint of weight greater than `m`'s weight. Let t_2 be the number of times in the cycle minus the number of times in `m`'s constraint's domain. Then at least $t_1 - t_2$ of the times of the events preassigned to `m`'s resource must occur in time groups limited by `m`. If this exceeds the number of time groups in `m`'s constraint times its `Maximum` attribute, then the excess, converted into a cost using `m`'s cost function in the usual way, gives the lower bound.

6.6.6. Limit workload monitors

A limit workload monitor has tag `KHE_LIMIT_WORKLOAD_MONITOR` and monitors a resource which is one point of application of one limit workload constraint. Functions

```
KHE_LIMIT_WORKLOAD_CONSTRAINT KheLimitWorkloadMonitorConstraint(
  KHE_LIMIT_WORKLOAD_MONITOR m);
KHE_RESOURCE KheLimitWorkloadMonitorResource(
  KHE_LIMIT_WORKLOAD_MONITOR m);
float KheLimitWorkloadMonitorWorkload(KHE_LIMIT_WORKLOAD_MONITOR m);
```

return the limit workload constraint, the monitored resource, and its current workload; and

```
void KheLimitWorkloadMonitorWorkloadAndLimits(
  KHE_LIMIT_WORKLOAD_MONITOR m, float *workload,
  int *minimum, int *maximum);
```

also returns the workload, plus the minimum and maximum values from the constraint.

Limit workload monitors contain a `ceiling` attribute, set and retrieved by

```
void KheLimitWorkloadMonitorSetCeiling(KHE_LIMIT_WORKLOAD_MONITOR m,
  int ceiling);
int KheLimitWorkloadMonitorCeiling(KHE_LIMIT_WORKLOAD_MONITOR m);
```

When `busy_count > ceiling`, the usual formula is overridden: the deviation is 0. For why this might be useful, consult Section 12.7.2. The default value of `ceiling` is `INT_MAX`, which effectively turns it off. If `m` is attached when `KheLimitWorkloadMonitorSetCeiling` is called, it will be detached and reattached by the call.

A limit workload monitor `m` may have non-zero `KheMonitorLowerBound(m)`. Add up the workloads of the tasks to which `m`'s resource is preassigned. If this exceeds the maximum of the corresponding limit workload constraint, converting the excess into a cost using `m`'s cost function in the usual way gives the lower bound.

6.7. Timetable monitors

A *timetable* is a record of what is going on at each time. As part of monitoring cost, KHE monitors the timetable of each resource and each event. Function

```
KHE_TIMETABLE_MONITOR KheResourceTimetableMonitor(KHE_SOLN soln,
  KHE_RESOURCE r);
```

returns the timetable monitor of resource `r`, and

```
KHE_TIMETABLE_MONITOR KheEventTimetableMonitor(KHE_SOLN soln,
  KHE_EVENT e);
```

returns the timetable monitor of event `e`. Type `KHE_TIMETABLE_MONITOR` is a subtype of type `KHE_MONITOR` with tag `KHE_TIMETABLE_MONITOR_TAG`. The cost of a timetable monitor is always 0, so it never appears in any list of defects.

When a timetable monitor is attached, a particular set of meets is known to it at any moment. For a resource timetable monitor it is the set of meets that are assigned a time and the resource. For an event timetable monitor it is the set of meets derived from the event that are assigned a time. The monitor offers these operations, which report which meets are running at each time:

```
int KheTimetableMonitorTimeMeetCount(KHE_TIMETABLE_MONITOR tm,
  KHE_TIME time);
KHE_MEET KheTimetableMonitorTimeMeet(KHE_TIMETABLE_MONITOR tm,
  KHE_TIME time, int i);
```

`KheTimetableMonitorTimeMeetCount` returns the number of known meets running at `time`,

and `KheTimetableMonitorTimeMeet` returns the `i`th of these meets. Closely related to them is

```
bool KheTimetableMonitorTimeAvailable(KHE_TIMETABLE_MONITOR tm,
  KHE_MEET meet, KHE_TIME time);
```

which returns `true` if moving `meet` within `tm`, or adding it to `tm`, so that its starting time is `time`, would neither place `meet` partly off the end of the timetable nor cause clashes.

A timetable monitor offers no operations which report its set of meets directly. For event timetables one can use functions `KheEventMeetCount` and `KheEventMeet` from Section 4.2 to obtain the meets derived from a particular event; the timetabled meets are just those with an assigned time. For resource timetables one can use `KheResourceAssignedTaskCount` and `KheResourceAssignedTask` from Section 4.9.1 to obtain all the tasks assigned the resource; the timetabled ones are just those whose enclosing meet has an assigned time.

The condition `KheTimetableMonitorTimeMeetCount(tm, time) >= 2` is true at each time when `tm` has a clash. To find out quickly which times these are, use

```
int KheTimetableMonitorClashingTimeCount(KHE_TIMETABLE_MONITOR tm);
KHE_TIME KheTimetableMonitorClashingTime(KHE_TIMETABLE_MONITOR tm, int i);
```

They return all times such that `tm` has a clash at that time, not in chronological order.

As usual, timetable monitors are created by `KheSolnMake` and exist for as long as the solution does. There is one for each resource, and one for each event. Unlike other monitors, however, timetable monitors are not attached initially. It is possible for the timetable returned by `KheResourceTimetableMonitor` or `KheEventTimetableMonitor` to be unattached and so not up to date (it will be empty in that case). It can be brought up to date by attaching it.

Link events monitors (but not spread events monitors) depend on event timetable monitors. All resource monitors except limit workload monitors depend on resource timetable monitors. When a monitor is attached, any unattached timetable monitor(s) it depends on are also attached. When the last monitor that depends on some timetable monitor is detached, that timetable monitor is detached. Thus, unless the user chooses to attach a timetable monitor directly, timetable monitors are attached only as needed by other monitors. Detaching a timetable monitor causes KHE to abort unless no attached monitors depend on it.

Although it would make sense to treat a timetable monitor as a group monitor, that option is not offered. The user who wants all the problems associated with a single resource or event to be channelled through a single monitor must create a group monitor, separate from the timetable monitor, and add the appropriate monitors to it in the usual way.

Timetable monitors may be debugged by calling `KheMonitorDebug` as usual. And

```
void KheTimetableMonitorPrintTimetable(KHE_TIMETABLE_MONITOR tm,
  int cell_width, int indent, FILE *fp);
```

prints a conventional tabular timetable, using `Days` and possibly `Weeks` time groups from the instance to determine its shape. Parameter `cell_width` is the width of each cell, in characters.

6.8. Time group monitors

A *time group* monitor is a monitor associated with one timetable monitor. It monitors what is happening at the times of its time group within the timetable; specifically, it keeps track of how many of the times of the time group are busy in that timetable (occupied by at least one meet). It also keeps track of how many idle times the time group contains, but only if there is a limit idle times monitor in the vicinity that needs to know.

Time group monitors are created and attached by KHE as required, and it is best not to meddle with that. However, there is no problem with retrieving information from them:

```
KHE_TIMETABLE_MONITOR KheTimeGroupMonitorTimetableMonitor(
  KHE_TIME_GROUP_MONITOR m);
KHE_TIME_GROUP KheTimeGroupMonitorTimeGroup(KHE_TIME_GROUP_MONITOR m);
int KheTimeGroupMonitorBusyCount(KHE_TIME_GROUP_MONITOR m);
```

These return `m`'s associated timetable monitor, the time group that `m` monitors, and the number of busy times in that time group.

When a limit idle times monitor is attached which monitors `tgm`'s time group within `tgm`'s timetable monitor, two other functions related to idle times calculations may be called:

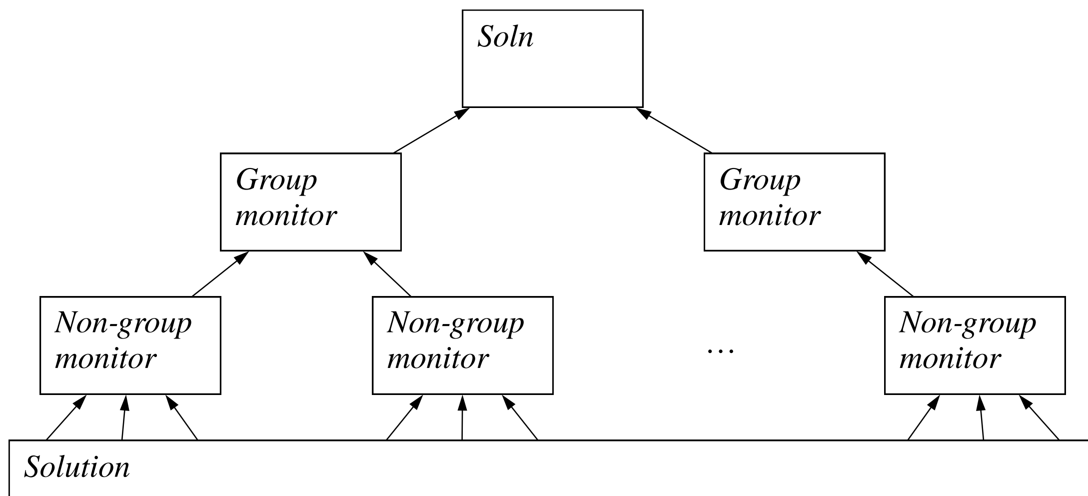
```
int KheTimeGroupMonitorIdleCount(KHE_TIME_GROUP_MONITOR m);
void KheTimeGroupMonitorFirstAndLastBusyTimes(
  KHE_TIME_GROUP_MONITOR tgm, KHE_TIME times[2], int *count);
```

The first returns the number of idle times. The second places the first and last busy times into `times`, and sets `*count` to the number of times it placed there. If there are no busy times, `*count` is 0; if there is one busy time, `*count` is 1; else `*count` is 2. This specification does not refer to idle times, but nevertheless the function will abort if there is no limit idle times monitor nearby.

6.9. Group monitors

Sometimes the cost of a *single* monitor is needed: for example, when reporting problems to the user. And the total cost of *all* monitors is always needed, since that is the cost of the solution.

Sometimes something in between these two extremes is needed: the cost of a set of related monitors. To support this, the monitors of a solution are organized into a directed acyclic graph, or *dag* for short, of arbitrary depth. Each monitor reports its cost to its parent monitors. The dag is often a tree, in which case the picture looks like this:



The leaves are the *non-group monitors*, the various monitors described previously which monitor the solution directly. The internal nodes are called *group monitors*, because they monitor a set of monitors (their children). The cost of a group monitor is the sum of the costs of its children.

The solution object itself is a group monitor (initially, the only one). It supports all the group monitor operations, plus the many other operations described earlier.

Group monitors have type `KHE_GROUP_MONITOR`, a concrete subtype of `KHE_MONITOR`, like `KHE_ASSIGN_TIME_MONITOR` etc. `KHE_GROUP_MONITOR` is a supertype of `KHE_SOLN`, so upcast

```
(KHE_GROUP_MONITOR) soln
```

is safe, although often unnecessary, since many operations on type `KHE_GROUP_MONITOR` have `KHE_SOLN` versions. For example, since `KHE_GROUP_MONITOR` is itself a subtype of `KHE_MONITOR`, the total cost of all monitors could be found by calling

```
KheMonitorCost((KHE_MONITOR) soln)
```

but of course the equivalent `KHE_SOLN` version, `KheSolnCost`, is easier to use.

When the solution changes at some point, the change is reported to the non-group monitors that monitor that point. Each updates its cost and reports any change to its parents, which update their cost and report to their parents, and so on until there are no parents. The dag usually has a single root, the solution object itself, but it does not have to be that way, because the links that join non-group and group monitors to their parent monitors can be added and deleted at will.

6.9.1. Basic operations on group monitors

Unlike other types of monitors, group monitors other than the solution object can be freely created and deleted. Function

```
KHE_GROUP_MONITOR KheGroupMonitorMake(KHE_SOLN soln, int sub_tag,
  char *sub_tag_label)
```

creates a new group monitor with no parents and no children. It is passed the solution as a parameter, and it remembers it, but it is not made a child of it. Functions

```
int KheGroupMonitorSubTag(KHE_GROUP_MONITOR gm);
char *KheGroupMonitorSubTagLabel(KHE_GROUP_MONITOR gm);
```

return the `sub_tag` and `sub_tag_label` attributes of `gm`. These are used to distinguish kinds of group monitors. If `sub_tag_label` is non-`NULL`, it is printed when debugging. The values of these attributes in solution objects are `-1` and `"Soln"`. The term 'sub-tag' is used because group monitors already have a tag attribute, whose value is `KHE_GROUP_MONITOR_TAG`.

A group monitor other than the solution object may be deleted by calling

```
void KheGroupMonitorDelete(KHE_GROUP_MONITOR gm);
```

Its children will no longer have it as a parent, and its parents will no longer have it as a child. For each parent of `gm`, the hole in the parent's list of child monitors is plugged by moving the last child monitor to `gm`'s position. For each child of `gm`, the hole in the child's list of parent monitors is plugged by moving the last parent monitor to `gm`'s position.

Every group monitor can have any number of child monitors, and every monitor (group or non-group) can have any number of parent monitors. Even the solution object can have parents, allowing monitoring of the total cost of a set of solutions. The operations for adding children to a group monitor and removing them are

```
void KheGroupMonitorAddChildMonitor(KHE_GROUP_MONITOR gm, KHE_MONITOR m);
void KheGroupMonitorDeleteChildMonitor(KHE_GROUP_MONITOR gm, KHE_MONITOR m);
```

Here `m` could be a non-group monitor or a group monitor. `KheGroupMonitorAddChildMonitor` makes `m` a child of `gm`, and `gm` a parent of `m`. It aborts if this would create a cycle in the dag (only possible when `m` is a group monitor). `KheGroupMonitorDeleteChildMonitor` removes `m` from `gm`, leaving `m` with one less parent and `gm` with one less child. The resulting holes are plugged as described above for deleting group monitors. It aborts if `m` is not a child of `gm`. There is also

```
bool KheGroupMonitorHasChildMonitor(KHE_GROUP_MONITOR gm, KHE_MONITOR m);
```

which returns `true` when `m` is a child of `gm`. It is useful when `m` may already be a child of `gm`:

```
if( !KheGroupMonitorHasChildMonitor(gm, m) )
  KheGroupMonitorAddChildMonitor(gm, m);
```

No-one is checking that one monitor does not become the child of another twice over; and if it does, its cost will be counted twice in the cost of its parent.

For a group monitor `m`, `KheLowerBound(m)` is the sum of the lower bounds of `m`'s children. It may increase when a descendant is added, and decrease when a descendant is removed.

Initially, all non-group monitors are made children of the solution object, and all of them except demand monitors are attached to the solution, so that `KheSolnCost` is the total cost of all non-demand monitors, which is indeed the cost of the solution. Care is needed when grouping not to inadvertently disconnect monitors from the solution, since then their costs will not be counted, or to connect them via multiple paths, since then their costs will be counted multiple times. It is usually best to make a new group monitor a child of the solution immediately:

```
gm = KheGroupMonitorMake(soln, sub_tag, sub_tag_label);
KheGroupMonitorAddChildMonitor((KHE_GROUP_MONITOR) soln,
  (KHE_MONITOR) gm);
```

And when deleting a group monitor, the best option may be helper function

```
void KheGroupMonitorBypassAndDelete(KHE_GROUP_MONITOR gm);
```

It calls `KheGroupMonitorDelete`, but first it makes `gm`'s children into children of `gm`'s parents, if any, thus keeping them linked in. There is also

```
void KheSolnBypassAndDeleteAllGroupMonitors(KHE_SOLN soln);
```

which applies `KheGroupMonitorBypassAndDelete` to every group monitor of `soln`.

Functions

```
int KheGroupMonitorChildMonitorCount(KHE_GROUP_MONITOR gm);
KHE_MONITOR KheGroupMonitorChildMonitor(KHE_GROUP_MONITOR gm, int i);
```

visit the child monitors of group monitor `gm` in the usual way. If `gm` is the solution object, these versions of the functions allow the user to avoid the upcast:

```
int KheSolnChildMonitorCount(KHE_SOLN soln);
KHE_MONITOR KheSolnChildMonitor(KHE_SOLN soln, int i);
```

Functions

```
int KheMonitorParentMonitorCount(KHE_MONITOR m);
KHE_GROUP_MONITOR KheMonitorParentMonitor(KHE_MONITOR m, int i);
```

visit the parent monitors of `m`. There is also

```
bool KheMonitorDescendant(KHE_MONITOR m1, KHE_MONITOR m2);
```

which returns `true` if `m1` is a descendant of `m2`, including when the two are equal.

6.9.2. Defects

Informally, a defect is a specific problem with a solution. In KHE, the word has a formal meaning as well: a *defect* is a monitor whose cost is non-zero.

It can be helpful to target defects directly, rather than wasting time changing parts of the solution where there are no defects. This is especially the case near the end of the solve process, when there may be thousands of monitors but only a handful of defects. To support this, KHE offers fast access to those child monitors of a group monitor which are defects:

```
int KheGroupMonitorDefectCount(KHE_GROUP_MONITOR gm);
KHE_MONITOR KheGroupMonitorDefect(KHE_GROUP_MONITOR gm, int i);
```

When a monitor's cost changes from zero to non-zero, the monitor is added to its parents' defect lists; and when its cost changes from non-zero to zero it is removed. These updates take a constant and negligible amount of time per parent. When the group monitor is the solution object

there are convenience versions:

```
int KheSolnDefectCount(KHE_SOLN soln);
KHE_MONITOR KheSolnDefect(KHE_SOLN soln, int i);
```

There is also

```
void KheGroupMonitorDefectDebug(KHE_GROUP_MONITOR gm,
  int verbosity, int indent, FILE *fp);
```

which is like `KheMonitorDebug` applied to `gm`, except that it prints only the defective children.

If a solution is changed and then changed back again to its original state, its cost returns to its original value, but there are two ways in which its defects can be different. First, they may appear in a different order. Second, although the number of defects which are demand monitors (Chapter 7) must return to its original value, the demand monitors that make up that number may change. This is because there are many maximum matchings in general, and KHE does not guarantee to find any particular one of them.

In practice, one wants to traverse a list of defects and try to repair them. Quite commonly, an attempt to repair a defect will remove it temporarily and then reinstate it if the repair was not successful. This will cause the defect to be shifted to the end of the defect list. A simple traversal of the defects from first to last will visit some defects more than once, and others not at all. To handle this problem, it is necessary to make a copy of the defects and traverse the copy. Although every defect will have non-zero cost at the time it is copied, as the list is traversed, after the solution changes or if the list includes demand monitors, one cannot assume that every monitor on the copy list will have non-zero cost.

To find the total cost of all monitors of a given type in the descendants of `gm`, call

```
KHE_COST KheGroupMonitorCostByType(KHE_GROUP_MONITOR gm,
  KHE_MONITOR_TAG tag, int *defect_count);
```

It returns the number of defects, in `*defect_count`, as well as the cost. It traverses the whole sub-dag of monitors of `gm` (actually, just the defects), so it is slow: it is intended for reporting, not for solving. It returns `0` when `tag` is `KHE_GROUP_MONITOR_TAG`, because it attributes cost to the monitors that originally generated it. Version

```
KHE_COST KheSolnCostByType(KHE_SOLN soln, KHE_MONITOR_TAG tag,
  int *defect_count);
```

may be called when the group monitor is the solution object. The values returned by these functions are displayed in a convenient tabular form by functions

```
void KheGroupMonitorCostByTypeDebug(KHE_GROUP_MONITOR gm,
  int verbosity, int indent, FILE *fp);
void KheSolnCostByTypeDebug(KHE_SOLN soln,
  int verbosity, int indent, FILE *fp);
```

which print one line for each kind of monitor under `gm` or `soln` for which there are defects.

6.9.3. Tracing

Sometimes a solver needs to know which monitors have experienced a change in cost recently. Ejection chain solvers, for example, need this information, and *monitor tracing* provides it.

Tracing involves objects of type `KHE_TRACE`. To create one, call

```
KHE_TRACE KheTraceMake(KHE_GROUP_MONITOR gm);
```

where `gm` is the group monitor to be traced. The solution may be traced by upcasting it:

```
t = KheTraceMake((KHE_GROUP_MONITOR) soln);
```

The group monitor that a trace object is for can be found by calling

```
KHE_GROUP_MONITOR KheTraceGroupMonitor(KHE_TRACE t);
```

To delete a trace object, call

```
void KheTraceDelete(KHE_TRACE t);
```

This will call `KheTraceEnd(t)` below if needed. KHE keeps a free list of trace objects in the solution object, so many trace objects can be created and deleted at virtually no cost.

Actual tracing is initiated and ended by calling

```
void KheTraceBegin(KHE_TRACE t);
void KheTraceEnd(KHE_TRACE t);
```

These must be called in matching pairs. `KheTraceBegin` removes any information left over from any preceding trace, and attaches `t` to its group monitor so that it can record what happens. `KheTraceEnd` detaches `t` from its group monitor. Different trace objects may be attached and detached quite independently of each other, even when they have the same group monitor.

After the trace ends, the following functions may be called:

```
KHE_COST KheTraceInitCost(KHE_TRACE t);
int KheTraceMonitorCount(KHE_TRACE t);
KHE_MONITOR KheTraceMonitor(KHE_TRACE t, int i);
KHE_COST KheTraceMonitorInitCost(KHE_TRACE t, int i);
```

`KheTraceInitCost` returns the initial cost of `t`'s group monitor (at the time the trace began); `KheTraceMonitorCount` returns the number of child monitors of `t`'s group monitor whose cost changed during the trace; `KheTraceMonitor` returns the `i`th of these child monitors; and `KheTraceMonitorInitCost(t, i)` returns the initial cost of `KheTraceMonitor(t, i)`.

These functions may be called during a trace as well as after it, returning values as though the trace had just ended. While it is not an error to call `KheGroupMonitorAddChildMonitor` or `KheGroupMonitorDeleteChildMonitor` while tracing the group monitor concerned, it is not recommended. A solution cannot be copied while one of its group monitors is being traced.

Chapter 7. Matchings and Evenness

Suppose a decision is made to run five Music meets simultaneously, when the school has only two Music teachers and two Music rooms. Clearly, when teachers and rooms are assigned later, there will be major problems, but until then the usual cost function will not reveal any problems.

More subtly, suppose there are eight teachers, and that three of them teach English only, three teach History only, and two teach both. Suppose a decision is make to run five English meets and five History meets simultaneously. Then there are enough English teachers to teach the five English meets, and there are enough History teachers to teach the five History meets, but there are not enough English and History teachers, taken together, to teach the ten meets.

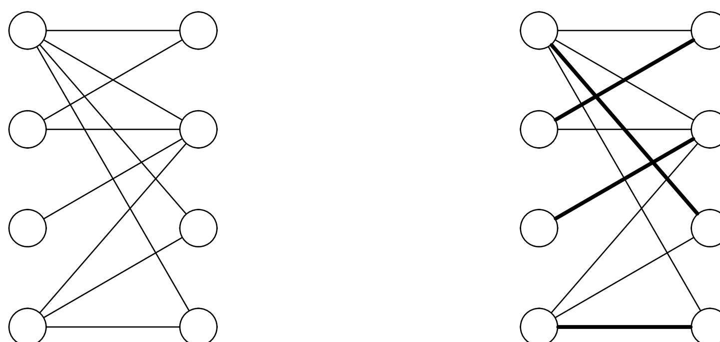
Matchings (officially, *unweighted bipartite matchings*) detect such problems. Although not compulsory, they are often helpful. This chapter describes them in general, how they apply to timetabling, and how to use them in KHE. Getting started can be as simple as calling

```
KheSolnMatchingBegin(soln);
KheSolnMatchingSetWeight(soln, KheCost(1, 0));
KheSolnMatchingAddAllWorkloadRequirements(soln);
KheSolnMatchingAttachAllOrdinaryDemandMonitors(soln);
```

after the solution is made a complete representation.

7.1. The bipartite matching problem

A *bipartite graph* is an undirected graph whose nodes are divided into two sets, such that every edge connects a node of one set to a node of the other. A *matching* is a subset of the edges such that no two edges touch the same node. A *maximum matching* is a matching containing as many edges as possible. The *bipartite matching problem* is the problem of finding a maximum matching in a bipartite graph. For example, here is a bipartite graph (at left), and the same graph with a maximum matching shown in bold (at right):



There is a standard polynomial-time algorithm for this problem.

In timetabling, where bipartite matching has been used for many years [2, 4, 13], it is usual for one of the two sets of nodes to represent variables (slots, events, etc.) demanding something

to be assigned to them, while the other set represents values (times, resources, etc.) which are available to supply these demands. Accordingly, these two sets will be referred to as the *demand nodes* and the *supply nodes*. A maximum matching assigns supply nodes to as many demand nodes as possible, given that each demand node requires any one of the supply nodes it is connected to, and each supply node may be assigned to at most one demand node. Although the problem is formally symmetrical between the two kinds of nodes, since each edge touches one of each, in timetabling it is not symmetrical. For example, it does not matter if some supply nodes are not matched, but it does matter if some demand nodes are not matched.

It is usually not a good idea to make the assignments indicated by a maximum matching, because there are other constraints not modelled by the matching, and it is desirable to find, not just any maximum matching, but one that satisfies these other constraints. The better way to use a maximum matching is as a monitor of the current state. Because the matching is maximum, it indicates that there must be at least a certain number of problems, in the form of unassigned demand nodes, in any solution incorporating the decisions already made, and that is valuable information when evaluating those decisions.

Some applications of matching to timetabling utilize the idea of a *tixel*, the author's term for one resource at one time (the name recalls the *pixel* of computer graphics). For example, teacher Smith during the first time on Mondays is one tixel; it may be represented by the ordered pair

(*Smith*, *Mon1*)

This is also called a *supply tixel*, because it can supply the demands of events for teachers. The events are said to contain *demand tixels*. For example, an event of duration 2 which requests student group 8*A*, one English teacher, and one room, is said to contain six demand tixels. This is shorthand for saying that it demands six supply tixels.

Underlying the high school timetabling problem is a matching that the author calls the *global tixel matching*. Its supply nodes are the supply tixels, one for each resource of the instance at each time. Its demand nodes are the demand tixels of the events of the instance. Edges connect demand tixels to those supply tixels that are suited to them. For example, a demand for student group 8A would be connected to supply tixels whose resource is 8A, and a demand for an English teacher at time *Mon1* would be connected to those supply tixels whose resource is an English teacher and whose time is *Mon1*. Each demand tixel wants to be assigned one supply tixel, and each supply tixel may only be assigned to one demand tixel, since otherwise there would be a timetable clash. So a matching is indeed what is required, and a maximum matching will have the least possible number of problems.

As decisions are made, in the form of assignments of times to meets or resources to tasks (or domain reductions, for example from the set of all qualified resources to a smaller set of preferred resources), the demand tixels affected by these decisions become connected to fewer supply tixels. When the maximum matching is recalculated (and fortunately there is an efficient algorithm for doing this incrementally as the graph changes) there may be more unmatched nodes than previously, suggesting that the decisions made may have been poor ones, and that alternatives should be explored.

The global tixel matching is useful for evaluating instances before solving begins. It can reveal, for example, that the supply of computer laboratories is insufficient to cover the demand, and other problems of that kind. It turns out to be very powerful late in the solve process, when

resources are being assigned after times have been assigned, provided it is enhanced with tixels expressing resource unavailabilities and workload limits (Section 7.4). However, it is quite weak before times are assigned, because it does not understand that the supply tixels assigned to events must be correlated in time: it does not perceive the contradiction in assigning, say, the two supply tixels (*Smith*,*Mon1*) and (*Lab6*, *Wed5*) to an event of duration 1.

An example given earlier, of scheduling five Music events simultaneously when there are only two Music teachers and two Music rooms, shows that useful checks can be made when deciding to run events simultaneously, even though their actual time remains undecided. Whatever time is ultimately assigned to simultaneous events, each resource of the instance can supply at most one tixel to satisfy their demands. So the demand tixels for one time of the events concerned may be matched with a set of supply nodes, one node for each resource of the instance. This is called *local tixel matching* by the author. The tixels are somewhat different, in that they share a common generic time rather than holding a variety of true times.

7.2. Setting up

By default, a solution contains no matching. To add a matching, call

```
void KheSolnMatchingBegin(KHE_SOLN soln);
```

To take it away again, call

```
void KheSolnMatchingEnd(KHE_SOLN soln);
```

`KheSolnMatchingEnd` can be omitted if the matching is needed for the lifetime of the solution, since the matching is deleted when its solution is deleted. There is also

```
bool KheSolnHasMatching(KHE_SOLN soln);
```

which returns `true` when `soln` has a matching. Most of the other operations of this chapter are undefined when no matching is present. Some may abort, others may do nothing.

`KheSolnMatchingBegin` adds exactly one matching to the solution. It is kept up to date thereafter as the solution changes, until `KheSolnMatchingEnd` is called or the solution is deleted. Adding a matching includes adding its demand nodes, each of which is represented by a monitor called a *demand monitor*. Removing a matching includes removing all demand monitors. A demand monitor contributes a cost to the solution just like other monitors do. The cost is 0 when the node is matched, and some non-negative value, set by the user, when it is unmatched.

Demand monitors may be attached and detached individually as usual. Detaching a demand monitor removes its node from the matching graph. Immediately after `KheSolnMatchingBegin` returns, the demand monitors it makes are all detached, so the matching graph has no demand nodes. Convenience functions defined below may be used to attach the demand monitors.

Rather than fiddling around calling `KheSolnHasMatching`, it is conventional to assume that a matching is present when KHE is being used for solving, but not when it is being used only to evaluate solutions. The rationale for this is that by comparison with the overall cost of a solve, it costs virtually nothing, and helps to make the solve environment uniform, if a matching is always present. If it isn't actually wanted, its demand monitors can be detached. On the other hand,

when evaluating solutions, at least when just their cost is required, matchings have no use, and if there are many solutions it is best to avoid the memory cost of the demand and supply nodes.

Behind the scenes, a lazy implementation is used: no matching is done until a query operation (for example, a request for the current cost of a demand monitor, or a debug print) occurs, allowing the time spent matching to be amortized over all operations carried out since the previous query. There is no way for the user to observe the laziness. The key operation, of bringing the matching up to date (making it maximum) runs in time roughly proportional to the number of unmatched nodes in the graph when it is called.

The cost of one unmatched node is set and retrieved by

```
void KheSolnMatchingSetWeight(KHE_SOLN soln, KHE_COST weight);
KHE_COST KheSolnMatchingWeight(KHE_SOLN soln);
```

For example, a call to

```
KheSolnMatchingSetWeight(soln, KheCost(1, 0));
```

gives each unmatched node a hard cost of 1. The initial weight is 0. A change of weight is reflected immediately in the cost reported by all demand monitors.

Although it would be trivial to allow the user to set the cost of each demand monitor individually, this has not been done, because it might suggest that the matching algorithm is capable of finding the matching which minimizes the total cost of unmatched nodes. In reality, there is no way to make the cost depend on which nodes are unmatched, nor on how appropriate the matching's assignments are. Those would be useful features, since then the cost of assign resources and prefer resources constraints could be reflected in the matching cost, but then a different problem, called *weighted bipartite matching*, would have to be solved, whose algorithm the author considers to be too slow for solving.

In the absence of weighted matching, choosing `weight` is not easy. The simple choice is `KheCost(1, 0)`, and it may well be the best. Another choice is one which guarantees that the weighted cost of the matching is a lower bound on the ultimate total cost of the violations of all relevant constraints, assuming that more assignments are added without changing the current ones. Each unassigned tixel in the matching must ultimately correspond with either a missing resource assignment at one time, or a resource clash at one time. So a suitable weight is the minimum of the following quantities: for each event resource, the sum of the combined weights of the assign resource constraints that apply to it; and for each resource, the sum of the combined weights of the avoid clashes constraints that apply to it. (Fortunately, both of these constraints incur a cost for each violating tixel.) Function

```
KHE_COST KheSolnMinMatchingWeight(KHE_SOLN soln);
```

works out this value. If there are no event resources and no resources, it returns 0.

The matching has a *type* that may be changed at any moment:

```
KHE_MATCHING_TYPE KheSolnMatchingType(KHE_SOLN soln);
void KheSolnMatchingSetType(KHE_SOLN soln, KHE_MATCHING_TYPE mt);
```

`KHE_MATCHING_TYPE` is the enumerated type

```
typedef enum {
  KHE_MATCHING_TYPE_EVAL_INITIAL,
  KHE_MATCHING_TYPE_EVAL_TIMES,
  KHE_MATCHING_TYPE_EVAL_RESOURCES,
  KHE_MATCHING_TYPE_SOLVE
} KHE_MATCHING_TYPE;
```

A full explanation of these values is given in the following section. Just briefly, however, `KHE_MATCHING_TYPE_SOLVE` implements an enhanced local tixel matching and is the best choice when solving; it is also the default value. The others are variants of global tixel matching. A change of type is reflected immediately in the cost reported by all attached demand monitors.

For the most part, matchings work quietly behind the scenes without attention from the user. However, there is an important optimization that only the user can invoke. Suppose that some changes are made to the solution as an experiment, then either retained or undone. Then KHE will run faster if that part of the program is bracketed by calls to these functions:

```
void KheSolnMatchingMarkBegin(KHE_SOLN soln);
void KheSolnMatchingMarkEnd(KHE_SOLN soln, bool undo);
```

Calls to these operations must occur in matching pairs, possibly nested. If `undo` is `true`, then `KheSolnMatchingMarkEnd` assumes without checking that all changes to `soln` since the corresponding call to `KheSolnMatchingMarkBegin` have been undone. It uses this information to bring the matching up to date more quickly than could be done without it. To encourage their use, both functions are well-defined even when there is no matching: in that case, they do nothing.

As an aid to debugging, function

```
void KheSolnMatchingDebug(KHE_SOLN soln, int verbosity,
  int indent, FILE *fp);
```

ensures that the matching is up to date, then prints its current state onto `fp`. Verbosity 1 prints just the number of unmatched demand monitors, verbosity 2 prints those monitors, and verbosity 3 prints all demand monitors and the supply nodes they are matched with.

7.3. Ordinary supply and demand nodes

This section explains how most of the supply and demand nodes of the matching, the ones associated with meets, are defined. Since they are linked together with edges that depend on the type of the matching, this section also explains `KHE_MATCHING_TYPE` in detail.

For each offset of a meet `meet` (for each integer between 0 inclusive and the duration of `meet` exclusive), the matching contains R *ordinary supply nodes*, where R is the total number of resources in the instance. If `meet` has duration d, this is dR supply nodes altogether. Each models the supply of one resource at one offset. These supply nodes cannot be accessed by the user.

Each task of `meet` contains `KheMeetDuration(meet)` demand nodes, which will be called *ordinary demand nodes* to distinguish them from the workload demand nodes to be defined later. Each models the demand made by its task at one offset. Ordinary demand nodes have type `KHE_ORDINARY_DEMAND_MONITOR` and may be accessed by

```
int KheTaskDemandMonitorCount(KHE_TASK task);
KHE_ORDINARY_DEMAND_MONITOR KheTaskDemandMonitor(KHE_TASK task, int i);
```

as usual. The first function's value is always equal to the duration of the enclosing meet. Like most monitors, these ones cannot be created or deleted by the user. They are created when the task is created, split and merged when it is split and merged, and deleted when it is deleted. Unlike other monitors, they are detached initially. This is so that, by default, KHE monitors only the official cost, not this extra stuff.

In addition to the operations applicable to all monitors, ordinary demand monitors offer

```
KHE_TASK KheOrdinaryDemandMonitorTask(KHE_ORDINARY_DEMAND_MONITOR m);
int KheOrdinaryDemandMonitorOffset(KHE_ORDINARY_DEMAND_MONITOR m);
```

returning the task that `m` monitors, and its offset within that task. Helper functions

```
void KheSolnMatchingAttachAllOrdinaryDemandMonitors(KHE_SOLN soln);
void KheSolnMatchingDetachAllOrdinaryDemandMonitors(KHE_SOLN soln);
```

ensure that all ordinary demand monitors are attached or detached; they visit every ordinary demand monitor of every task of every meet of `soln`, check whether it is currently attached, then attach or detach it if required.

Although the list of monitors in a task is fixed, each may be attached or detached individually, and they may be linked by edges to supply nodes in different ways, depending on the matching type, as will now be explained.

An ordinary demand node's *own meet* is the meet its task lies in. Its *root meet* is the meet reached by following the chain of assignments (possibly empty) out of its own meet to a meet that contains no assignment. Its *own offset* is its offset in its own meet, and its *root offset* is its offset in its root meet (the sum of its own offset and the offsets along the assignment path).

When linking an ordinary demand node to ordinary supply nodes, there are at least two ways to take time into account:

A. Link it only to ordinary supply nodes lying in cycle meets at offsets that represent the times of the time domain of its own meet, shifted by its own offset.

B. Link it only to ordinary supply nodes lying in its root meet at its root offset.

Informally, (A) evaluates the initial state of time assignment, whereas (B) evaluates its current state in a way that ensures that simultaneous demands compete for the same supply nodes, as in local tixel matching. And there are at least two ways to take resources into account:

1. Link it to supply nodes representing the resources of its task's domain.

2. Link it to supply nodes representing the resources of its task's root task's domain. If the root task is a cycle task, this will link only to supply nodes representing that resource.

Informally, (1) evaluates the initial state of resource assignment, whereas (2) evaluates the current state. The four non-empty matching types produce the four conjunctions of these conditions:

	A	B
1	KHE_MATCHING_TYPE_EVAL_INITIAL	KHE_MATCHING_TYPE_EVAL_TIMES
2	KHE_MATCHING_TYPE_EVAL_RESOURCES	KHE_MATCHING_TYPE_SOLVE

Type (B2) is suited to solving; the others are suited to evaluation before or after solving.

7.4. Workload demand nodes

In addition to ordinary demand nodes, matchings may contain *workload demand nodes*, used to take account of avoid unavailable times constraints, limit busy times constraints, and limit workload constraints, collectively called *workload demand constraints* here. For example, suppose the cycle contains 40 times, and teacher *Smith* has a required workload limit of 30 times and is unavailable at time *Mon1*. Then ten workload demand nodes should be created, one demanding supply tixel (*Smith*,*Mon1*), and the other nine demanding *Smith* at one unrestricted time.

It is important to include workload demand nodes, since otherwise the problems reported by the matching will be unrealistically few. They are the same for all matching types, and in most cases it is enough to call helper function

```
void KheSolnMatchingAddAllWorkloadRequirements(KHE_SOLN soln);
```

This may be done at any time, and does what is usually wanted. However, it is partly heuristic, so KHE offers the option of controlling the details.

For the purposes of matchings only, a *workload requirement* is a requirement imposed on a resource that it be occupied attending meets for at most a given number of the times of some time group. Within a solution at any moment, a sequence of workload requirements is associated with each resource. They may be visited in order by calling

```
int KheSolnMatchingWorkloadRequirementCount(KHE_SOLN soln,
  KHE_RESOURCE r);
void KheSolnMatchingWorkloadRequirement(KHE_SOLN soln, KHE_RESOURCE r,
  int i, int *num, KHE_TIME_GROUP *tg, KHE_MONITOR *m);
```

The first returns the number of workload requirements associated wth `r` in `soln`, and the second returns the `i`'th requirement, in the form of a number of times and a time group. If the third return parameter, `*m`, is non-`NULL`, it is the *originating monitor*: the monitor that gave rise to this requirement. The originating monitor is stored in workload demand monitors created as a consequence of this requirement, to assist in analysing defects; it is not otherwise used.

Each resource has no requirements initially. To change the requirements of resource `r`, begin with a call to

```
void KheSolnMatchingBeginWorkloadRequirements(KHE_SOLN soln,
  KHE_RESOURCE r);
```

continue with any number of calls to

```
void KheSolnMatchingAddWorkloadRequirement(KHE_SOLN soln,
  KHE_RESOURCE r, int num, KHE_TIME_GROUP tg, KHE_MONITOR m);
```

where m may be NULL, and end with a call to

```
void KheSolnMatchingEndWorkloadRequirements(KHE_SOLN soln,
  KHE_RESOURCE r);
```

All three functions must be called, in order. The first clears r's workload requirements, the second appends a requirement that r attend events for at most num of the times of tg (num may not exceed the number of times in tg), and the third replaces any existing workload demand nodes for r with new ones derived from the workload requirements. The new monitors are attached automatically as they are created. KheMatchingMonitorSetAllWorkloadRequirements calls these functions. The sections below describe the calls it makes, and how workload requirements are converted into workload demand nodes.

The workload demand nodes created by KheSolnMatchingEndWorkloadRequirements are monitors of type KHE_WORKLOAD_DEMAND_MONITOR. Like other monitors of resources, they appear on the list of monitors visited by functions KheResourceMonitorCount and KheResourceMonitor from Section 6.6.

In addition to the operations applicable to all monitors, workload demand monitors offer

```
KHE_RESOURCE KheWorkloadDemandMonitorResource(
  KHE_WORKLOAD_DEMAND_MONITOR m);
KHE_TIME_GROUP KheWorkloadDemandMonitorTimeGroup(
  KHE_WORKLOAD_DEMAND_MONITOR m);
KHE_MONITOR KheWorkloadDemandMonitorOriginatingMonitor(
  KHE_WORKLOAD_DEMAND_MONITOR m);
```

These return the resource that the workload demand monitor is for, the time group of the workload requirement that led to m, and the originating monitor (possibly NULL) of the workload requirement that led to m.

7.4.1. Constructing workload requirements

This section explains how KheSolnMatchingAddAllWorkloadRequirements works. For each resource r, it first calls KheSolnMatchingBeginWorkloadRequirements(soln, r), and then visits each required workload demand monitor m of weight greater than 0 applicable to r, in order of decreasing weight. What it does with each monitor is explained below. It then finishes its work on r with a call to KheSolnMatchingEndWorkloadRequirements(soln, r).

If m is an avoid unavailable times monitor, or a limit busy times monitor whose Maximum attribute is 0, then for each time t in m's constraint's domain it calls

```
KheSolnMatchingAddWorkloadRequirement(soln, r, 0,
  KheTimeSingletonTimeGroup(t), m);
```

If m is a limit busy times monitor with Maximum greater than 0, then for each time group tg in m's constraint it calls

```
KheSolnMatchingAddWorkloadRequirement(soln, r, k, tg);
```

where k is the Maximum attribute. The Minimum attribute is ignored.

A limit workload monitor is like a limit busy times monitor whose time group contains all the times of the cycle, so `KheSolnMatchingAddWorkloadRequirement` is called once with this time group. The number passed to this call requires careful calculation, involving the workloads of all events. The remainder of this section explains this calculation.

Let k be the integer eventually passed to `KheSolnMatchingAddWorkloadRequirement`. Initialize k to the `Maximum` attribute of the limit workload constraint. For each event resource *er*, let $d(er)$ be its duration and $w(er)$ be its workload. The basic idea is that if `r` is assigned to *er*, then $d(er) - w(er)$ should be added to k. For example, a resource with workload limit 30 that is assigned to an event resource with duration 3 and workload 2 needs a workload requirement of 31, not 30. And if `r` is assigned to an event with duration 6 but workload 12, then k needs to be decreased by 6.

In some cases, preassignments or domain restrictions make it certain that `r` will be assigned to some event, and in those cases the adjustment can be done safely in advance. For example, if every staff member attends a weekly meeting with duration 1 and workload 0, then their workload requirements can all be increased by 1 to compensate. Similarly, if `r` will definitely not be assigned to some event, then the event's duration and workload have no effect on `r`.

The residual problem cases are those event resources *er* whose workload and duration differ, which `r` may be assigned to but not necessarily. In these cases, an inexact model is used which preserves the guarantee that the number of unmatched nodes is a lower bound on the final number, but the number is weaker (that is, smaller) than the ideal.

If $w(er) > d(er)$, then *er* is ignored. This case can only make the problem harder, so ignoring it means that the number returned will be smaller than the ideal. If $w(er) < d(er)$, then $d(er) - w(er)$ is added to k, just as though `r` was assigned to *er*. If `r` is ultimately assigned to *er*, then this will be exact; if it is not, then again it will weaken the bound, by overestimating `r`'s available workload.

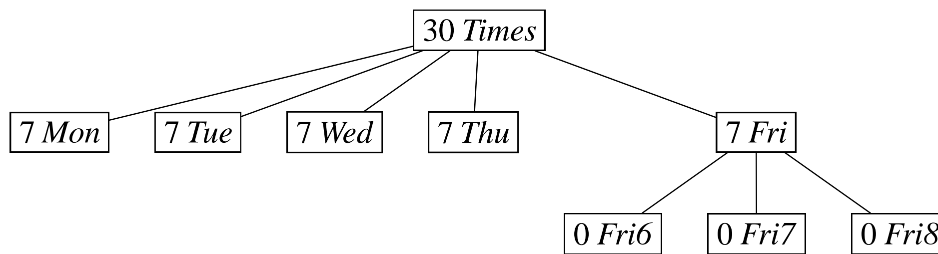
These tests are actually applied to clusters of events known to be running simultaneously, because of required link events constraints or preassignments and other time domain restrictions. Each resource can be assigned to at most one of the event resources of the events of a cluster, so only one of the events, the one whose modelling is least exact, needs to be taken account of.

7.4.2. From workload requirements to workload demand nodes

KHE converts workload requirements to workload demand nodes automatically, during the call to `KheSolnMatchingEndWorkloadRequirements` (defined above). The following explanation of how this is done, adapted from [9], is included for completeness.

When converting workload requirements into workload demand nodes, the relationships between the requirements' sets of times affect the outcome. In general, an exact conversion seems to be possible only when these sets of times satisfy the *subset tree condition*: each pair of sets of times is either disjoint, or else one is a subset of the other.

For example, suppose the cycle has five days of eight times each, and resource r is required to be occupied for at most thirty times altogether and at most seven on any one day, and to be unavailable at times *Fri6*, *Fri7*, and *Fri8*. These requirements form a tree (in general, a forest):



A postorder traversal of this tree may be used to deduce that workload demand nodes for *r* are needed for one *Mon* time, one *Tue* time, one *Wed* time, one *Thu* time, one *Fri6* time, one *Fri7* time, one *Fri8* time, and three arbitrary times. In general, each tree node contributes a number of demand nodes equal to the size of its set of times minus its number minus the number of demand nodes contributed by its descendants, or none if this number is negative.

The tree is built by inserting the workload requirements in order, ignoring requirements that fail the subset tree condition. For example, a failure would occur if, in addition to the above requirements, there were limits on the number of morning and afternoon times. The constraints which give rise to such requirements are still monitored by other monitors, but their omission from the matching causes it to report fewer unmatchable nodes than the ideal. Fortunately, such overlapping requirements do not seem to occur in practice, at least, not as required constraints.

7.5. Diagnosing failure to match

KHE's usual methods of organizing monitors, such as grouping and tracing, may be applied to demand monitors. This section offers three other ways to visit unmatched demand monitors.

7.5.1. Visiting unmatched demand nodes

The unmatched demand nodes may be visited by functions

```
int KheSolnMatchingDefectCount(KHE_SOLN soln);
KHE_MONITOR KheSolnMatchingDefect(KHE_SOLN soln, int i);
```

Each monitor is either an ordinary demand monitor or a workload demand monitor; a call to `KheMonitorTag` followed by a downcast will produce the specific type. Then functions defined earlier give access to the part of the solution being monitored by these monitors.

Unmatched demand nodes with higher indexes tend to have become unmatched more recently than demand nodes with lower indexes. When the number of unmatched demand nodes increases, it is reasonable to take the last unmatched demand node as an indication of what went wrong. However, it will usually be better to use grouping and tracing to localize problems.

7.5.2. Hall sets

Hall sets are the definitive method of diagnosing failure to match. They are fine for occasional use, such as for generating a report to the user, but too slow for repeated use during solving.

Suppose there is a set D of demand nodes, whose outgoing edges all lead to nodes in some set S of supply nodes. Then every node in D must be matched with a node in S, or not matched at

all. If $|D| > |S|$, then at least $|D| - |S|$ nodes of D will be unmatched in any maximum matching.

It turns out that every case of an unmatched node can be explained in this way, often utilizing sets D and S that are small enough to understand in user terms: they might represent the demand and supply of Science laboratories, for example. Such a D and S, with every edge out of D leading to S, and $|D| > |S|$, is called a *Hall set* after the mathematician P. Hall. Given a maximum matching, every unmatched demand node lies in a Hall set.

The following functions examine the Hall sets of a matching. They all begin by building the Hall sets if the ones currently stored are not up to date. This means that any change to the solution invalidates everything returned by all previous calls to these functions.

The number of Hall sets is returned by

```
int KheSolnMatchingHallSetCount(KHE_SOLN soln);
```

This is not usually the same as the number of unmatched demand nodes, since there may be several of those in one Hall set. No node lies in two Hall sets. The number of supply and demand nodes in the `i`'th Hall set may be found by calling

```
int KheSolnMatchingHallSetSupplyNodeCount(KHE_SOLN soln, int i);
int KheSolnMatchingHallSetDemandNodeCount(KHE_SOLN soln, int i);
```

By the way that Hall sets are defined, `KheSolnMatchingHallSetDemandNodeCount(soln, i)` must be larger than `KheSolnMatchingHallSetSupplyNodeCount(soln, i)`.

The `j`'th supply node of the `i`'th Hall set can only be an ordinary supply node, but, in case other kinds of supply nodes are added in future, the following function is used to find the meet it lies in, its offset within that meet, and the resource it represents:

```
bool KheSolnMatchingHallSetSupplyNodeIsOrdinary(KHE_SOLN soln,
  int i, int j, MEET *meet, int *meet_offset, KHE_RESOURCE *r);
```

At present this always returns `true`. A report to the user should distinguish the cases when `*meet` is and is not a cycle meet. The `j`'th demand node of the `i`'th Hall set is returned by

```
KHE_MONITOR KheSolnMatchingHallSetDemandNode(KHE_SOLN soln,
  int i, int j);
```

It will be either an ordinary demand node or a workload demand node as usual. Finally,

```
void KheSolnMatchingHallSetsDebug(KHE_SOLN soln,
  int verbosity, int indent, FILE *fp);
```

prints the Hall sets of `m`'s matching onto `fp` with the given verbosity and indent. The verbosity must be at least 1 but otherwise does not affect what is printed.

7.5.3. Finding competitors

Given an unmatched demand monitor `m` returned by `KheSolnMatchingHallSetDemandNode` or `KheSolnMatchingDefect`, a *competitor* of that monitor is either `m` itself or a monitor whose removal would allow `m` to match. Competitors are similar to the demand nodes of Hall sets, ex-

cept that they relate to a single unmatched demand node. They are themselves always matched. Finding competitors amounts to redoing the search for an augmenting path for the failed node and noting the demand nodes that are visited along the way.

Functions

```
void KheSolnMatchingSetCompetitors(KHE_SOLN soln, KHE_MONITOR m);
int KheSolnMatchingCompetitorCount(KHE_SOLN soln);
KHE_MONITOR KheSolnMatchingCompetitor(KHE_SOLN soln, int i);
```

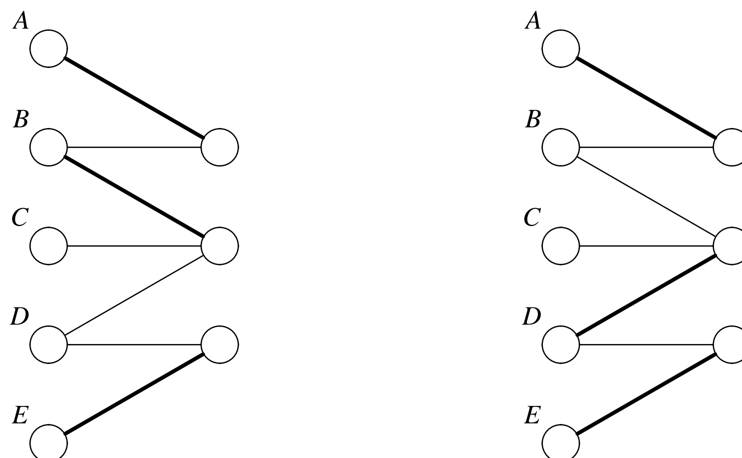
may be used together to visit the competitors of unmatched demand monitor `m`:

```
KheSolnMatchingSetCompetitors(soln, m);
for( i = 0;  i < KheSolnMatchingCompetitorCount(soln);  i++ )
{
  competitor_m = KheSolnMatchingCompetitor(soln, i);
  ... visit competitor_m ...
}
```

The competitors are visited in breadth-first order, beginning with `m` (which the user may choose to skip by initializing `i` in the loop above to `1` rather than `0`). There may be any number of competitors other than `m`, including none, and they may be ordinary demand monitors and workload demand monitors.

The solution contains one set of competitors which remains constant except when reset by a call to `KheSolnMatchingSetCompetitors`. If the solution changes, this set of competitors remains well-defined as a set of monitors, but becomes out of date as a set of competitors.

Competitors are useful because they can be found quickly, but they are not definitive in the way that Hall sets are: in unusual cases, a given unmatched monitor may have different competitors in different maximum matchings. For example, consider these two matchings:



Both are maximum, since all three supply nodes are matched in each; but the competitors of C in the first matching are A and B, while the competitors of C in the second are D and E.

It is important not to change the solution when traversing competitors. Even if it is changed back again, the unmatched demand nodes may be different afterwards. In the usual case where the aim is to move one meet that is competing for some scarce resources, the right approach is to

use the loop to find those meets, store them, and move them after it ends.

In applications such as ejection chains it is important to understand what the defect really is. In the case of unmatched demand nodes, the true defect is the Hall set. This may be approximated in practice by the set of competitors. Thus, a repair should operate on the set of competitors independently of their order or which one happens to be the unmatched one.

7.6. Evenness monitoring

Suppose that a school has seven Mathematics teachers, and that at some time there are seven Mathematics lessons running simultaneously. All seven teachers must be utilized at that time, which, although feasible, will restrict the options for resource assignment later.

Unless the teachers are very overworked, there must be other times when few Mathematics lessons are running. The Mathematics lessons are distributed unevenly through the cycle.

KHE offers a kind of monitor, of type `KHE_EVENNESS_MONITOR`, which monitors this kind of evenness. These work similarly to demand monitors; they are created and removed by

```
void KheSolnEvennessBegin(KHE_SOLN soln);
void KheSolnEvennessEnd(KHE_SOLN soln);
```

although the call to `KheSolnEvennessEnd` may be omitted when evenness monitoring is wanted for the lifetime of the solution. Evenness monitors are created by `KheSolnEvennessBegin` but not attached initially. There is one evenness monitor for each resource partition of the instance and each time of the cycle, which keeps track of how many tasks whose domains lie within that partition (as determined by `KheResourceGroupPartition`) are running at that time. The monitor reports a deviation when this number exceeds some limit, which is usually set at one less than the number of resources in the partition. Thus, the deviation would be zero when six Mathematics teachers are needed, and one when seven are needed. Function

```
bool KheSolnHasEvenness(KHE_SOLN soln);
```

returns `true` when evennness monitors are present.

Like demand monitoring, evenness monitoring depends on the resources demanded at each time. Unlike demand monitoring, however, domains that cross partition boundaries are not taken into account, and evenness is only monitored at the root level of the layer tree. Despite these simplifications, evenness monitoring is potentially useful for giving early warning of demand problems, especially when used in conjunction with domain tightening (Section 11.3.3).

When present, evenness monitors may be found in the list of all monitors kept in the solution, and attached and detached in the usual way. More useful in practice are functions

```
void KheSolnAttachAllEvennessMonitors(KHE_SOLN soln);
void KheSolnDetachAllEvennessMonitors(KHE_SOLN soln);
```

which visit each evenness monitor and ensure that it is attached or detached. The usual operations on monitors may be carried out by upcasting to type `KHE_MONITOR` as usual. There are also operations specific to evenness monitors:

```
KHE_RESOURCE_GROUP KheEvennessMonitorPartition(KHE_EVENNESS_MONITOR m);
KHE_TIME KheEvennessMonitorTime(KHE_EVENNESS_MONITOR m);
int KheEvennessMonitorCount(KHE_EVENNESS_MONITOR m);
```

These return the partition being monitored, the time being monitored, and the number of tasks whose domains lie within that partition that are currently running at that time (or 0 if `m` is unattached). It would be useful to be able to retrieve the specific tasks that go to make up this count, but that information is not kept. If it is needed, it is necessary to search the cycle meet overlapping the time, and all the meets assigned to that cycle meet directly or indirectly, to find the tasks running at the monitored time whose domains lie within the monitored partition.

Each evenness monitor also contains a limit, such that when the count goes above that limit a deviation is reported. This limit can be retrieved and changed at any time by calling

```
int KheEvennessMonitorLimit(KHE_EVENNESS_MONITOR m);
void KheEvennessMonitorSetLimit(KHE_EVENNESS_MONITOR m, int limit);
```

Its default value is the number of resources in the partition, minus this same number divided by six and rounded down. Thus, when there are less than six resources, the value is the number of resources; when there are between six and eleven resources, the value is one less than the number of resources; and so on. This seems to work reasonably well in practice. Another way to ignore unevenness in small partitions would be to detach their monitors.

The deviation is `KheEvennessMonitorCount(m) - KheEvennessMonitorLimit(m)`, or 0 if this number is negative. This is converted into a cost by multiplying by a weight (there is no choice of cost function). The weight may be retrieved, and set at any time, by

```
KHE_COST KheEvennessMonitorWeight(KHE_EVENNESS_MONITOR m);
void KheEvennessMonitorSetWeight(KHE_EVENNESS_MONITOR m, KHE_COST weight);
```

The default weight is the smallest non-zero weight, `KheCost(0, 1)`. Helper function

```
void KheSolnSetAllEvennessMonitorWeights(KHE_SOLN soln, KHE_COST weight);
```

sets the weights of all evenness monitors at once.

Evenness is not monitored in the current version of `KheGeneralSolve` (Section 8.1), because tests run by the author showed run time increases of about 20%, for little or no gain. Although it has some beneficial effects, evenness monitoring tends to disrupt node regularity and reduce diversity, since it causes very similar solutions to have slightly different costs.

Part B

Solving

A solver is an operation that makes large-scale changes to a solution. Solvers operate at a high level and should not be cluttered with implementation details: their source files will include `khe.h` as usual, but should not include header file `khe_interns.h` which gives access to KHE's internals. Thus, the user of KHE is as well equipped to write a solver as its author.

Many solvers are packaged with KHE. They are the subject of this part of the manual, all of which is implemented using `khe.h` but not `khe_interns.h`.

Chapter 8. Introducing Solving

This chapter introduces solving. It defines an interface for solvers, presents a few high-level ones, and explains some general concepts, including setting options and gathering statistics.

8.1. General solving

A *solver* is a function that finds solutions, or partial solutions, to instances. A *general solver* solves an instance completely, unlike, say, a *time solver* which only finds time assignments, or a *resource solver* which only finds resource assignments.

The recommended interface for general solvers, defined in `khe.h`, is

```
typedef KHE_SOLN (*KHE_GENERAL_SOLVER)(KHE_SOLN soln,
  KHE_OPTIONS options);
```

A general solver may split meets, build layer trees and task trees, assign times and resources, and so on without restriction. It will usually return the solution it is given, but it may return a different solution to the same instance, in which case it should delete the solution it is given.

Its second parameter, `options`, is a pointer to a set of options which may be used to vary the behaviour of the solver. Options are the subject of Section 8.4.

The main general solver distributed with KHE is

```
KHE_SOLN KheGeneralSolve2014(KHE_SOLN soln, KHE_OPTIONS options);
```

This is a single-threaded general solver which works by calling functions defined elsewhere in this guide. It returns the solution it is given.

The author's intention is that the best solver (all things considered) that he creates in any given year, if better than his previous solvers, should be called `KheGeneralSolve` with the year appended, and that KHE's main program should call it, either directly or as the solver passed to some parallel solver. `KheGeneralSolve2014` is the first of these solvers.

`KheGeneralSolve2014` assumes that `soln` is as returned by `KheSolnMake`, so it begins with `KheSolnSplitCycleMeet` and `KheSolnMakeCompleteRepresentation`. Then it calls other solvers defined elsewhere in this guide: it builds a layer tree and task tree, attaches demand monitors, calls `KheCycleNodeAssignTimes` to assign times, and `KheTaskingAssignResources` to assign resources. Finally, it calls `KheSolnEnsureOfficialCost` and returns.

`KheGeneralSolve2014` is affected indirectly by many options, via the functions it calls. The only options it consults directly are `monitor_evenness`, which it uses to decide whether to install evenness monitors (Section 7.6), and `time_assignment_only`, which when set causes it to exit early, immediately after time assignment.

8.2. Parallel solving

Function

```
void KheArchiveParallelSolve(KHE_ARCHIVE archive, int thread_count,
  int make_solns, KHE_GENERAL_SOLVER solver, KHE_OPTIONS options,
  int keep_solns, KHE_SOLN_GROUP soln_group);
```

creates a pool of `thread_count` threads and uses them to solve the instances of `archive`. They include the thread that called `KheArchiveParallelSolve`, so `thread_count` must be at least 1.

`KheArchiveParallelSolve` creates `make_solns` solutions for each instance of `archive`, by creating that many solutions and calling `solver` on each solution with a copy of `options`. The solutions passed to `solver` are identical except that the diversifier of the first is 0, the diversifier of the second is 1, and so on. The solver may use these values to create diverse solutions.

If `soln_group` is non-`NULL`, `KheArchiveParallelSolve` keeps the best `keep_solns` out of the `make_solns` solutions it made for each instance, and adds them to `soln_group`, deleting the others. Otherwise it deletes all the solutions it made.

A variant of `KheArchiveParallelSolve` that may sometimes be more convenient is

```
KHE_SOLN KheInstanceParallelSolve(KHE_INSTANCE ins, int thread_count,
  int make_solns, KHE_GENERAL_SOLVER solver, KHE_OPTIONS options);
```

Behind the scenes it is the same, but it solves a single instance rather than an entire archive, and it returns any one best solution rather than storing a set of best solutions in a solution group.

Parallelism is obtained via functions `pthread_create` and `pthread_join` from the Posix threads library. KHE has been carefully designed to ensure that operations carried out in parallel on distinct solutions cannot interfere with each other. If you do not have Posix, a simple workaround documented in KHE's makefile will allow you to compile KHE without it. The only difference is that `KheArchiveParallelSolve` and `KheInstanceParallelSolve` will find their solutions sequentially rather than in parallel.

8.3. Benchmarking

For benchmarking (that is, for gathering statistics while a solver runs), KHE offers

```
void KheBenchmark(KHE_ARCHIVE archive, KHE_GENERAL_SOLVER solver,
  char *solver_name, char *author_name, char test_label,
  KHE_STATS_TABLE_TYPE table_type);
```

It solves `archive`, possibly several times, using `solver`, writing the results into files in directory `"stats"` of the current directory. Some files are archives, others contain tables of statistics recording the performance of `solver`, printed by KHE's statistics functions (Section 8.5).

Parameter `solver_name` is a brief name for `solver`, suited for use in the header of a table column; `author_name` is the name of the author of the solver; and `test_label` (a character between `'A'` and `'Z'`) determines which tests are performed and which files are written. These may change from time to time. See the top of file `khe_sm_benchmark.c` for current details.

Parameter table_type determines the format of any tables written. Its values are

```
typedef enum {
  KHE_STATS_TABLE_PLAIN,
  KHE_STATS_TABLE_LOUT,
  KHE_STATS_TABLE_LATEX
} KHE_STATS_TABLE_TYPE;
```

which request plain text, Lout, or LaTeX format.

KheBenchmark takes it upon itself to skip some instances of the archive it is given. To see which are skipped, consult function KheBenchmarkTryInstance in file khe_sm_benchmark.c. If it comes upon such an instance, it includes a row for it in the tables it prints, but it does not attempt to solve it, and it leaves the entries for that row blank.

8.4. Options

All solvers take an options parameter of type KHE_OPTIONS, a pointer to a set of options which can be used to vary their behaviour. Function

```
KHE_OPTIONS KheOptionsMake(void);
```

returns a new options object whose options all have their default values. For each option, there is one function to retrieve the option and another to set it. These functions are documented alongside the solvers that their options affect, and the full list of options appears below. Some simple solvers do not use any options; in that case, the options argument may be NULL. Function

```
void KheOptionsDelete(KHE_OPTIONS options);
```

may be called to delete an options object when it is no longer needed.

Options can be classified into two kinds, although the distinction between them is not absolute. One kind is there for the convenience of the end user, to allow him to try out different possibilities. Options of this kind are not set by any of KHE's solvers. The other kind is there because some of KHE's solvers need to vary the behaviour of other solvers that they call. These ones are set by KHE's solvers.

Because options (especially the second kind) can change, when solving in parallel different options objects must be passed to each solve. These can be created by copying using

```
KHE_OPTIONS KheOptionsCopy(KHE_OPTIONS options);
```

It will call KheEjectorCopy to copy any ejectors stored inside it. This is necessary because a single ejector cannot safely be accessed by two solvers in parallel. Split analysers also cannot be used in parallel, so KheOptionsCopy creates a new split analyser object for the copy.

The following subsections present the complete list of options. Only brief indications of their meaning are given here, with references to the places where they are described in detail.

8.4.1. General options

This subsection describes options used widely or by KHE's general solvers.

The `diversify` option determines whether some solvers consult the solution's diversifier (Section 4.5), and so produce a different result for different solutions when their diversifiers are different. It is retrieved and set by

```
bool KheOptionsDiversify(KHE_OPTIONS options);
void KheOptionsSetDiversify(KHE_OPTIONS options, bool diversify);
```

Its default value is `true`. Many of the solvers packaged with KHE consult this option.

The `monitor_evenness` option determines whether `KheGeneralSolve2014` (Section 8.1) installs evenness monitors (Section 7.6). It is retrieved and set by

```
bool KheOptionsMonitorEvenness(KHE_OPTIONS options);
void KheOptionsSetMonitorEvenness(KHE_OPTIONS options,
  bool monitor_evenness);
```

Its default value is `false`.

The `time_assignment_only` option determines whether `KheGeneralSolve2014` (Section 8.1) exits early, leaving the solution in its state after time assignment. It is retrieved and set by

```
bool KheOptionsTimeAssignmentOnly(KHE_OPTIONS options);
void KheOptionsSetTimeAssignmentOnly(KHE_OPTIONS options,
  bool time_assignment_only);
```

Its default value is `false`.

8.4.2. Structural solver options

This subsection describes options used by KHE's structural solvers.

The `structural_time_equiv` option holds a time-equivalence object (Section 9.2). It is retrieved and set by

```
KHE_TIME_EQUIV KheOptionsStructuralTimeEquiv(KHE_OPTIONS options);
void KheOptionsSetStructuralTimeEquiv(KHE_OPTIONS options,
  KHE_TIME_EQUIV structural_time_equiv);
```

The default value is a time-equivalence object created by `KheOptionsMake` or `KheOptionsCopy` and deleted by `KheOptionsDelete`. There seems to be no reason to ever call `KheOptionsSetStructuralTimeEquiv`, but the user of the time-equivalence object will need to call `KheTimeEquivSolve` at some point.

The `structural_split_analyser` option holds a split analyser object (Section 9.7.1). It is retrieved and set by

```
KHE_SPLIT_ANALYSER KheOptionsStructuralSplitAnalyser(KHE_OPTIONS options);
void KheOptionsSetStructuralSplitAnalyser(KHE_OPTIONS options,
  KHE_SPLIT_ANALYSER structural_split_analyser);
```

The default value is a split analyser object created by KheOptionsMake or KheOptionsCopy and deleted by KheOptionsDelete. There seems to be no reason to ever call KheOptionsSetStructuralSplitAnalyser.

8.4.3. Time solver options

This subsection describes options used by KHE's time solvers, except for ejection chain time repair algorithms, whose options appear in Section 8.4.5.

The time_cluster_meet_domains option determines whether KheCycleNodeAssignTimes (Section 10.8.3) clusters meet domains using KheSolnClusterAndLimitMeetDomains (Section 10.3.3) before assigning times and unclusters them afterwards. It is retrieved and set by

```
bool KheOptionsTimeClusterMeetDomains(KHE_OPTIONS options);
void KheOptionsSetTimeClusterMeetDomains(KHE_OPTIONS options,
  bool time_cluster_meet_domains);
```

Its default value is false.

The time_tighten_domains option determines whether KheCycleNodeAssignTimes (Section 10.8.3) tightens resource domains (Section 11.3.4). It is retrieved and set by

```
bool KheOptionsTimeTightenDomains(KHE_OPTIONS options);
void KheOptionsSetTimeTightenDomains(KHE_OPTIONS options,
  bool time_tighten_domains);
```

Its default value is true.

The time_node_repair option determines whether KheCycleNodeAssignTimes (Section 10.8.3) ends by calling KheEjectionChainNodeRepairTimes (Section 10.7.2). If so, it calls it twice, before and after removing regularity-enhancing features. It is retrieved and set by

```
bool KheOptionsTimeNodeRepair(KHE_OPTIONS options);
void KheOptionsSetTimeNodeRepair(KHE_OPTIONS options,
  bool time_node_repair);
```

Its default value is true.

The time_node_regularity option determines whether KheNodeLayeredAssignTimes (Section 10.8.2) tries for node regularity. It is retrieved and set by

```
bool KheOptionsTimeNodeRegularity(KHE_OPTIONS options);
void KheOptionsSetTimeNodeRegularity(KHE_OPTIONS options,
  bool time_node_regularity);
```

Its default value is true.

The time_layer_swap option determines whether or not KheNodeLayeredAssignTimes (Section 10.8.2) tries more than one ordering of its layers. It is retrieved and set by

```
bool KheOptionsTimeLayerSwap(KHE_OPTIONS options);
void KheOptionsSetTimeLayerSwap(KHE_OPTIONS options,
  bool time_layer_swap);
```

Its default value is `false`.

The `time_layer_repair` option determines whether or not `KheNodeLayeredAssignTimes` (Section 10.8.2) repairs its assignment of times to each layer immediately after assigning the layer. It is retrieved and set by

```
bool KheOptionsTimeLayerRepair(KHE_OPTIONS options);
void KheOptionsSetTimeLayerRepair(KHE_OPTIONS options,
  bool time_layer_repair);
```

Its default value is `true`.

If `time_layer_repair` is `true`, then option `time_layer_repair_backoff` determines whether exponential backoff is used to decide which layers to repair. It is retrieved and set by

```
bool KheOptionsTimeLayerRepairBackoff(KHE_OPTIONS options);
void KheOptionsSetTimeLayerRepairBackoff(KHE_OPTIONS options,
  bool time_layer_repair_backoff);
```

Its default value is `false`, meaning to repair every layer.

The `time_layer_repair_long` option affects `KheEjectionChainLayerRepairTimes` (Section 10.7.2), determining whether it targets just the current layer, or every layer up to and including the current layer. It is retrieved and set by

```
bool KheOptionsTimeLayerRepairLong(KHE_OPTIONS options);
void KheOptionsSetTimeLayerRepairLong(KHE_OPTIONS options,
  bool time_layer_repair_long);
```

Its default value is `false`, meaning to target just the current layer.

The `time_kempe_stats` option holds an object of type `KHE_KEMPE_STATS`, used for recording statistics about Kempe meet moves (Section 10.2.2). It is retrieved and set by

```
KHE_KEMPE_STATS KheOptionsTimeKempeStats(KHE_OPTIONS options);
void KheOptionsSetTimeKempeStats(KHE_OPTIONS options,
  KHE_KEMPE_STATS time_kempe_stats);
```

Its default value is a new `KHE_KEMPE_STATS` object, both when an options object is created and when it is copied. So there is not usually any need to call `KheOptionsSetTimeKempeStats`.

8.4.4. Resource solver options

This subsection describes options used by KHE's resource solvers, except for ejection chain resource repair algorithms, whose options appear in Section 8.4.5.

The `resource_invariant` option determines whether resource solvers limit themselves to producing results that preserve the *resource assignment invariant* (Section 11.2), which states that the number of unmatched demand tixels may not increase. It is retrieved and set by

```
bool KheOptionsResourceInvariant(KHE_OPTIONS options);
void KheOptionsSetResourceInvariant(KHE_OPTIONS options,
  bool resource_invariant);
```

Its default value is `false`. Many resource solvers consult this option: `KheTaskTreeMake`, `KheTaskingMakeTaskTree`, `KheTaskingTightenToPartition`, `KheResourcePairReassign`, `KheResourcePairRepairSplitAssignments`, and `KheEjectionChainRepairResources`. It would be ideal if they all did, but they don't at present.

The `resource_rematch` option tells `KheTaskingAssignResources` whether to call `KheResourceRematch`. It is retrieved and set by

```
bool KheOptionsResourceRematch(KHE_OPTIONS options);
void KheOptionsSetResourceRematch(KHE_OPTIONS options,
  bool resource_rematch);
```

It has default value `true`.

The `resource_pair` option affects `KheResourcePairRepair` as explained in Section 11.9.2. It is retrieved and set by

```
KHE_OPTIONS_RESOURCE_PAIR KheOptionsResourcePair(KHE_OPTIONS options);
void KheOptionsSetResourcePair(KHE_OPTIONS options,
  KHE_OPTIONS_RESOURCE_PAIR resource_pair);
```

It has default value `KHE_OPTIONS_RESOURCE_PAIR_SPLITS`. Some rudimentary statistics are gathered in three integer values: `resource_pair_calls`, `resource_pair_successes`, and `resource_pair_truncs`. These may be retrieved and set as usual:

```
int KheOptionsResourcePairCalls(KHE_OPTIONS options);
void KheOptionsSetResourcePairCalls(KHE_OPTIONS options,
  int resource_pair_calls);
int KheOptionsResourcePairSuccesses(KHE_OPTIONS options);
void KheOptionsSetResourcePairSuccesses(KHE_OPTIONS options,
  int resource_pair_successes);
int KheOptionsResourcePairTruncs(KHE_OPTIONS options);
void KheOptionsSetResourcePairTruncs(KHE_OPTIONS options,
  int resource_pair_truncs);
```

See Section 11.9.2 for the details.

8.4.5. Ejection chain options

This section describes options relevant to ejector objects and ejection chain repair algorithms. For full details, consult Chapter 12.

Functions

```
KHE_EJECTOR KheOptionsEjector(KHE_OPTIONS options, int index);
void KheOptionsSetEjector(KHE_OPTIONS options, int index,
  KHE_EJECTOR ej);
```

retrieve and set one ejector object for each non-negative `index`. At each index the default value is `NULL`. Functions

```
char *KheOptionsEjectorSchedulesString(KHE_OPTIONS options);
void KheOptionsSetEjectorSchedulesString(KHE_OPTIONS options,
  char *ejector_schedules_string);
```

retrieve and set a string describing the schedules to apply to an ejector. For the default value, consult Section 12.6. Functions

```
bool KheOptionsEjectorPromoteDefects(KHE_OPTIONS options);
void KheOptionsSetEjectorPromoteDefects(KHE_OPTIONS options,
  bool ejector_promote_defects);
```

retrieve and set the `ejector_promote_defects` option of ejectors. Its default value is `true`. Functions

```
bool KheOptionsEjectorFreshVisits(KHE_OPTIONS options);
void KheOptionsSetEjectorFreshVisits(KHE_OPTIONS options,
  bool ejector_fresh_visits);
```

retrieve and set the `ejector_fresh_visits` option of ejectors. Its default value is `false`.

The `ejector_repair_times` option determines whether augment functions are permitted to change the assignments of meets. It is retrieved and set by

```
bool KheOptionsEjectorRepairTimes(KHE_OPTIONS options);
void KheOptionsSetEjectorRepairTimes(KHE_OPTIONS options,
  bool ejector_repair_times);
```

It is set by the various ejection chain functions, so setting by the caller of those functions will have no effect. Its default value is `true`.

Option `ejector_vizier_node` determines whether `KheEjectionChainNodeRepairTimes` and `KheEjectionChainLayerRepairTimes` (Section 10.7.2) insert a vizier node (Section 9.5.4) temporarily while they run. It is retrieved and set by

```
bool KheOptionsEjectorVizierNode(KHE_OPTIONS options);
void KheOptionsSetEjectorVizierNode(KHE_OPTIONS options,
  bool ejector_vizier_node);
```

Its default value is `false`.

The `ejector_nodes_before_meets` option determines whether augment functions that try both node swaps and meet moves try the node swaps first. It is retrieved and set by

```
bool KheOptionsEjectorNodesBeforeMeets(KHE_OPTIONS options);
void KheOptionsSetEjectorNodesBeforeMeets(KHE_OPTIONS options,
  bool ejector_nodes_before_meets);
```

Its default value is `false`.

The `ejector_use_kempe_moves` option determines whether augment functions that move meets use Kempe meet moves in addition to ejecting and basic ones (Section 10.2.2). It is retrieved and set by

```
KHE_OPTIONS_KEMPE KheOptionsEjectorUseKempeMoves(KHE_OPTIONS options);
void KheOptionsSetEjectorUseKempeMoves(KHE_OPTIONS options,
  KHE_OPTIONS_KEMPE ejector_use_kempe_moves);
```

where type `KHE_OPTIONS_KEMPE` is

```
typedef enum {
  KHE_OPTIONS_KEMPE_NO,
  KHE_OPTIONS_KEMPE_AUTO,
  KHE_OPTIONS_KEMPE_YES
} KHE_OPTIONS_KEMPE;
```

`KHE_OPTIONS_KEMPE_NO` means to not use them, and `KHE_OPTIONS_KEMPE_YES` means to use them wherever possible (this is the default value). `KHE_OPTIONS_KEMPE_AUTO` means to use them only when moving meets that lie in nodes that lie in layers of large duration relative to the cycle duration, reasoning that swaps are virtually always needed when such meets are moved.

The `ejector_use_fuzzy_moves` option determines whether augment functions that move meets try fuzzy meet moves (Section 10.7.4) in addition to the other kinds of meet moves. If they do, to conserve running time they only do so at depth 1 on the ejection chain, i.e. only when repairing a defect of the current best solution, not when repairing a defect introduced by a previous repair. The option is retrieved and set by

```
bool KheOptionsEjectorUseFuzzyMoves(KHE_OPTIONS options);
void KheOptionsSetEjectorUseFuzzyMoves(KHE_OPTIONS options,
  bool ejector_use_fuzzy_moves);
```

Its default value is `false`. At present the `width`, `depth`, and `max_meets` parameters of `KheFuzzyMeetMove` are fixed constants.

The `ejector_use_split_moves` option determines whether augment functions that move meets try split meet moves in addition to the other kinds of meet moves. The option is retrieved and set by

```
bool KheOptionsEjectorUseSplitMoves(KHE_OPTIONS options);
void KheOptionsSetEjectorUseSplitMoves(KHE_OPTIONS options,
  bool ejector_use_split_moves);
```

Its default value is `false`, but some of the solvers change it on their own authority.

The `ejector_ejecting_not_basic` option determines whether augment functions that assign and move meets use ejecting assignments and moves, not basic ones (Section 10.2.2). It is retrieved and set by

```
bool KheOptionsEjectorEjectingNotBasic(KHE_OPTIONS options);
void KheOptionsSetEjectorEjectingNotBasic(KHE_OPTIONS options,
  bool ejector_ejecting_not_basic);
```

Its default value is `true`.

The `ejector_limit_node` option holds a node. When it is non-`NULL`, it causes augment functions that assign and move meets to limit their repairs to the descendants of that node. It is

retrieved and set by

```
KHE_NODE KheOptionsEjectorLimitNode(KHE_OPTIONS options);
void KheOptionsSetEjectorLimitNode(KHE_OPTIONS options,
  KHE_NODE ejector_limit_node);
```

Its default value is `NULL`.

The `ejector_repair_resources` option determines whether augment functions are permitted to change the assignments of tasks. It is retrieved and set by

```
bool KheOptionsEjectorRepairResources(KHE_OPTIONS options);
void KheOptionsSetEjectorRepairResources(KHE_OPTIONS options,
  bool ejector_repair_resources);
```

It is set by the various ejection chain functions, so setting by the caller of those functions will have no effect. Its default value is `true`.

The `ejector_limit_defects` option is an integer limit on the number of defects handled by the main loop of the ejector. Each time the main list of defects is copied and sorted, if its size exceeds this limit, defects are dropped from the end until it doesn't. It is retrieved and set by

```
int KheOptionsEjectorLimitDefects(KHE_OPTIONS options);
void KheOptionsSetEjectorLimitDefects(KHE_OPTIONS options,
  int ejector_limit_defects);
```

Its default value is `INT_MAX`.

8.5. Gathering statistics

KHE offers a module for gathering statistics. It can calculate running times and generate files containing tables in several formats, and graphs in Lout format.

8.5.1. Running time and date

To find out how long something takes to run, objects of type `KHE_STATS_TIMER` (the usual pointer to a private record) are used. Each records one moment in time. To create and delete these timer objects, the functions are

```
KHE_STATS_TIMER KheStatsTimerMake(void);
void KheStatsTimerDelete(KHE_STATS_TIMER st);
```

`KheStatsTimerMake` returns a new timer, initialized by calling `KheStatsTimerReset` on it, and `KheStatsTimerDelete` deletes `st`, reclaiming the memory it used. There is also

```
KHE_STATS_TIMER KheStatsTimerCopy(KHE_STATS_TIMER st);
```

which copies `st`, producing a new timer holding the same time as `st`. The other functions are

```
void KheStatsTimerReset(KHE_STATS_TIMER st);
float KheStatsTimerNow(KHE_STATS_TIMER st);
```

KheStatsTimerReset resets the time held within st to the time when KheStatsTimerReset was called. KheStatsTimerNow compares the time recorded in st (when KheStatsTimerReset was last called) with the time now and reports the difference in seconds. Both functions may be called any number of times on the same timer. Any number of timers may be used independently.

Because wall clock times are used, times measured within one thread of a parallel solve will not in general measure the time consumed by that thread. However, a parallel solver can be called between KheStatsTimerReset and KheStatsTimerNow, and then they will reliably measure the elapsed time of the parallel solve.

Also offered is

```
char *KheStatsDateToday(void);
```

which returns the current date as a string in static memory.

For the sake of compilations that do not have the Unix system functions called by these functions, file khe.h has a KHE_USE_TIMING preprocessor flag. Its default value is 1; changing it to 0 will turn off all calls to Unix timing system functions. If that is done, all functions will still compile and run without error, but KheStatsTimerNow will always return -1.0, and KheStatsDateToday will return "?".

8.5.2. Files of tables and graphs

The main thing that the stats module does is generate files of tables and graphs. Any number of files may be generated simultaneously (not in parallel, because the stats module has no locking, but by one thread). One file may contain any number of tables and graphs, although only one may be generated at a time within any one file.

To begin and end a file, call

```
void KheStatsFileBegin(char *file_name);
void KheStatsFileEnd(char *file_name);
```

This writes a file called file_name in sub-directory stats of the current directory (which the user must have created previously). The file is opened by KheStatsFileBegin and closed by KheStatsFileEnd. To generate the actual tables and graphs, see the following subsections.

8.5.3. Tables

To generate tables, make matching pairs of calls to the following functions in between the calls to KheStatsFileBegin and KheStatsFileEnd:

```
void KheStatsTableBegin(char *file_name, KHE_STATS_TABLE_TYPE table_type,
  int col_width, char *corner, bool with_average_row, bool with_total_row,
  bool highlight_cost_minima, bool highlight_time_minima,
  bool highlight_int_minima);
void KheStatsTableEnd(char *file_name);
```

Only one table at a time can be generated into a given file, so a table is not identified separately from its file. The table is begun by KheStatsTableBegin, and finished, including being written out to the file, by KheStatsTableEnd. Where the file format permits, a label will be associated

with the table: the file name for the first table, the file name followed by an underscore and 2 for the second table, and so on. The value of the table is created in between these two calls, by calling functions to be presented shortly. Because the entire table is saved in memory until `KheStatsTableEnd` is called, these other calls may occur in any order. In particular it is equally acceptable to generate the table row by row or column by column.

The format of the table is specified by `table_type`:

```
typedef enum {
  KHE_STATS_TABLE_PLAIN,
  KHE_STATS_TABLE_LOUT,
  KHE_STATS_TABLE_LATEX
} KHE_STATS_TABLE_TYPE;
```

The choices are plain text, Lout, or LaTeX. Parameter `col_width` determines the width in characters of each column in plain text; it is ignored by the other formats. Parameter `corner` is printed in the top left-hand corner of the table. It must be non-`NULL`, but it can be the empty string.

Each entry in the table has a type, which may be either *string*, *cost*, *time* (really just an arbitrary `float`), or *int*. If `with_average_row` is `true`, the table ends with an extra row. Each entry in this row contains the average of the non-blank, non-string entries above it, if they all have the same type; otherwise the entry is blank. If `with_total_row` is `true`, the effect is the same except that totals are printed, not averages.

If `highlight_cost_minima` is `true`, the minimum values of type *cost* in each row appear in bold font, or marked by an asterisk in plain text. Parameters `highlight_time_minima` and `highlight_int_minima` are the same except that they highlight values of type *time* or *int*.

A caption can be added by calling

```
void KheStatsCaptionMake(char *file_name, char *fmt, ...);
```

at any time between `KheStatsTableBegin` and `KheStatsTableEnd`, as often as desired. This does what `printf` would do with the arguments after `file_name`. The results of all calls are saved and printed as a caption by `KheStatsTableEnd`.

In any given table, each row except the first (header) row must be declared, by calling

```
void KheStatsRowAdd(char *file_name, char *row_label, bool rule_below);
```

The rows appear in the order of the calls. Parameter `row_label` both identifies the row and appears in the first (header) column of the table. If `rule_below` is `true`, the row will have a rule below it. The header row always has a rule below it, and there is always a rule below the last row (not counting any average or total row).

In the same way, non-header columns are declared, in order, by calls to

```
void KheStatsColAdd(char *file_name, char *col_label, bool rule_after);
```

where `col_label` both identifies the column and appears in the first (header) row of the table, and setting `rule_after` to `true` causes a rule to be printed after the column.

To add an entry to the table, call any one of these functions:

```
void KheStatsAddEntryString(char *file_name, char *row_label,
  char *col_label, char *str);
void KheStatsAddEntryCost(char *file_name, char *row_label,
  char *col_label, KHE_COST cost);
void KheStatsAddEntryTime(char *file_name, char *row_label,
  char *col_label, float time);
void KheStatsAddEntryInt(char *file_name, char *row_label,
  char *col_label, int val);
```

These add an entry to `file_name`'s table at row `row_label` and column `col_label`, aborting if these are unknown or an entry has already been added there. If no entry is ever added at some position, the table will be blank there. The entry's format depends on the call. For example,

```
KheStatsAddEntryCost(file_name, row_label, col_label, KheSolnCost(soln));
```

adds a solution cost to the table which will be formatted in the standard way.

All strings passed to these functions that require long-term storage are copied, so mutating strings are not a concern. On the other hand, there is no locking, so calls which create tables should be single-threaded, as should calls which modify the same table.

8.5.4. Graphs

To generate graphs in Lout format, make matching pairs of calls to the following functions in between the calls to `KheStatsFileBegin` and `KheStatsFileEnd`:

```
void KheStatsGraphBegin(char *file_name);
void KheStatsGraphEnd(char *file_name);
```

As for tables, only one graph can be generated into a given file at a time, and so the graph is identified by the file name. To set options which control the overall appearance of the graph, call

```
void KheStatsGraphSetWidth(char *file_name, float width);
void KheStatsGraphSetHeight(char *file_name, float height);
void KheStatsGraphSetXMax(char *file_name, float xmax);
void KheStatsGraphSetYMax(char *file_name, float ymax);
void KheStatsGraphSetAboveCaption(char *file_name, char *val);
void KheStatsGraphSetBelowCaption(char *file_name, char *val);
void KheStatsGraphSetLeftCaption(char *file_name, char *val);
void KheStatsGraphSetRightCaption(char *file_name, char *val);
```

These determine the width and height of the graph (in centimetres), the maximum x and y values, and the small captions above, below, to the left of, and to the right of the graph. If calls to these functions are not made, the options remain unspecified, causing Lout's graph package to substitute default values for them in its usual way. The caption values must be valid Lout source.

A caption can be added by calling the same function as for tables:

```
void KheStatsCaptionMake(char *file_name, char *fmt, ...);
```

at any time between `KheStatsGraphBegin` and `KheGraphTableEnd`.

Any number of *datasets* may be displayed on one graph; each dataset is a sequence of points. Often there is just one dataset. To create a dataset, call

```
void KheStatsDataSetAdd(char *file_name, char *dataset_label,
  KHE_STATS_DATASET_TYPE dataset_type);
```

where `dataset_label` is used to identify the dataset, and `dataset_type` determines how the data are presented. At present the stats module offers just one choice:

```
typedef enum {
  KHE_STATS_DATASET_HISTO
} KHE_STATS_DATASET_TYPE;
```

but the Lout graph package offers many others, so it would not be difficult to expand the choices here. `KHE_STATS_DATASET_HISTO` prints a histogram. The x values of the dataset's points should be increasing integers; the y values are the frequencies. Function

```
void KheStatsPointAdd(char *file_name, char *dataset_label,
  float x, float y);
```

adds a point to a dataset. The points are generated in the order received, so in practice, successive calls to `KheStatsPointAdd` on the same dataset should have increasing `x` values.

8.6. Exponential backoff

One strategy for making solvers faster is to do a lot of what is useful, and not much of what isn't useful. When something is always useful, it is best to simply do it. When something might be useful but wastes a lot of time when it isn't, it is best to try it, observe whether it is useful, and do more or less of it accordingly. Solvers that do this are said to be *adaptive*.

For example, suppose there is a choice of two or more methods of doing something. In that case, information can be kept about how successful each method has been recently, and the choice can be weighted towards recently successful methods.

However, this section is concerned with a different situation, involving just one method. Suppose there is a sequence of *opportunities* to apply this method, and that as each opportunity arrives, the solver can choose to apply the method or not. Typically, the method will be a repair method: repair is optional. If the solver *accepts* the opportunity, the method is then run and either *succeeds* (does something useful) or *fails* (does nothing useful). Otherwise, the solver *declines* the opportunity. So opportunities are classified as successful, failed, or declined.

Exponential backoff from computer network implementation is a form of adaptation suited to this situation. It works as follows. If the solver applies the method and it is successful, then it forgets all history and will accept the next opportunity. But if the solver applies the method and it fails, then it remembers the total number of failed opportunities F (including this one) since the last successful opportunity, and does not accept another opportunity until after it has declined 2^{F-1} opportunities. Declined opportunities do not count as failures.

Here are some examples. Each character is one opportunity; `S` is a successful opportunity (or the start of the sequence), `F` is a failed one, and `.` is a declined one. Each successful opportunity

makes a fresh start, so the examples all begin with `S` and contain only `F` and `.` thereafter:

```
S
SF.
SF.F..
SF.F..F....
SF.F..F....F........
```

and so on. Every complete trace of exponential backoff can be broken at each `S` into sub-traces like these. Methods that always succeed are tried at every opportunity. Methods that always fail are tried only about $\log_2 n$ times, where n is the total number of opportunities.

Other rules for which opportunities to accept could be used, rather than waiting until 2^{F-1} opportunities have been declined. For example, every opportunity could be accepted, which amounts to having no backoff at all. The principles are the same, only the rule changes.

KHE offers four operations which together implement exponential backoff:

```
KHE_BACKOFF KheBackoffBegin(KHE_BACKOFF_TYPE backoff_type);
bool KheBackoffAcceptOpportunity(KHE_BACKOFF bk);
void KheBackoffResult(KHE_BACKOFF bk, bool success);
void KheBackoffEnd(KHE_BACKOFF bk);
```

`KheBackoffBegin` creates a new backoff object, passing a `backoff_type` value which determines which rule is used, of type

```
typedef enum {
  KHE_BACKOFF_NONE,
  KHE_BACKOFF_EXPONENTIAL
} KHE_BACKOFF_TYPE;
```

The two values select no backoff and exponential backoff. `KheBackoffAcceptOpportunity` is called when an opportunity arises, and returns `true` if that opportunity should be accepted. In that case, the next call must be to `KheBackoffResult`, reporting whether or not the method was successful. `KheBackoffEnd` reclaims the memory consumed by the backoff object.

Suppose that the program pattern without exponential backoff is

```
while( ... )
{
  ...
  if( opportunity_has_arisen )
    success = try_repair_method(soln);
  ...
}
```

Then the modified pattern for including exponential backoff is

```
bk = KheBackoffBegin(KHE_BACKOFF_EXPONENTIAL);
while( ... )
{
  ...
  if( opportunity_has_arisen && KheBackoffAcceptOpportunity(bk) )
  {
    success = try_repair_method(soln);
    KheBackoffResult(bk, success);
  }
  ...
}
KheBackoffEnd(bk);
```

Each successful `KheBackoffAcceptOpportunity` is followed by a call to `KheBackoffResult`.

All backoff objects hold a few statistics, kept only for printing by `KheBackoffDebug` below, and a boolean flag which is `true` if the next call must be to `KheBackoffResult`. When exponential backoff is requested, a backoff object also maintains two integers, C and M. C is the number of declines since the last accept (or since the backoff object was created). M is the maximum number of opprtunities that may be declined, defined by

$$M = \begin{cases} 0 & \text{if } F = 0 \\ 2^{F-1} & \text{if } F \geq 1 \end{cases}$$

where F is the number of failures since the last success (or since the backoff object was created). The next call to `KheBackoffAcceptOpportunity` will return `true` if $C \geq M$. The implementation will not increase M if that would cause an overflow. Overflow is very unlikely, since an enormous number of opportunities would have to occur first.

Function

```
char *KheBackoffShowNextDecision(KHE_BACKOFF bk);
```

returns `"ACCEPT"` when the next call to `KheBackoffAcceptOpportunity` will return `true`, and `"DECLINE"` when it will return `false`. There is also

```
void KheBackoffDebug(KHE_BACKOFF bk, int verbosity, int indent, FILE *fp);
```

Verbosity 1 prints the current state, including a ‘`!`’ when the flag is set, on one line. Verbosity 2 prints some statistics: the number of opportunities so far, and how many are successful, failed, and declined, in a multi-line format. A function for testing this module appears in `khe.h`.

Chapter 9. Structural Solvers

This chapter documents the solvers packaged with KHE that modify the structure of a solution: split and merge its meets, add nodes and layers, and so on. These solvers may alter time and resource assignments, but they only do so occasionally and incidentally to their structural work.

9.1. Layer tree construction

KHE offers a solver for building a layer tree holding the meets of a given solution:

```
KHE_NODE KheLayerTreeMake(KHE_SOLN soln);
```

The root node of the tree, holding the cycle meets, is returned. The function has no special access to data behind the scenes. Instead, it works by calling basic operations and helper functions:

- It calls `KheMeetSplit` to satisfy split events constraints and other influences on the number and duration of meets, as far as possible. It is usual to call `KheLayerTreeMake` when each event is represented in `soln` by a single meet of the full duration (that is, after `KheSolnMake` and `KheSolnMakeCompleteRepresentation`), but some meets may be already split. In any case, `KheLayerTreeMake` does not create, delete, or merge meets.

- It calls `KheMeetBoundMake` with a `NULL` meet bound group to set the time domains of meets to satisfy preassigned times, prefer times constraints, and other influences on time domains, as far as possible. For each meet, one call to `KheMeetBoundMake` is made for each possible duration. It is usual to call `KheLayerTreeMake` at a moment when the time domains of the meets are not restricted by meet bounds, but some meets may already have bounds. In any case, `KheLayerTreeMake` only adds bounds, never removes them, so it either leaves a domain unchanged, or reduces it to a subset of its initial value.

- It calls `KheMeetAssign` in trivial cases where there is no doubt that the assignments will be final. Precisely, if there are two events of equal duration linked by a link events constraint and split into meets of equal durations, and the algorithm places one in a parent node and the other in a child of that parent, then, provided the child node itself has no children (which would render the case non-trivial), the meets of the child node will be assigned to meets of the parent node, and the child node will be deleted in accordance with the convention given in Chapter 10, that meets whose assignments will never change should not lie in nodes.

- It calls `KheMeetAssignFix` to fix all the assignments it makes (as defined immediately above). These can be unfixed afterwards if desired.

- It calls `KheNodeMake` and `KheNodeAddMeet` to ensure that for each event there is one node holding the meets of that event, unless these meets receive the trivial assignments just described. There is also a node (the root node returned by `KheLayerTreeMake`, also accessible as `KheSolnNode(soln, 0)`) holding the cycle meets. Any other meets (usually none) are not placed into nodes. `KheLayerTreeMake` requires `soln` to contain no nodes initially.

- It calls `KheNodeAddParent` to reflect link events constraints (even between events whose durations differ), as far as possible, and the need to ultimately assign every meet to a cycle meet. When `KheLayerTreeMake` returns, every node is a descendant of the root node.

- Some instances contain events which have already been split, with the fragments presented as distinct events. It is best if the nodes holding the meets derived from these fragments are merged. So for each pair of distinct events which appear to be part of one course because they share a spread events constraint or avoid split assignments constraint, if certain other conditions (Section 9.1.5) are satisfied, the nodes holding the meets of those two events are merged by a call to `KheNodeMerge`.

These elements interact in ways that make most of them impossible to separate. For example, the splitting of an event into meets needs to be influenced not just by the event's own split events constraints and distribute split events constraints, but also by the constraints of the events that it is linked to by link events constraints.

Logically, order events constraints should also affect the construction of layer trees. In the version of KHE documented here they are not consulted, but this will change.

Although `KheLayerTreeMake` does not call `KheLayerMake`, resource layers (sets of events that share a common preassigned resource which has a hard avoid clashes constraint) strongly influence its behaviour. It ensures that the events of each layer are split into meets which can be packed into the cycle meets without overlapping in time, except in the unlikely case where the total duration of the events of the layer exceeds the total number of times in the cycle.

For each `meet` with a pre-existing assignment to some `target_meet`, `KheLayerTreeMake` tries to place `meet` into a child node of `target_meet`'s node. In exceptional circumstances, this may not be possible, and then the pre-existing assignment is removed by `KheLayerTreeMake`. Suppose there is an event with two meets, both assigned to other meets. If those two other meets are both derived from the same event, or if they are both cycle meets, then all is well; but if not, one of the original meets will be unassigned. This is done because `KheLayerTreeMake` tracks relations between events, not meets, and cannot cope with the idea of one event being assigned partly to one event and partly to another. A meet will also be unassigned when there is a cycle of assignments, but that should never occur in practice.

The above attempts to be a complete specification of `KheLayerTreeMake`, sufficient for using it. For the record, the following subsections explain how it works in detail.

9.1.1. Overview

`KheLayerTreeMake` uses a constructive heuristic which runs quickly. It works by examining the relevant constraints and taking actions to satisfy them, giving priority to those with higher weight. It does not search through a large space of possible solutions to find the best. This is appropriate, because in practice good solutions are easy to find. The problem is more about giving due weight

to the many influences on the solution than about real solving.

`KheLayerTreeMake` begins by unassigning meets to remove cases where two meets derived from a single event are assigned to meets not both derived from the same event or both cycle meets, and splitting meets whose duration exceeds the number of times in the instance into meets of duration within that bound. This allows the remainder of the algorithm to assume that each event is preassigned to at most one other event, and that there are no oversize meets.

In practice, it is likely that the constraints of an instance will cooperate harmoniously, but for completeness it is necessary to handle cases where they do not. For example, there is nothing to prevent a link events constraint from linking two events, one of which is required by a split events constraint to split into three meets, while the other is required to split into one.

There is a data structure, described in the following sections, which embodies all the requirements that the final layer tree must satisfy, including how events are to be split into meets, and how meets are to be grouped into nodes. It is an invariant that at least one layer tree must satisfy all these requirements. Initially, the data structure embodies no requirements at all. A long series of *jobs* is then applied to it, each inspired by some constraint or other feature of the instance to request that the data structure add some new requirements to the ones it currently embodies. If no layer trees would satisfy both the old and new requirements, the job is *rejected* (it is ignored); otherwise, it is *accepted* (its requirements are added). There are also cases in which some of the requirements of a job are accepted but others have to be rejected. The jobs are sorted by decreasing priority, which is usually the combined weight of the constraint that inspired the job. In this way, contradictory requests are resolved by giving preference to requests of higher priority.

Here is the full list of job types, with brief descriptions. How each job modifies the data structure will be explained later. The jobs not derived from constraints have high priority.

Pre-existing splits. Each already split event *e* generates a job requiring the meets that *e* is ultimately split into to be packable into (created by further splitting of) the pre-existing meets.

Preassigned times. XHSTT specifies that a meet derived from an event with a preassigned time must be assigned that time. Several simultaneous meets derived from one event are unlikely to be wanted, so this job requests that a preassigned event be not split further than its pre-existing splits, and that the meets' time domains be set to singleton domains.

Pre-existing assignments and link events constraints. These are interpreted as requests to create parent-child links between nodes.

Avoid clashes constraints. Each resource subject to a required avoid clashes constraint gives rise to a job which requests that the layer tree recognize that the events to which the resource is preassigned cannot overlap in time.

Split events constraints and distribute split events constraints. These request restrictions on the number of meets that an event may be split into, and their durations.

Spread events constraints. If the events of an event group of a spread events constraint are split into too many or too few meets, then a non-zero number of deviations of the constraint becomes inevitable. The job tries to tighten the requirements on the number of meets of the events concerned, to the point where this problem cannot arise.

Prefer times constraints. This kind of job requests that the time domain of the meets of an event which have a certain duration be reduced to satisfy a prefer times constraint. This may lead to an empty domain for meets of that duration; if so, then there can be no meets of that duration

at all, which may prevent the job from being accepted.

After all jobs have been applied, the data structure is traversed and a layer tree is built. Finally, `KheLayerTreeMake` examines each pair of events connected by a spread events or avoid split assignments constraint, and if those events' nodes satisfy the conditions given in Section 9.1.5), it merges them by calling `KheNodeMerge`.

9.1.2. Linking

The data structure used by `KheLayerTreeMake` must be close enough to the layer tree to make it straightforward to derive an actual layer tree at the end. In fact, it needs to represent the set of layer trees that satisfy the requirements of all the jobs accepted so far. This section explains how this is done for linking, and later sections explain the parts that handle splitting and layering.

If meet s_1 can be assigned to meet s_2 at offset o_1, and s_2 can be assigned to s_3 at offset o_2, then it is always possible to assign s_1 directly to s_3 at offset $o_1 + o_2$. Thus, the relation of assignability between meets is transitive. Although it is not safe to assign a meet to itself, it does no harm to pretend here that assignability is reflexive as well.

In some cases, two meets are assignable to each other. They must have equal durations and time domains, but that is not unusual. By a well-known fact about reflexive and transitive relations, two-way assignability is an equivalence relation between meets.

Similar relations can be defined between events. Let $A(e_1, e_2)$ hold when the meets of e_1 can be assigned to the meets of e_2 at non-overlapping offsets. Define

$$S(e_1, e_2) = A(e_1, e_2) \wedge A(e_2, e_1)$$

Again, A is reflexive and transitive, and S is an equivalence relation.

The data structure used for linking events includes a representation of relations A and S. The equivalence classes defined by S are represented by nodes of a graph, containing the events of the class and connected to other equivalence classes by directed edges representing A. A could be an arbitrary directed acyclic graph, but in fact it is limited to a tree: each equivalence class is recorded as assignable to at most one other equivalence class. Relational nodes will always be called classes, to avoid confusion with layer tree nodes.

The child classes of each equivalence class are organized into layers. That additional structure is not needed for linking, however, so its description will be deferred to Section 9.1.4.

Initially, each event lies in its own class, plus there is one class with no events, representing the cycle meets. Every event class is a child of the cycle meets class. Thus, initially relation S is empty, and relation A records only the basic fact that every event is assignable to the cycle meets to begin with. This is quite true, since, at this initial stage, before any jobs are accepted, the data structure believes that each event's domain is the entire cycle, that each event is free to split into meets of duration 1, and that there are no layers.

Basing the data structure on events, rather than on meets, seems to be right, but it does cause differences between the meets of one event to be overlooked. For example, the data structure believes that all meets derived from the same event have the same time domain.

Jobs that link events together do so by proposing elements of A and S to the data structure, which accepts them when it can. An S proposal is a request to merge the equivalence classes

containing its two events into one (if they are not already the same); an A proposal is a request to replace one parent link by another (which must still imply the first by transitivity). A proposal could be rejected for various reasons: it might lead to a directed acyclic graph which is not a tree, or cause events from the same layer to overlap in time, or lead to unacceptable restrictions on how events are to be split (as in the example at the start of this chapter), and so on.

Pre-existing assignments are proposed first as elements of S, and if that fails as elements of A. The second proposal at least cannot fail to be accepted, because these jobs have maximum priority and do not contradict each other. A link events constraint job first proposes all pairs of linked events of equal duration as elements of S, and then all pairs regardless of duration as elements of A. In general, an A proposal could require that the whole set of classes lying on a cycle of A links be evaluated for merging, but this particular way of making proposals ensures that, in fact, only pairwise merges need to be evaluated.

Each equivalence class has a *class leader*, one of its own events. When an equivalence class is created, its leader is the sole event it initially contains, and when two classes are merged, one of the two leaders is chosen to be the leader of the merged class. For convenience, we pretend that the cycle meets are derived from a single *cycle event* which is the leader of their class.

If class C contains an event e which is assigned to an event outside C, then the event e is assigned to lies in the parent class of C. There may not be two such events in C unless they are assigned to the same event at the same offset. The leader must be one of these events. The data structure only becomes aware of assignments when the jobs representing them are accepted.

If C does not contain an event which is assigned to another event outside the class, then it must contain at least one event which is not assigned at all, since otherwise there would be a cycle of assignments within the class. Any such unassigned event may be the leader.

These conditions are trivially satisfied when a class is created, by making its sole event the leader. When two classes are merged, there are various possibilities, including failure to merge when the two leaders are assigned to distinct events outside both classes.

When constructing the final layer tree, all the unassigned events of each class except the leader are placed in layer tree nodes which are made children of the node containing the leader. Similarly, the nodes containing the leaders of child classes become children of the node containing the leader of the parent class. In reality, of course, it is the meets derived from these events by the splitting algorithm to be described next that are placed into these nodes.

9.1.3. Splitting

Given an event e of duration d, any mathematical partition of d is a possible outcome of splitting e. For example, if e has duration 6, the possible outcomes are the eleven partitions

6	4 2	3 3	3 1 1 1	2 2 1 1	1 1 1 1 1 1
5 1	4 1 1	3 2 1	2 2 2	2 1 1 1 1	

One element of a partition is called a *part*, and is the duration of one meet.

Any condition that limits how an event is split defines a subset of this set of partitions. For example, if a split events constraint states that an event of duration 6 should be split into exactly four meets, that is equivalent to requiring the partition to be either 3 1 1 1 or 2 2 1 1.

Each equivalence class holds a set of events of equal duration that are assignable to each

other. These will eventually be partitioned into meets in the same way. In addition to the events, the class holds the requirements that the final partition must satisfy. These define a subset of the set of all partitions of the duration, but it is not possible to store the subset directly, because for large durations it may be very large. One partition *is* stored, however: the lexically minimum one satisfying the requirements. (A lexically minimum partition has minimum largest part, and so on recursively. For example, 1 1 1 1 1 1 is the lexically minimum partition of 6.) It is an invariant that the set of partitions satisfying the requirements may not be empty.

In the special case of the equivalence class that represents the cycle meets, the requirements are fixed to allow exactly one partition: the one representing the durations of the cycle meets.

The requirements on partitions are of two kinds. First, there are the *local requirements*. These are mainly lower and upper bounds on the total number of parts, and on the number of parts of each possible duration, modelled on the corresponding fields of the split events and distribute split events constraints. Another kind of local requirement arises when a pre-existing split job is accepted: if an event of duration 6 is already split into meets of duration 4 and 2, say, when the algorithm begins, then, to be acceptable, a partition must be packable into partition 4 2. One partition is *packable* into another if splitting some parts of the second partition and discarding others can produce the first. For example, 2 1 1 is packable into 2 2 2, but neither of 3 1 1 1 and 2 2 1 1 is packable into the other.

Second, there are the *structural requirements*. Each parent class has an arbitrary number of child classes, whose events will eventually be assigned to the parent class's events. So the lexically minimum partition of each child class must be packable into the parent class. In these calculations the constraint always flows upwards: the child's lexically minimum partition is taken as given, and the parent's minimum partition is adjusted (if possible) to ensure that the child's is packable into it. When a child class's minimum partition changes, the parent's requirements must be re-tested. In this way, a change to a partition propagates upwards through the structure until it either dies out or causes some class to have no legal partitions. In the second case, the job which originated the changes must be rejected.

Some of the child classes may be organized into layers. In that case, each layer's classes, taken together, must be packable into the parent class. Each layer is represented by a split layer object, as explained in detail in the next section. That object contains a minimum partition which must be packable into the parent class, just like the minimum partitions of child classes.

Deciding whether any partitions satisfy even the local requirements is non-trivial: is it safe to place two events into one class, when one is already split into partition 4 2 and the other is already split into partition 3 2 1? Some simple checks are made, then a full generate-and-test enumeration is begun and interrupted at the first success. The enumeration produces the lexically minimum acceptable partition first, which is then stored and propagated upwards. Fortunately, packability can be tested very quickly in practice, despite being an NP-complete bin packing problem, because event durations are usually small.

At the end, after the last job is processed, each event of each class is split into meets whose durations form the lexically minimum partition of that class.

9.1.4. Layering

The relation between meets and layers (sets of events that share a common preassigned resource

with a required avoid clashes constraint) is a many-to-many relation: a layer may contain any number of meets, and a meet may lie in any number of layers.

Suppose that meet s_1 lies in layer l and is assigned to meet s_2. KHE enforces the rule that any assignment of s_2 may not be such as to cause s_1 to overlap in time with any other meet of l. In a sense, s_2 (actually, that part of it assigned s_1) becomes a member of l while s_1 is assigned to it. We say that s_1 lies *directly* in l, and s_2 lies *indirectly* in l.

An event lies directly in a layer if any of its meets lie directly in the layer. An equivalence class lies directly in a layer if any of its events lie directly in the layer, and it lies indirectly in the layer if any of its child classes lie in the layer, either directly or indirectly. This is because the events of child classes will eventually be assigned to the events of the class.

The layering aspect of `KheLayerTreeMake` is based on an object called a *split layer*, which represents one element of the many-to-many relation between equivalence classes and layers. In other words, there is one split layer object for each case of an equivalence class lying in a layer, directly or indirectly. Its attributes are the class, the resource defining the layer, the set of all child classes of the class that lie in the layer, and a partition, whose value will be defined shortly.

When an equivalence class lies directly in a layer (when it contains an event that lies directly in the layer), none of its child classes can lie in the layer, since that would mean that two events of the same layer overlap in time. So in that case the set of child classes must be empty. To keep it that way, the partition contains as many 1's as the duration of the class. This makes it clear that there is no room for any child classes in the layer, without constraining the division of the class's events into sub-events in any way.

When an equivalence class lies indirectly in a layer, some of its child classes lie in the layer. Their total duration must not exceed the duration of the class, and their meets, taken together, must be packable into the class, since they are disjoint in time. So in this case the set of child classes may be (in fact, must be) non-empty, and the partition holds the multiset union of the lexically minimum partitions of the child classes.

The job which adds a layer to the data structure adds its events one by one. In the unlikely event that the duration of the layer exceeds the number of times in the cycle, or bin packing problems prevent an event being added, the job rejects the event, which amounts to ignoring the presence of the preassigned resource in that event.

Adding an event to a layer means that the event's class and all its ancestors must get split layer objects for the layer. For all these classes, moving upwards until either there are no more ancestors or a class already has a split layer object for the layer, either add a new split layer object holding just the current child class, or add the child class to an existing split layer object.

While the upward propagation adds new split layer objects, there is no possibility of failure, since a layer containing a single event is no more constraining than the event alone (the event is already present, only its membership of a layer is changing). But if an existing split layer object is reached, the class must be added to it, and so its partition grows, possibly leading to an empty set of acceptable partitions in the parent, causing rejection of the request.

9.1.5. Merging

As mentioned earlier, when instances contain events which have already been split, it is best to merge the nodes containing those events. The advantages include ensuring that how the instance

is presented does not affect the way it is solved, exposing symmetries which could be expensive if left hidden, and taking a step towards regularity.

Node merging is carried out after the main part of the layer tree construction algorithm is complete and a layer tree is present. For each pair of events that share a spread events or avoid split assignments constraint, the first meet of each event is found and the chain of fixed assignments is followed to the first unfixed meet and from there to the node. The two nodes thus found are candidates for merging. If they both exist, and they are distinct, and the first meet in each contains the same preassigned resources (counting resources in meets assigned to the meet, directly or indirectly, as well as resources in the meet itself), then the nodes are merged.

Only nodes which share at least one preassigned resource are merged. This ensures that it is right to assign non-overlapping times to the meets of a node, which is what solvers usually do.

Requiring the same preassigned resources turns out to be important, because of the way that layers are built from nodes, not from meets. If some of the meets of a node contain a resource but others do not, then when the nodes containing that resource are formed into a layer later, the layer's duration may be longer than the cycle length, making it awkward to timetable.

9.2. Time-equivalence

Two sets of meets are *time-equivalent* if it can be shown, by following fixed meet assignments, that each set of meets must occupy the same set of times as the other while fixed assignments remain in place. This may be true even when none of the meets is assigned a time.

Two events are time-equivalent if their sets of meets are time-equivalent. Usually, this is because they are joined by a link events constraint which is being handled structurally, for example by `KheLayerTreeMake` (Section 9.1).

Two resources are time-equivalent if they have the same resource type (call it `rt`), `KheResourceTypeDemandIsAllPreassigned(rt)` (Section 3.5.1) is `true`, and the sets of meets containing their preassigned tasks are time-equivalent. Time-equivalent resources are busy at the same times. They are usually students who choose the same courses.

It is clear that time-equivalence between sets of meets is an equivalence relation, as is time-equivalence between events and between resources. So the events and resources of an instance can be partitioned into time-equivalence classes. These classes are calculated by a *time-equivalence solver*, which can be created and deleted by calling

```
KHE_TIME_EQUIV KheTimeEquivMake(void);
void KheTimeEquivDelete(KHE_TIME_EQUIV te);
```

However, a call to `KheOptionsStructuralTimeEquiv` (Section 8.4.2) is the usual way to obtain a time-equivalence object. To perform the calculation for a particular `soln`, call

```
void KheTimeEquivSolve(KHE_TIME_EQUIV te, KHE_SOLN soln);
```

The equivalence classes of events are event groups which can be visited by

```
int KheTimeEquivEventGroupCount(KHE_TIME_EQUIV te);
KHE_EVENT_GROUP KheTimeEquivEventGroup(KHE_TIME_EQUIV te, int i);
```

in the usual way. The equivalence class for a given event is returned efficiently by

```
KHE_EVENT_GROUP KheTimeEquivEventEventGroup(KHE_TIME_EQUIV te,
  KHE_EVENT e);
```

If `e` is not time-equivalent to any other event, a singleton event group containing `e` is returned. There is also

```
int KheTimeEquivEventEventGroupIndex(KHE_TIME_EQUIV te, KHE_EVENT e);
```

which returns the value `i` such that `KheTimeEquivEventGroup(te, i)` contains `e`.

Similarly, the equivalence classes of resources are resource groups which can be visited by

```
int KheTimeEquivResourceGroupCount(KHE_TIME_EQUIV te);
KHE_RESOURCE_GROUP KheTimeEquivResourceGroup(KHE_TIME_EQUIV te, int i);
```

in the usual way. The equivalence class for a given resource is returned efficiently by

```
KHE_RESOURCE_GROUP KheTimeEquivResourceResourceGroup(KHE_TIME_EQUIV te,
  KHE_RESOURCE r);
```

If `r` is not time-equivalent to any other resource, including the case when its resource type is not all preassigned, a singleton group containing `r` is returned. Again,

```
int KheTimeEquivResourceResourceGroupIndex(KHE_TIME_EQUIV te,
  KHE_RESOURCE r);
```

returns the value `i` such that `KheTimeEquivResourceGroup(te, i)` contains `r`.

All of these results reflect the state of the solution at the time of the most recent call to `KheTimeEquivSolve(te)`; they are not updated as the solution changes.

9.3. Layers

Layers were introduced in Section 5.3, but no easy way to build a set of layers was provided. This section remedies that deficiency and adds some useful aids to solving with layers.

9.3.1. Layer construction

The usual rationale for the existence of a layer is that its nodes' meets must not overlap in time because they contain preassignments of a common resource. Function

```
KHE_LAYER KheLayerMakeFromResource(KHE_NODE parent_node,
  KHE_RESOURCE r);
```

builds a layer of this kind. It calls `KheLayerMake` to make a new child layer of `parent_node`, and `KheLayerAddResource` to add `r` to this layer. Then, each child node of `parent_node` which contains a meet preassigned `r` (either directly within the node, indirectly within descendant nodes, or in meets assigned, directly or indirectly, to those meets) is added to the layer.

The *layering* of node `parent_node` is a particular set of layers which is useful when

assigning times to the child nodes of `parent_node`, created by calling function

```
void KheNodeChildLayersMake(KHE_NODE parent_node);
```

This will delete any existing child layers of `parent_node` and add the layers of the layering.

The layering is built as follows. First, for each resource of the instance that possesses a required avoid clashes constraint, one layer is built by calling `KheLayerMakeFromResource` above. If it turns out to be empty, it is immediately deleted again. Each pair of these layers such that one's node set is a subset of the other's is merged with `KheLayerMerge`. Finally, each child of `parent_node` not in any layer goes into a layer (with no resources) by itself.

The layers emerge from `KheNodeChildLayersMake` in whatever order they happen to be. The user will probably need to sort them, by calling `KheNodeChildLayersSort` (Section 5.3), passing it a user-defined comparison function. Section 10.8.2 has an example of a comparison function that seems to work well in practice.

After sorting, there may be value in calling

```
void KheNodeChildLayersReduce(KHE_NODE parent_node);
```

This merges some layers of marginal utility into others, as follows. Suppose there is a layer *L* whose nodes all appear in earlier layers. Then if the meets of the nodes are assigned layer by layer, *L*'s nodes will all be assigned before time assignment reaches *L*. Arguably, *L* could be deleted without harm. However, it does contain one piece of useful information: it knows that the meets to which its resources are preassigned will all be assigned times after *L* is assigned. If this information is to be preserved, *L*'s resources need to be moved forwards to the first earlier layer that is true of. For each node *N* of *L*, find the minimum over all layers containing *N* of the index of the layer. This is the index of the layer during whose time assignment *N* will be assigned. Then find the maximum, over all nodes *N* of *L*, of these minima. This is index of the layer whose assignment will complete the assignment of all the nodes of *L*. If this is smaller than *L*'s index, `KheNodeChildLayersReduce` deletes *L* and moves its resources to this earlier layer.

Two important facts about layers and layerings must be borne in mind. First, they reflect the state of the layer tree at a particular moment. If, after they are built, the tree is restructured (if nodes are moved, etc.) they become out of date and useless. Second, building a layering is slow and should not be done within the inner loops of a solver.

Altogether, it seems best to regard layers as temporary structures, created when required by `KheChildLayersMake` and destroyed by `KheChildLayersDelete`. In between these two calls, nodes may be merged and split, but it is best not to move them. A useful convention, supported by several of KHE's solvers that use layers, is to assume that if child layers are present, then they are up to date. Such solvers begin by calling `KheChildLayersMake` if there are no layers, and end by calling `KheChildLayersDelete`, but only if they called `KheChildLayersMake`.

9.3.2. Layer coordination

High schools usually contain *forms* or *years*, which are sets of students of the same age who follow the same curriculum, at least approximately. These students may be grouped into classes, each represented by one student group resource. At some times, the student group resources of one form might attend the same events, or linked events. For example, they might all attend a

common Sport event, or they might all attend Mathematics at the same times so that they can be regrouped by ability at Mathematics. At other times, they might attend quite different events, but over the course of the cycle they all attend the same amount of each different kind of event: so many times of English, so many of Science, so many of a shared elective, and so on.

As an aid to producing a regular timetable, it might be helpful to *coordinate* the timetables of student groups from the same form: run all the form's English classes simultaneously, all its Mathematics classes simultaneously, and so on. Where resources are insufficient to support this, changes can be made later. In this way, a regular timetable is produced to begin with, and irregularities are introduced only where necessary.

The XML format does not explicitly identify forms, or even say which resource type contains the student group resources. This is in fact an advantage, because it forces us to look for structure that aids regularity. We then coordinate the timetabling of resources that possess the useful structure, without knowing or caring whether they are in fact student group resources.

Coordination will only work when the chosen resources attend similar events. This was the rule when inferring resource partitions (Section 3.5.5), so we take the resource partition as the structural equivalent of the form. The events should occupy all or most of the times of the cycle, otherwise coordination eliminates too many options for spreading them in time. 'Forms' of teachers and rooms are rarely useful, just because they do not satisfy these conditions.

After `KheLayerTreeMake` returns, it is the nodes lying directly below the root node that need to be coordinated, not events or meets. Two child nodes may be coordinated by moving one of them so that it is a child node of the other. KHE offers solver function

```
void KheCoordinateLayers(KHE_NODE parent_node, bool with_domination);
```

which carries out such moves on some of the children of `parent_node`, as follows.

`KheCoordinateLayers` is only interested in resources whose layers have duration at least 90% of the duration of `parent_node`. For each pair of such resources lying in the same resource partition, it checks whether their two layers are similar by building the layers with `KheLayerMakeFromResource` and calling `KheLayerSimilar` (Section 9.3). If so, it uses `KheNodeMove` (Section 9.5.3) to make each node of the second layer a child of the corresponding node of the first, unless the two nodes are the same, forcing these nodes to be simultaneous. It does not assign meets, or remove them from nodes. Finally, it removes the two layers it made.

If `with_domination` is `false`, the behaviour is as described. If `with_domination` is `true`, a slight generalization is used. Suppose that one of the two layers has duration equal to the duration of `parent_node`, and all but one of its nodes is similar to some node in the other layer. Then the dissimilar nodes of the other layer (possibly none) might as well be made children of the one dissimilar node of that layer, since if the other nodes are coordinated they must run simultaneously with it anyway. (The durations of their meets may be incompatible; that is not checked at present, although it should be.) So that is done.

In unusual cases the duration of a layer can be larger after coordinating than before. At the end, if any layers have duration larger than the parent node's duration, `KheCoordinateLayers` tries to reduce the duration of those layers to the parent node's duration, by finding cases where one node of a layer can be safely moved to below another.

9.4. Runarounds

Layer coordination can lead to problems assigning resources. For example, suppose that the five student groups of the Year 7 form each attend one Music event, and that the school has two Music teachers and two Music rooms. Each event is easily accommodated individually, but when the Year 7 layers are coordinated, they run simultaneously and exceed resource limits.

These problems do not arise in large faculties with sufficient resources to accommodate an entire form at once. Thus they do not invalidate the basic idea of node layer coordination. What is needed is a local fix for these problems. This is what *runarounds* provide: a way to spread the events concerned through the times they need, without abandoning coordination altogether.

9.4.1. Minimum runaround duration

Consider the case above where there are not enough Music resources to run the Year 7 Music events simultaneously. If these events lie in nodes that are children of a common parent (one may lie in the parent itself), it is easy to detect this problem: carry out a time assignment at the parent, and see whether the cost of the solution increases. This is assuming that the matching monitors, which detect unsatisfiable resource demands, are attached.

More generally, we can ask how large the duration of the parent node has to be in order to ensure that there is no cost increase. This quantity is called the *minimum runaround duration* of the node. It will be equal to the duration when there is no problem, and larger when there is a problem. It can be calculated as follows. While a time assignment of the child nodes produces a state of higher cost than the unassigned state, add new meets to the parent node. The duration of the parent node when this process ends is its minimum runaround duration. Function

```
bool KheMinimumRunaroundDuration(KHE_NODE parent_node,
   KHE_TIME_SOLVER time_solver, KHE_TIME_OPTIONS options,
   int *duration);
```

sets `*duration` to the minimum runaround duration of `parent_node` and returns `true`, except in an unlikely case, documented below, when it returns `false` with `*duration` undefined.

`KheMinimumRunaroundDuration` first unassigns all the child meets and saves the unassigned cost. It then carries out the time assignment trials just described. For each trial after the first it adds one fresh meet to `parent_node` for each of its original meets, utilizing their durations and time domains, but with no event resources. So the result's duration must be a multiple of the duration of `parent_node`. Before returning, it unassigns all the children and removes the meets it added, leaving the tree in its initial state, unless some child meets were assigned to begin with.

Parameter `time_solver` is a time assignment solver which is called to carry out each trial. A simple solver, such as `KheSimpleAssignTimes` from Section 10.4, should be sufficient here.

Increasing the duration at each trial by the full duration of the node may seem excessive, and there are cases where fewer additional meets would be enough. However, those cases require the child nodes' assignments to overlap in ways that do not work out well in practice, because they may lead to split assignments in the tasks affected.

How many trials are needed? In reasonable instances, each child node's duration should be no greater than the parent node's duration. Thus, after as many trials as there are child nodes plus one, there should be enough room in the parent node to assign every child meet at an offset

which does not overlap with any other, or with the original parent meets. This is the number of trials that `KheMinimumRunaroundDuration` carries out. It stops early if one succeeds with cost no greater than the unassigned cost. It returns `false` only when each trial either did not assign all the child meets (that is, the call on `time_solver` returned `false`) or did assign them all, but at a higher cost than the unassigned cost.

9.4.2. Building runarounds

Nodes may be classified into three types. A *fixed node* has no child nodes. There is no possibility of spreading the events of a fixed node and its descendants through more times than the node's duration. A *problem node* has minimum runaround duration larger than its duration, like the node of Music events used as an example above. It must have child nodes, and timetabling them simultaneously is known to be inferior to spreading them out further. The remaining nodes are *free nodes*: they have child nodes which may run simultaneously, or not, as convenient.

Using `KheNodeMerge` to merge problem nodes with other problem nodes and free nodes can eliminate problem nodes without greatly disrupting regularity. For example, merging a Music problem node of duration 2 and minimum runaround duration 6 with a free node of duration 4 produces a merged node of duration 6 which can usually be timetabled without problems.

If a merged node can be timetabled without the cost of the solution increasing, it may be kept, and is then called a *runaround node*. (The term *runaround* is used by manual timetablers known to the author to describe this kind of timetable, where events like the Music events are 'run around' with other events.) Otherwise it must be split up again and some other merging tried instead. It only remains, then, to decide which sets of nodes to try to merge.

Regularity is easier to attain when nodes have the same duration, so if there are already many nodes of a certain duration, it is helpful if a merged node also has that duration. Nevertheless, a node should not be added to a merge merely to make up some duration: merging limits the choices open to later phases of the solve, so it should be done only when necessary.

A minimum runaround duration could be very large, close to the duration of the whole cycle. For example, suppose there is a single teacher, the school chaplain, who gives each of the five Year 7 student groups 6 times of religious instruction per week. Those events have a minimum runaround duration of 30. When the minimum runaround duration of a node is larger than a certain value, the algorithm given below ignores the node: its events will be awkward to timetable, but runarounds as defined here are not the answer.

To build runaround nodes from the child nodes of `parent_node`, call

```
void KheBuildRunarounds(KHE_NODE parent_node,
  KHE_NODE_TIME_SOLVER mrd_solver, KHE_TIME_OPTIONS mrd_options,
  KHE_NODE_TIME_SOLVER runaround_solver,
  KHE_TIME_OPTIONS runaround_options);
```

where `mrd_solver` and `mrd_options` are passed to `KheMinimumRunaroundDuration` when minimum runaround durations need to be calculated, and `runaround_solver` and `runaround_options` are used to timetable merged nodes. `KheSimpleAssignTimes` is sufficient for `mrd_solver`, and `KheRunaroundNodeAssignTimes` works well as `runaround_solver`. All nodes are unassigned afterwards.

It would not do to merge (for example) a node that includes both Year 7 and Year 8 events with a node that includes only Year 7 ones. So `KheBuildRunarounds` first works out which resources are preassigned to events in or below which nodes (taking account only of preassigned resources which have required avoid clashes constraints, and whose events occupy at least 90% of the duration of `parent_node`), and partitions the child nodes of `parent_node` into disjoint subsets, such the nodes in each subset have the same preassigned resources.

For each disjoint subset independently, `KheBuildRunarounds` tries to build a merged node around each of the subset's problem nodes in turn, largest minimum runaround duration first. When doing this, it prefers to build a node of a particular duration u, and it prefers to use other problem nodes (again, largest minimum runaround duration first), but it will also use free nodes (minimum duration first). It is heuristic, but it usually works well. It is not limited to sequences of pairwise mergings, as clustering algorithms often are. Here is the algorithm in detail:

1. The input is a set of nodes N (one disjoint subset as above), plus u, a desirable duration for a merged node, and v, a maximum duration for a merged node. The output is M, the final set of nodes. Write $d(n)$ for the duration of node n, $r(n)$ for its minimum runaround duration, and $d(X)$ for the total duration of the set of nodes X.

2. Initialize M to empty. Sort N to put free nodes first, in decreasing duration order, problem nodes next, in increasing minimum runaround duration order, and fixed nodes last.

3. If N is empty, stop. Otherwise delete the last element of N and call it n.

4. If n is fixed, problem with $r(n) \geq v$, or free, move it to M and return to Step 3.

5. Here n must be a problem node satisfying $r(n) < v$. Within each of the following cases, some non-empty subsets X of N are defined. In each case, $r(n) \leq d(n) + d(X)$, so a merged node consisting of n merged with X is likely to work well. For each case in turn, and for each set X defined within each case in turn, remove X from N, merge n and X, and timetable the resulting merged node. If that is successful (all events timetabled with no increase in solution cost), add the merged node to M and return to Step 3. If it fails, split the merged node up again, return the nodes of X to their former places in N, and try the next set X; or if there are no more sets, add n to M and return to Step 3.

   Case 1. For each $x \in N$ from last to first such that $r(n) \leq d(n) + d(x) = u \leq v$, let $X = \{x\}$.

   Case 2. For each i from 1 to $|N|$ such that X_i, the last i elements of N, satisfies the condition $r(n) \leq d(n) + d(X_i) \leq v$, let $X = X_i$.

`KheBuildRunarounds` calls `KheMinimumRunaroundDuration` to find minimum runaround durations, passing `mrd_solver` to it. It calls `KheNodeMerge` to merge nodes, `runaround_solver` to timetable merged nodes, and `KheNodeSplit` to undo failed merges. It uses one-fifth of the duration of `parent_node` for v. For u, it builds a frequency table of the durations of child nodes of `parent_node`. It then chooses the duration for which the frequency times the duration is maximum. This weights the choice away from small durations, which are not very useful.

9.5. Rearranging nodes

Earlier sections of this chapter contain the major solvers which work with nodes. This section contains a miscellany of smaller helper funtions which rearrange nodes.

9.5.1. Node merging

Two nodes may be merged by calling

```
bool KheNodeMergeCheck(KHE_NODE node1, KHE_NODE node2);
bool KheNodeMerge(KHE_NODE node1, KHE_NODE node2, KHE_NODE *res);
```

The nodes may be merged if they have the same parent node, possibly `NULL`.

The meets of the result, `*res`, are the meets of `node1` followed by the meets of `node2`, and the child nodes of `*res` are the child nodes of `node1` followed by the child nodes of `node2`. The two nodes must either lie in the same layers and have the same parent, or have no parent, otherwise `KheNodeMerge` aborts. This implies that node merging cannot violate the cycle rule, or any rule. As usual with merging, `node1` and `node2` are undefined afterwards (actually, `node1` is recycled as `*res` and `node2` is freed), but one may write, for example,

```
KheNodeMerge(node1, node2, &node1);
```

to re-use variable `node1` to hold the result.

Merging permits the meets of the child nodes of the two nodes to be assigned to the meets of either node, rather than to just one as before. For example, suppose the layer tree rooted at `node1` contains the Science events of several groups of Year 7 students, and the layer tree rooted at `node2` contains the Music events of the same groups of students. Then originally the Science events must be simultaneous and the Music events must be simultaneous, but afterwards the two kinds of events may intermingle. This may be useful if there are few Music teachers and Music rooms, so that the Music events must be spread out in time. This kind of arrangement is well known to manual timetablers; it has various names, including *runaround*.

There is no operation to split a node into two nodes. However, `KheNodeMerge` may be undone using marks and paths as usual.

9.5.2. Node meet splitting and merging

Node meet splitting and merging (not to be confused with node merging above) split the meets of a node as much as possible, and merge them together as much as possible:

```
void KheNodeMeetSplit(KHE_NODE node, bool recursive);
void KheNodeMeetMerge(KHE_NODE node, bool recursive);
```

Both operations always succeed, although they may do nothing.

For every offset of every meet of `node`, `KheNodeMeetSplit` calls `KheMeetSplit`, passing it the `recursive` parameter. In this way, the meets become as split up as possible.

`KheNodeMeetMerge` sorts the meets so that meets assigned to the same target meets are adjacent, with their target offsets in increasing order, using `KheMeetIncreasingAsstCmp` from

Section 5.2. Unassigned meets go at the end. It then tries to merge each pair of adjacent meets. Any calls to `KheMeetMerge` it makes are passed the `recursive` parameter.

9.5.3. Node moving

A node may be made the child of `parent_node`, instead of its current parent, by calling

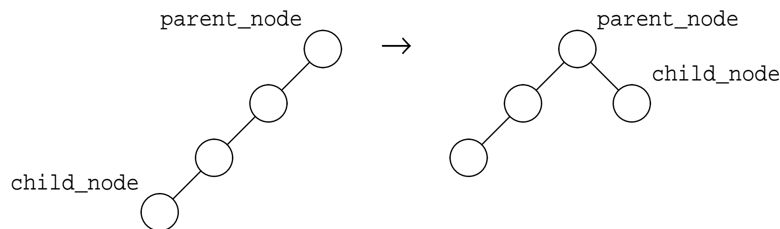
```
bool KheNodeMoveCheck(KHE_NODE child_node, KHE_NODE parent_node);
bool KheNodeMove(KHE_NODE child_node, KHE_NODE parent_node);
```

This does the same as the sequence

```
KheNodeDeleteParent(child_node);
KheNodeAddParent(child_node, parent_node);
```

except that this sequence will fail if any of `child_node`'s meets are assigned initially, whereas `KheNodeMove` deals with such assignments and can fail only the cycle rule.

In most cases, `KheNodeMove` begins by deassigning those meets of `child_node` that are assigned. However, there is one interesting exception. Suppose that `child_node`'s new parent node is an ancestor of `child_node`'s current parent node:



In each case where a complete chain of assignments reaches from a meet `meet` of `child_node` to a meet of `parent_node`, `meet` will be assigned afterwards, to the meet at the end of the chain, with offset equal to the sum of the offsets along the chain. This is valid (it does not change the timetable). Where there is no complete chain, `meet` will be unassigned afterwards.

For example, suppose node `p` has accumulated children to make the timetable regular, but now the children's original freedom to be assigned elsewhere needs to be restored:

```
while( KheNodeChildCount(p) > 0 )
  KheNodeMove(KheNodeChild(p, 0), KheNodeParent(p));
```

`KheNodeMove` preserves the current timetable during these relinkings.

9.5.4. Vizier nodes

A *vizier* (Arabic *wazir*) is a senior official, the one who actually runs the country while the nominal ruler gets the adulation. In a similar way, a *vizier node* sits below another node and does what that other node nominally does: act as the common parent of the subordinate nodes, and hold the meets that those nodes' meets assign themselves to.

Any node can have a vizier, but only the cycle node really has a use for one. By connecting everything to the cycle node indirectly via a vizier, it becomes trivial to try time repairs in

which the meets of the vizier node change their assignments, effecting global alterations such as swapping everything on Tuesday morning with everything on Wednesday morning. Function

```
KHE_NODE KheNodeVizierMake(KHE_NODE parent_node);
```

inserts a new vizier node directly below `parent_node`. Afterwards, `parent_node` has exactly one child node, the vizier; it may be accessed using `KheNodeChild(parent_node, 0)` as usual, and it is also the return value. For every meet `pm` of the parent node, the vizier has one meet `vm` with the same duration as `pm` and assigned to `pm` at offset 0. The domain of `vm` is `NULL`; its assignment is not fixed. Each child node of `parent_node` becomes a child of the vizier; each child layer of `parent_node` becomes a child layer of the vizier; each meet assigned to a meet of the parent node becomes assigned to the corresponding meet of the vizier. If `parent_node` has zones, the vizier is given new corresponding zones, and the parent node's zones are removed.

All this leaves the timetable unchanged, including constraints imposed by domains and zones. The vizier takes over without affecting anyone's existing rights and privileges. A vizier node is not different from any other node; only its role is special.

`KheNodeSwapChildNodesAndLayers` (Section 5.2) is used to move the child nodes and layers to the vizier node, so they are the exact same objects after the call as before. But although the zones added to the vizier correspond exactly with the original zones, they are new objects.

To remove a vizier node, call

```
void KheNodeVizierDelete(KHE_NODE parent_node);
```

Here `parent_node` must have no child layers, no zones, and exactly one child node, assumed to be the vizier. It calls `KheNodeSwapChildNodesAndLayers` again, to make the child nodes of the vizier into child nodes of `parent_node`, and the child layers of the vizier into child layers of `parent_node`. Any assignments to meets in the child nodes of the vizier must be to meets in the vizier, and they are converted into assignments to meets in `parent_node` where possible (when the target meet in the vizier is itself assigned). New zones are created in `parent_node` based on the zones and meet assignments in the vizier. Finally the vizier and its meets are deleted.

Zones are not preserved across calls to `KheNodeVizierMake` and `KheNodeVizierDelete` in the exact way that child nodes and child layers are. The zones added to the vizier node by `KheNodeVizierMake` are new objects, although they do correspond exactly with the zones in `parent_node`. The zones added to `parent_node` by `KheNodeVizierDelete` are also new, and there will be a zone in a given parent meet at a given offset only if there was a meet in the vizier which was assigned that parent meet and was running (with a zone) at that offset. If vizier meets overlap in time (not actually prohibited), that will further confuse the reassignment of zones. It may be best to follow `KheNodeVizierDelete` by a call to some function which ensures that every offset of every parent meet has a zone, for example `KheNodeExtendZones` (Section 9.6).

Function `KheNodeMeetSplit` (Section 9.5.2) is useful with vizier nodes. Splitting a vizier's meets non-recursively opens the way to fine-grained swaps, between half-mornings instead of full mornings, and so on. A wild idea, that the author has not tried, is to have an unsplit vizier with its own split vizier. Then the larger swaps and the smaller ones are available together.

9.5.5. Flattening

Although layer coordination and runaround building are useful for promoting regularity, there may come a point where these kinds of voluntary restrictions prevent assignments which satisfy more important constraints, and so they must be removed.

What is needed is to flatten the layer tree. Two functions are provided for this. The first is

```
void KheNodeBypass(KHE_NODE node);
```

This requires `node` to have a parent, and it moves the children of `node` so that they are children of that parent. The second is

```
void KheNodeFlatten(KHE_NODE parent_node);
```

It moves nodes as required to ensure that all the proper descendants of `parent_node` initially are children of `parent_node` on return.

Both functions use `KheNodeMove` to move nodes. They cannot fail, because `KheNodeMove` fails only when there is a problem with the cycle rule, which cannot occur here. Both functions are 'interesting exceptions' (Section 9.5.3) where assignments are preserved. By convention (Chapter 10), meets with fixed, final assignments should not lie in nodes. If that convention is followed, these functions do not affect such meets.

9.6. Adding zones

Suppose a layer of child nodes of node *n* has its meets assigned to the meets of *n* at various offsets. Define one zone for each child node *c* of the layer, whose meet-offsets are the ones at which *c*'s meets are running. Helper function

```
void KheLayerInstallZonesInParent(KHE_LAYER layer);
```

installs these zones, first deleting any existing zones of the parent node of `layer`, then installing one zone for each child node of `layer` containing at least one assigned meet. Such zones form an image of how one child layer (the first to be assigned, usually) is assigned. An algorithm can use them as a template when assigning the other child layers, or when repairing the assignments of any child layers, including the first layer.

`KheLayerInstallZonesInParent` installs zones representing the assignments of one layer into the layer's parent node. If the duration of the parent node exceeds the duration of the layer, some offsets in some parent node meets will not be assigned any zone. This seems likely to be a problem, or at least a lost opportunity. What to do about it is not clear.

Arguably, zones should be derived from all layers, not just one, in a way that gives every offset a zone. But that is not easy to do, even heuristically. Anyway, there are advantages in using zones derived from a good assignment of some layer, since the assignment proves that those zones work well. This suggests taking the zones installed by `KheLayerInstallZonesInParent` and extending them until every offset has a zone. Accordingly, function

```
void KheNodeExtendZones(KHE_NODE node);
```

ensures that every offset of every meet of `node` has a zone, by assigning one of `node`'s existing

zones to each offset in each meet of `node` that does not have a zone—unless `node` has no zones to begin with, in which case it does nothing.

For each (zone, meet) pair where the meet has at least one offset without a zone, the algorithm finds one option for adding some of the zone to the meet (how much to add, and where), and assigns a priority to the option. Then it selects an option of minimum priority, carries it out, and repeats. It runs out of options only when every offset in every meet has a zone.

An option for adding some of a given zone to a given meet is found as follows. If the zone is already present in the meet, it is best to add it at offsets adjacent to the offsets it already occupies, if possible. If the zone is not already present, it is best to add it adjacent to existing offsets or the ends of the meet, in a continuous run, to avoid fragmentation of the offsets it occupies as well as the offsets it doesn't occupy. Constraints on zone durations arise either way. Within the limits imposed by them, it is best to aim for an ideal zone duration, which in a completely unoccupied meet is the meet duration divided by the total number of zones, but which is adjusted to take account of existing zone durations, with higher being a better option than lower. As the option is decided on, it is assigned a priority based on whether it utilizes an underutilized zone, avoids fragmentation, and approximates to the ideal zone duration.

9.7. Meet splitting and merging

This section presents features which modify the meet splits made by layer tree construction.

9.7.1. Analysing split defects

Given a defect (a monitor of non-zero cost), it is usually easy to see what needs to be done to repair it: if there is a clash, move one of the clashing meets away; if there is a split assignment, try to find a resource to assign to all the tasks; and so on.

Split defects, that is, split events and distribute split events monitors of non-zero cost, are awkward to analyse in this way, partly because split events monitors monitor both the number of meets and their durations, and partly because several split events and distribute split events monitors may cooperate in constraining how a given event is split into meets.

KHE offers a *split analyser* which analyses the split events and distribute split events monitors of a given event, and comes up with a sequence of suggestions as to how any defects among those monitors could be repaired using splits or merges (or both: for example, if there are too few meets of a given duration, that could be corrected by splitting larger meets or by merging smaller ones). To create and subsequently delete a split analyser object, call

```
KHE_SPLIT_ANALYSER KheSplitAnalyserMake(void);
void KheSplitAnalyserDelete(KHE_SPLIT_ANALYSER sa);
```

In practice, it is better to obtain a split analyser object from the `structural_split_analyser` option (Section 8.4.2). To carry out the analysis for a particular solution and event, call

```
void KheSplitAnalyserAnalyse(KHE_SPLIT_ANALYSER sa,
  KHE_SOLN soln, KHE_EVENT e);
```

After doing this, the sequence of suggestions for `e` which are splits may be retrieved by calling

```
int KheSplitAnalyserSplitSuggestionCount(KHE_SPLIT_ANALYSER sa);
void KheSplitAnalyserSplitSuggestion(KHE_SPLIT_ANALYSER sa, int i,
  int *merged_durn, int *split1_durn);
```

for `i` between `0` and `KheSplitAnalyserSuggestionSplitCount(sa) - 1` as usual. Each split suggestion suggests splitting any meet of duration `*merged_durn` into two fragments, one with duration `*split1_durn`. Similarly, the sequence of merge suggestions may be retrieved by

```
int KheSplitAnalyserMergeSuggestionCount(KHE_SPLIT_ANALYSER sa);
void KheSplitAnalyserMergeSuggestion(KHE_SPLIT_ANALYSER sa, int i,
  int *split1_durn, int *split2_durn);
```

Each suggests merging any two meets with durations `*split1_durn` and `*split2_durn`.

Each suggestion is distinct from the others. No notice is taken of constraint weights, except that constraints of weight zero are ignored. The suggestions are updated only by calls to `KheSplitAnalyserAnalyse`; they are unaffected by later changes to the solution. So they go out of date after a split or merge, but become up to date again if that split or merge is undone.

Function

```
void KheSplitAnalyserDebug(KHE_SPLIT_ANALYSER sa, int verbosity,
  int indent, FILE *fp);
```

places a debug print of `sa` onto `fp` with the given verbosity and indent, including suggestions.

9.7.2. Merging adjacent meets

It sometimes happens that at the end of a solve, two meets derived from the same event are adjacent in time and not separated by a break. If the same resources are assigned to both, they can be merged, which may remove a spread defect and thus reduce the overall cost. Function

```
void KheMergeMeets(KHE_SOLN soln);
```

unfixes meet splits in all meets derived from events and carries out all merges that reduce solution cost. For each event `e`, it takes the meets derived from `e` that have assigned times and sorts them chronologically. Then, for each pair of adjacent meets in the sorted order, it tries `KheMeetMerge`, keeping the merge if it succeeds and reduces cost.

`KheMergeMeets` can be called at any time. The best time to call it is probably at the very end of solving, or possibly after time assignment.

9.8. Monitor attachment and grouping

Sometimes, how monitors are grouped and attached is important: when using ejection chains (Chapter 12), for example, or Kempe and ejecting meet moves (Section 10.2.2). This section lays out some general concepts and conventions for monitor attachment and grouping.

Solutions often contain structural constraints: nodes, restricted domains, fixed assignments, and so on. A solver is expected to respect such constraints, unless its specification explicitly states otherwise. They are part of the solution, and every solver should be able to deal with them. In

the same way, a solver may find that some monitors have been deliberately detached before it starts running. For example, all monitors of soft constraints may have been detached, because the caller wants the solver to concentrate on hard constraints. A solver should not change the attachments of monitors to the solution, unless its specification explicitly states otherwise. Its aim is to minimize `KheSolnCost(soln)`, however that is defined by `soln`'s monitor attachments.

There are two ways to exclude a monitor from contributing to the solution cost: by detaching it using `KheMonitorDetachFromSoln`, and by ensuring that there is no path from it to the solution group monitor. The first way should always be used, because it is the efficient way.

Some solvers need specific groupings. The Kempe meet move operation (Section 10.2.2) is an example: its precondition specifies that a particular group monitor must be present. This is permissible, and as with all preconditions it imposes a requirement on the caller of the operation to ensure that the precondition is satisfied when the operation is called. But such requirements should not prohibit the presence of other group monitors. For example, the implementation of the Kempe meet move operation begins with a tiny search for the group monitor it requires. If other group monitors are present nearby, that is not a problem. If this example is followed, multiple requirements for group monitors will not conflict.

There is a danger that group monitors will multiply, slowing down the solve and confusing its logic. It is best if each function that creates a group monitor takes responsibility for deleting it later, even if this means creating the same group monitors over and over again. Timing tests conducted by the author show that adding and deleting the group monitors used by the various solvers in this guide takes an insignificant amount of time.

Two monitors (or defects) are *correlated* when they monitor the same thing, not formally usually, but in reality. For example, if two events are joined by a link events constraint, and one is fixed to the other, then two spread events monitors, one for each event, will be correlated.

Correlated defects are bad for ejection chains. The cost of each defect separately might not be large enough to end the chain if removed, causing the chain to terminate in failure, whereas if it was clear that there was really only one problem, the chain might be able to repair it and continue. So correlated monitors should be grouped, whenever possible. These groups are the equivalence classes of the correlation relation, which is clearly an equivalence relation. A grouping of correlated monitors is called a *primary grouping*.

A function which creates a primary grouping works as follows. Monitors not relevant to the grouping remain as they were. Relevant monitors are deleted from any parents they have, and partitioned into groups of correlated monitors. For each group containing two or more monitors, a group monitor called a *primary group monitor* is made, the monitors are made children of it, and it is made a child of the solution object. For each group containing one monitor, that monitor is made a child of the solution, and no group monitor is made. Any group monitors other than the solution object which lose all their children because of these changes are deleted, possibly causing further deletions of childless group monitors.

A function which deletes a primary grouping visits all monitors relevant to the grouping and deletes those parents of those monitors whose `sub_tag` indicates that they are part of the primary grouping. The deleting is done by calls to `KheGroupMonitorBypassAndDelete`.

Function `KheEjectionChainPrepareMonitors` (Section 12.7.3) creates primary groupings of some correlated monitors, and detaches others, in preparation for ejection chain repair.

Secondary groupings are useful groupings that are not primary groupings (that do not group monitors which monitor the same thing). Instead, they group diverse sets of monitors for particular purposes, such as efficient access to defects.

Secondary groupings are often built on primary groupings: if a monitor that a secondary grouping handles is a descendant of a primary group monitor, the primary group monitor goes into the secondary grouping, replacing the individual monitors which are its children.

A secondary grouping makes one group monitor, called a *secondary group monitor*, not many. The secondary group monitor is not made a child of the solution object, nor are its children unlinked from any other parents that they may have. So it does not disturb existing calculations in any way; rather, it adds a separate calculation on the side. A secondary grouping can be removed by passing the secondary group monitor to `KheGroupMonitorDelete`.

It is convenient to have standard values for the sub-tags and sub-tag labels of the group monitors created by grouping functions, both primary and secondary. So KHE defines type

```
typedef enum {
  KHE_SUBTAG_SPLIT_EVENTS,             /* "SplitEventsGroupMonitor"           */
  KHE_SUBTAG_DISTRIBUTE_SPLIT_EVENTS, /* "DistributeSplitEventsGroupMonitor" */
  KHE_SUBTAG_ASSIGN_TIME,              /* "AssignTimeGroupMonitor"            */
  KHE_SUBTAG_PREFER_TIMES,             /* "PreferTimesGroupMonitor"           */
  KHE_SUBTAG_SPREAD_EVENTS,            /* "SpreadEventsGroupMonitor"          */
  KHE_SUBTAG_LINK_EVENTS,              /* "LinkEventsGroupMonitor"            */
  KHE_SUBTAG_ORDER_EVENTS,             /* "OrderEventsGroupMonitor"           */
  KHE_SUBTAG_ASSIGN_RESOURCE,          /* "AssignResourceGroupMonitor"        */
  KHE_SUBTAG_PREFER_RESOURCES,         /* "PreferResourcesGroupMonitor"       */
  KHE_SUBTAG_AVOID_SPLIT_ASSIGNMENTS, /* "AvoidSplitAssignmentsGroupMonitor" */
  KHE_SUBTAG_AVOID_CLASHES,            /* "AvoidClashesGroupMonitor"          */
  KHE_SUBTAG_AVOID_UNAVAILABLE_TIMES, /* "AvoidUnavailableTimesGroupMonitor" */
  KHE_SUBTAG_LIMIT_IDLE_TIMES,         /* "LimitIdleTimesGroupMonitor"        */
  KHE_SUBTAG_CLUSTER_BUSY_TIMES,       /* "ClusterBusyTimesGroupMonitor"      */
  KHE_SUBTAG_LIMIT_BUSY_TIMES,         /* "LimitBusyTimesGroupMonitor"        */
  KHE_SUBTAG_LIMIT_WORKLOAD,           /* "LimitWorkloadGroupMonitor"         */
  KHE_SUBTAG_ORDINARY_DEMAND,          /* "OrdinaryDemandGroupMonitor"        */
  KHE_SUBTAG_WORKLOAD_DEMAND,          /* "WorkloadDemandGroupMonitor"        */
  KHE_SUBTAG_KEMPE_DEMAND,             /* "KempeDemandGroupMonitor"           */
  KHE_SUBTAG_NODE_TIME_REPAIR,         /* "NodeTimeRepairGroupMonitor"        */
  KHE_SUBTAG_LAYER_TIME_REPAIR,        /* "LayerTimeRepairGroupMonitor"       */
  KHE_SUBTAG_TASKING,                  /* "TaskingGroupMonitor"               */
  KHE_SUBTAG_ALL_DEMAND                /* "AllDemandGroupMonitor"             */
} KHE_SUBTAG_STANDARD_TYPE;
```

for the sub-tags, and the strings in comments, obtainable by calling

```
char *KheSubTagLabel(KHE_SUBTAG_STANDARD_TYPE sub_tag);
```

for the corresponding sub-tag labels. There is also

```
KHE_SUBTAG_STANDARD_TYPE KheSubTagFromTag(KHE_MONITOR_TAG tag);
```

which returns the appropriate sub-tag for a group monitor whose children have the given `tag`.

Functions for creating secondary groupings appear elsewhere in this guide. They include `KheKempeDemandGroupMonitorMake`, needed by Kempe and ejecting meet moves (Section 10.2.2), and several functions used by ejection chain repair algorithms (Section 12.7.4).

When building secondary groupings, these public functions are often helpful:

```
bool KheMonitorHasParent(KHE_MONITOR m, int sub_tag,
  KHE_GROUP_MONITOR *res_gm);
void KheMonitorAddSelfOrParent(KHE_MONITOR m, int sub_tag,
  KHE_GROUP_MONITOR gm);
void KheMonitorDeleteAllParentsRecursive(KHE_MONITOR m);
```

Consult the documentation in the source code to find out what they do.

Chapter 10. Time Solvers

A *time solver* assigns times to meets, or changes their assignments. This chapter presents a specification of time solvers, and describes the time solvers packaged with KHE.

10.1. Specification

If time solvers share a specification, where possible, it is easy to replace one by another, pass one as a parameter to another, and so on. This section recommends such a specification.

In hierarchical timetabling, 'time assignment' means the assignment of the meets of child nodes to the meets of a parent node, so the recommended interface is

```
typedef bool (*KHE_NODE_TIME_SOLVER)(KHE_NODE parent_node,
  KHE_OPTIONS options);
```

This typedef appears in `khe.h`. The recommended meaning is that such a *node time solver* should assign or reassign some or all of the meets of the proper descendants of `parent_node`: it might assign the unassigned meets of the child nodes of `parent_node`, or reassign the meets of proper descendants of `parent_node`, and so on. It is free to reorganize the tree below `parent_node`, provided that every descendant of `parent_node` remains a descendant. It must not change anything in or above `parent_node`. In the tree below `parent_node` it may add, delete, split, and merge meets. Some solvers (e.g. ejection chains) do actually do this, so the caller must take care to avoid the error (very easily made, as the author can testify) of assuming that the set of meets after a time solver is called is the same as before. The `options` parameter is as in Section 8.4.

A solver should return `true` when it has changed the solution (usually for the better, but not necessarily), and when it is not sure whether it did or not. It should return `false` when it did not change the solution. The caller may use this information to evaluate the helpfulness of the solver, or to decide whether to follow it with a repair step, and so on.

A second time solver type is defined in `khe.h`:

```
typedef bool (*KHE_LAYER_TIME_SOLVER)(KHE_LAYER layer,
  KHE_OPTIONS options);
```

Instead of assigning or reassigning meets in the proper descendants of some parent node, a *layer time solver* assigns or reassigns meets in the nodes of `layer` and their descendants, like a node time solver for the parent node of `layer`, but limited to `layer`. The solver is free to reorganize the layer tree below the nodes of `layer` (but not to alter the nodes of `layer`), provided every descendant of each node of `layer` remains a descendant of that node.

If all time solvers follow these rules, then meets that do not lie in nodes will never be visited by them. The recommended convention is that meets should not lie in nodes if and only if they already have assignments that should never be changed.

Time assignment solvers (and solvers generally) are free to use the back pointers of the solution entities they target. However, since there is potential for conflict here when one solver calls another, the following conventions are recommended.

If solver `S` does not use back pointers (if it never sets any), then this should be documented, and solvers that call `S` may assume that back pointers will be unaffected by it. If `S` uses back pointers (if it sets at least one), then this should be documented, and solvers that call `S` must assume that back pointers in the solution objects targeted by `S` will not be preserved. As a safety measure, solvers should set the back pointers that they have used to `NULL` before returning.

10.2. Helper functions

The functions presented in this section assign and unassign meets, but are not complete time solvers in themselves. Instead, they are helper functions that time solvers might find useful.

10.2.1. Node assignment functions

This section presents several functions which affect the assignments of the meets of one node.

These functions swap the assignments of the meets of two nodes:

```
bool KheNodeMeetSwapCheck(KHE_NODE node1, KHE_NODE node2);
bool KheNodeMeetSwap(KHE_NODE node1, KHE_NODE node2);
```

Both `node1` and `node2` must be non-`NULL`. Both functions return `true` if the nodes have the same number of meets, and a sequence of `KheMeetSwap` operations applied to corresponding meets would succeed. `KheNodeMeetSwapCheck` just makes the check, while `KheNodeMeetSwap` performs the meet swaps as well. If `node1` and `node2` are the identical same node, `false` is returned. As usual when swapping, the code fragment

```
if( KheNodeMeetSwap(node1, node2) )
  KheNodeMeetSwap(node1, node2);
```

is guaranteed to change nothing, whether the first swap succeeds or not.

To maximize the chances of success it is naturally best to sort the meets before calling these functions, probably like this:

```
KheNodeMeetSort(node1, &KheMeetDecreasingDurationCmp);
KheNodeMeetSort(node2, &KheMeetDecreasingDurationCmp);
```

This sorting has been omitted from `KheNodeMeetSwapCheck` and `KheNodeMeetSwap` for efficiency, since each node's meets need to be sorted only once, yet the node may be swapped many times. The user is expected to sort the meets of every relevant node, perhaps like this:

```
for( i = 0;  i < KheSolnNodeCount(soln);  i++ )
  KheNodeMeetSort(KheSolnNode(soln, i), &KheMeetDecreasingDurationCmp);
```

before any swapping begins. Some other functions, for example `KheNodeRegular` (Section 5.2), also sort meets, so care is needed.

These functions propagate one node's assignments to another:

```
bool KheNodeMeetRegularAssignCheck(KHE_NODE node, KHE_NODE sibling_node);
bool KheNodeMeetRegularAssign(KHE_NODE node, KHE_NODE sibling_node);
```

`KheNodeMeetRegularAssignCheck` calls `KheNodeMeetRegular` (Section 5.2) to check that the two nodes are regular, and if they are, it goes on to check that each meet in `sibling_node` is assigned, and that each meet of `node` is either already assigned to the same meet and offset that the corresponding meet of `sibling_node` is assigned to, or else may be assigned to that meet and offset. `KheNodeMeetRegularAssign` makes all these checks too, and then carries out the assignments if the checks all pass.

To unassign all the meets of `node`, call

```
void KheNodeMeetUnAssign(KHE_NODE node);
```

Even preassigned meets are unassigned, so some care is needed here.

10.2.2. Kempe and ejecting meet moves

The *Kempe meet move* is a well-known generalization of moves and swaps. It originates as a move of one meet, say from time t_1 to time t_2 (in reality, from one meet and offset to another meet and offset). If this initial move creates clashes with other meets, then they are moved from t_2 to t_1. If that in turn creates clashes with other meets, then they are moved from t_1 to t_2, and so on until all clashes are removed. The result is usually a move or swap, but it can be more complex.

Curiously, the Kempe meet move is not unlike an ejection chain algorithm. Instead of removing a single defect at each step, it removes an arbitrary number, but it tries only one repair: moving to t_2 on odd-numbered steps and to t_1 on even-numbered steps.

Suppose the original meet m_1 has duration d_1. Usually, the Kempe meet move only moves meets of duration d_1, and only from t_1 to t_2 (on odd-numbered steps) and from t_2 to t_1 (on even-numbered steps). However, when m_1 is being moved to a different offset in the same target meet, the Kempe meet move does not commit itself to this until it has examined the first meet, call it m_2, which has to be moved on the second step. If m_2 was immediately adjacent to m_1 in time before m_1 was moved on the first step, it is acceptable for m_2 to have a duration d_2 which is different from d_1. In that case, all meets moved on odd-numbered steps must have duration d_1, and all meets moved on even-numbered steps must have duration d_2, and each meet is moved to the opposite end of the block of adjacent times that m_1 and m_2 were together assigned to originally.

Kempe meet moves need to know what clashes they have caused, and this is done via the matching, partly because it is the fastest way, and partly because it works at any level of the layer tree, unlike avoid clashes monitors, which work only at the root. Accordingly, preassigned demand monitors must be attached, and grouped (directly or indirectly) under a group monitor with sub-tag `KHE_SUBTAG_KEMPE_DEMAND`, by calling

```
KHE_GROUP_MONITOR KheKempeDemandGroupMonitorMake(KHE_SOLN soln);
```

before making any Kempe meet moves. This is a secondary grouping, as defined in Section 9.8. The group monitor's children are the ordinary demand monitors of the preassigned tasks of `soln`. (As usual in KHE, a *preassigned task* is a task derived from a preassigned event

resource.) No primary groupings are relevant here so primary group monitors never replace the ordinary demand monitors. The operation will abort if it cannot find a group monitor with this sub-tag among the parents of the first demand monitor of the first preassigned task of the meet it moves. If that meet has no preassigned tasks, it will search the meets assigned to it, directly and indirectly. There may be no preassigned tasks at all, in which case there can be no clashes. In that case, the Kempe meet move operation does exactly what an ordinary meet move would do.

Use of the matching raises the question of whether Kempe meet moves should try to remove demand defects other than *simple clashes*, where a resource which possesses a hard avoid clashes constraint is preassigned to two meets which are running at the same time. The author's view is that it should not. When there is a simple clash caused by one meet moving to a time, the only possible resolution is for the other to move away. With demand defects in general, there may be multi-way clashes which can be resolved by moving one of several meets away, and that is not what the Kempe meet move is about.

Assuming that the grouping is done correctly, then, a call to

```
bool KheKempeMeetMove(KHE_MEET meet, KHE_MEET target_meet,
  int offset, bool preserve_regularity, int *demand, bool *basic,
  KHE_KEMPE_STATS kempe_stats);
```

will make a Kempe meet move. It is similar to `KheMeetMove` in moving the current assignment of `meet` to `target_meet` at `offset`, but it requires `meet` to be already assigned so that it knows where to move clashing meets back to. It does not use back pointers or visit numbers. It sets `*demand` to the total demand of the meets it moves, to give the caller some idea of the disruption it caused, and it sets `*basic` to `true` if it did not find any meets that needed to be moved back the other way, so that what it did was just a basic meet move. The `kempe_stats` parameter is used for collecting statistics about Kempe meet moves, as described below; it may be `NULL` if statistics are not wanted. There is also

```
bool KheKempeMeetMoveTime(KHE_MEET meet, KHE_TIME t,
  bool preserve_regularity, int *demand, bool *basic,
  KHE_KEMPE_STATS kempe_stats);
```

which moves `meet` to the cycle meet and offset representing time `t`.

If `preserve_regularity` is `false`, these functions ignore zones. One way to take zones into account is to call `KheMeetMovePreservesZones` (Section 5.4) first. In theory this is inadequate when meets of different durations are moved, but the inadequacy will virtually never arise in practice. The other way is to set `preserve_regularity` to `true`, and then the functions will use `KheNodeIrregularity` (Section 5.4) to measure the irregularity of the nodes affected, before and after; the operation will fail if the total irregularity of the nodes affected has increased.

`KheKempeMeetMove` succeeds, returning `true`, if it moves `meet` to `target_meet` at `offset`, possibly moving other meets as well, to ensure that the final state has no new simple clashes and no new cases of a preassigned resource attending a meet at a time when it is unavailable. It fails, returning `false`, in these cases:

- Some call to `KheMeetMove`, which is used to make the individual moves, returns `false`. This includes the case where `meet` is already assigned to `target_meet` at `offset`, which, as previously documented, is defined to fail for the practical reason that the move accomplishes

nothing and pursuing it can only waste time.

- Moving some meet makes some preassigned resource busy when it is unavailable.

- A meet which needs to be moved is not currently assigned to the expected target meet (either `meet`'s original target meet or `target_meet`, depending on whether the current step is odd or even), or has the wrong duration or offset. This prevents the changes from spreading beyond the expected area of the solution.

- `preserve_regularity` is `true` but the operation increases irregularity (discussed above).

- Some meet needs to be moved, but it has already moved during this operation, indicating that the classical graph colouring reason for failure has occurred.

If `KheKempeMeetMove` fails, it leaves the solution in the state it was in at the failure point. In practice, it must be enclosed in `KheMarkBegin` and `KheMarkEnd` (Section 4.10), so that undoing can be used to clean up the mess. This could easily have been incorporated into `KheKempeMeetMove`, producing a version that left the solution unchanged if it failed. However, the caller will probably want to enclose the operation in `KheMarkBegin` and `KheMarkEnd` anyway, since it may need to be undone for other reasons, so cleanup is left to the caller.

The `kempe_stats` parameter is an object (the usual pointer to a private record) used to record statistics about Kempe meet moves. If statistics are wanted, then to create and delete a Kempe stats object, call

```
KHE_KEMPE_STATS KheKempeStatsMake(void);
void KheKempeStatsDelete(KHE_KEMPE_STATS kempe_stats);
```

Actually the usual way to obtain a `KHE_KEMPE_STATS` object is from the `time_kempe_stats` attribute of `KHE_OPTIONS` (Section 8.4.3), which is initialized by `KheKempeStatsMake`. Each time the object is passed to a successful call to `KheKempeMeetMove` or `KheKempeMeetMoveTime`, its statistics are updated. They can be retrieved at any time using the following functions.

A *step* of a Kempe meet move is a move of one meet. The statistics include a histogram of the number of successful Kempe meet moves with `step_count` steps, for each `step_count`, retrievable by calling

```
int KheKempeStatsStepHistoMax(KHE_KEMPE_STATS kempe_stats);
int KheKempeStatsStepHistoFrequency(KHE_KEMPE_STATS kempe_stats,
  int step_count);
int KheKempeStatsStepHistoTotal(KHE_KEMPE_STATS kempe_stats);
float KheKempeStatsStepHistoAverage(KHE_KEMPE_STATS kempe_stats);
```

These return the maximum `step_count` for which there is at least one Kempe meet move, or `0` if none; the number of Kempe meet moves with `step_count` steps; the total number of steps over all Kempe meet moves; and the average number of steps. This last is only safe to call if `KheKempeStatsStepHistoTotal > 0`.

A *phase* of a Kempe meet move is a move of one or more meets in one direction. For example, a Kempe move that turns out to be an ordinary move has one phase; one that turns out to move one meet in one direction, then two in the other, has two phases; and so on. The statistics

include a histogram of the number of successful Kempe meet moves with phase_count phases, for each phase_count, retrievable by calling

```
int KheKempeStatsPhaseHistoMax(KHE_KEMPE_STATS kempe_stats);
int KheKempeStatsPhaseHistoFrequency(KHE_KEMPE_STATS kempe_stats,
  int phase_count);
int KheKempeStatsPhaseHistoTotal(KHE_KEMPE_STATS kempe_stats);
float KheKempeStatsPhaseHistoAverage(KHE_KEMPE_STATS kempe_stats);
```

These return the maximum phase_count for which there is at least one Kempe meet move, or 0 if none; the number of Kempe meet moves with phase_count phases; the total number of phases over all Kempe meet moves; and the average number of phases. This last is only safe to call if KheKempeStatsPhaseHistoTotal > 0.

Functions

```
bool KheEjectingMeetMove(KHE_MEET meet, KHE_MEET target_meet,
  int offset, bool preserve_regularity, int *demand, bool *basic);
bool KheEjectingMeetMoveTime(KHE_MEET meet, KHE_TIME t,
  bool preserve_regularity, int *demand, bool *basic);
```

offer a variant of the Kempe meet move called the *ejecting meet move*. This begins by moving meet to target_meet at offset, and then finds the meets that need to be moved back the other way exactly as for Kempe meet moves (using the same group monitor), but instead of moving them, it unassigns them and stops. KheEjectingMeetMove does not require meet to be assigned initially (the move may be an assignment), not does it carry out any checking of the durations and offsets of the meets it unassigns. All other details are as for Kempe meet moves. Similarly,

```
bool KheBasicMeetMove(KHE_MEET meet, KHE_MEET target_meet,
  int offset, bool preserve_regularity, int *demand);
bool KheBasicMeetMoveTime(KHE_MEET meet, KHE_TIME t,
  bool preserve_regularity, int *demand);
```

are variants in which even the unassignments are omitted. They are the same as KheMeetMove and KheMeetMoveTime as far as changing the solution goes, differing from them only in optionally preserving regularity, and in reporting demand. No group monitor is needed.

The rest of this section describes KheKempeMeetMove's implementation. It is an important operation, so its implementation must be robust, and must squeeze every drop of utility out of the basic idea. KheEjectingMeetMove is just a cut-down version of KheKempeMeetMove.

A *frame* is a set of adjacent positions in a target meet, defined by the target meet, a start offset into the target meet, and a stop offset, which may equal the duration of the target meet, but be no larger. The set of positions runs from the start offset inclusive to the stop offset exclusive. A meet *lies in* a frame when it is assigned to that frame's target meet, and the set of positions it occupies in that target meet is a subset of the set of positions defined by the frame.

The Kempe meet move operation defines four frames. On odd-numbered steps, including the move of the original meet, every move is of a meet lying in a frame called the *odd-from frame* to a frame called the *odd-to frame*. Similarly, every meet move on even-numbered steps is from the *even-from frame* to the *even-to frame*.

The odd-from frame and the odd-to frame have the same duration, and the even-from frame and the even-to frame have the same duration. When a meet is moved, its new target meet is the target meet of the to frame of its step, and its offset in that target meet is defined by requiring its offset in its to frame to equal its former offset in its from frame. This completely determines where the meet is moved to, and ensures that the timetable of moved meets is replicated in the to frame exactly as it was in the from frame.

The implementation will now be described, assuming that the four frames are given. How they are defined will be described later.

First, if there are no preassigned tasks within `meet` or within meets assigned to `meet`, directly or indirectly, then `KheKempeMeetMove` calls `KheMeetMove` and returns its result. Otherwise, it finds the group monitor it needs as described above and begins to trace it. It then carries out a sequence of steps. As each step begins, there is a given set of meets to move, and the step tries to move them. An empty set signals success.

On odd-numbered steps, `KheKempeMeetMove` moves the given set of meets from their offsets in the odd-from frame to the same offsets in the odd-to frame. This will fail if any of the meets do not lie entirely within the odd-from frame, and if any call to `KheMeetMove` returns `false`. Even-numbered steps are the same, using the even-from frame and even-to frame.

The set of meets to move on the first step contains just `meet`. At the end of each step, the set of meets for the next step is found, as follows. The monitor trace is used to find the preassigned demand monitors whose cost increased during the current step. For each of these monitors, `KheMonitorFirstCompetitor` and `KheMonitorNextCompetitor` (Section 7.5.3) are used to find the demand monitors competing with them for supply. These can be of four kinds:

1. A workload demand monitor derived from an avoid unavailable times monitor signals that a preassigned resource has moved to an unavailable time, so fail.

2. Any other workload demand monitor signals a workload overload other than an unavailable time, so ignore it. At a higher level, this defect might cause failure, but, as explained above, the Kempe meet move itself only takes notice of simple clashes and unavailabilities.

3. A demand monitor derived from an unpreassigned task does not signal a simple clash, so ignore it, on the same reasoning as the previous item.

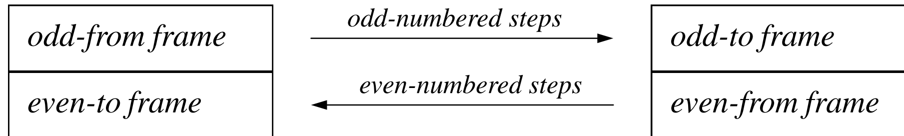
4. A demand monitor derived from a preassigned task signals a simple clash. The appropriate enclosing meet of the task (the one on the chain of assignments leading out of the task's meet just before the expected target meet) is found. If there is no such meet, or it was moved on a previous step, fail. If it was moved on the current step, or is already scheduled to move on the next step, ignore it. Otherwise schedule it to be moved on the next step.

A task is taken to be preassigned when a call to `KheTaskIsPreassigned` (Section 4.9.3), with `as_in_event_resource` set to `false`, returns `true`.

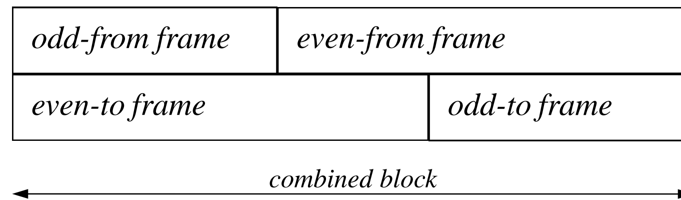
It remains to explain how the four frames are defined.

Given the call `KheKempeMeetMove(meet, target_meet, offset, ...)`, the target meet of the odd-from frame and the even-to frame is `KheMeetAsst(meet)`, and the target meet of the even-from frame and the odd-to frame is `target_meet`. These may be equal, or not.

The odd frames have the same duration, and the even frames have the same duration. Usually, all frames have the same duration, the odd-from frame and the even-to frame are equal, and the even-from frame and the odd-to frame are equal. This is the *separate case*:



But there is another possibility, the *combined case*. Suppose the odd-from frame and the even-from frame are adjacent in time (suppose they have the same target meet, and the start offset of either equals the stop offset of the other). Call the union of their two sets of offsets the *combined block*. In that case, the durations of the odd-from frame and the even-from frame may differ. The odd-to frame occupies the opposite end of the combined block from the odd-from frame, and the even-to frame occupies the opposite end from the even-from frame:



Four diagrams could be drawn here, showing cases where the odd-from frame has shorter and longer duration than the even-from frame, and where it appears to the left and right of the even-from frame. But in all these cases, meets move between the frames in the same way.

To find these frames, first make the initial move of `meet` to `target_meet` at `offset`. This is an odd-numbered move, so it moves a meet from the odd-from frame to the odd-to frame. But it is defined by the caller, so no frames are needed. If it fails, then fail. Otherwise, find the resulting clashing meets. This may cause failure in various cases, as explained above; if successful, all the clashing meets will currently be assigned to `target_meet` at various offsets. If there are no clashing meets, the initial move suffices, so return success. Otherwise, let the *initial clash frame* be the smallest frame enclosing the clashing meets. The even-from frame will be a superset of this frame, to allow all the clashing meets to move legally on the second step.

Next, see whether the separate case applies, as follows. The initial meet must lie inside the odd-to frame after it moves. Since the even-from frame must equal the odd-to frame in the separate case, let the even-from frame be the initial clash frame, enlarged as little as possible to include the initial meet after it moves. Then the odd-from frame is defined completely by the requirements that its duration must equal the duration of the even-from frame, and that the offset of the initial meet in the odd-from frame before it moves must equal its offset in the odd-to frame, and so in the even-from frame, after it moves. Once the odd-from frame is defined in this way, check that it does not protrude out either end of its target meet, nor overlap with the even-from frame. If it passes this check, set the odd-to frame equal to the even-from frame, and set the even-to frame equal to the odd-from frame. The separate case applies.

Otherwise, see whether the combined case applies, as follows. If the initial meet's original target meet is not `target_meet`, or its original position overlaps the initial clash frame, then the combined case does not apply, and so the entire operation fails. Otherwise, set the even-from frame to the initial clash frame, and set the odd-from frame to the smallest frame which both

includes the initial meet's original position and also abuts the even-from frame. This frame must exist; no further checks are needed. Set the odd-to frame to occupy the opposite end of the combined block from the the odd-from frame, and set the even-to frame to occupy the opposite end of the combined block from the even-from frame. The combined case applies.

10.3. Meet bound groups and domain reduction

The functions described in this section do not assign meets. Instead, they reduce meet domains.

10.3.1. Meet bound groups

Meet domains are reduced by adding meet bound objects to meets (Section 4.8.4). Frequently, meet bound objects need to be stored somewhere where they can be found and deleted later. The required data structure is trivial—just an array of meet bounds—but it is convenient to have a standard for it, so KHE defines a type `KHE_MEET_BOUND_GROUP` with suitable operations.

To create a meet bound group, call

```
KHE_MEET_BOUND_GROUP KheMeetBoundGroupMake(void);
```

To add a meet bound to a meet bound group, call

```
void KheMeetBoundGroupAddMeetBound(KHE_MEET_BOUND_GROUP mbg,
  KHE_MEET_BOUND mb);
```

To visit the meet bounds of a meet bound group, call

```
int KheMeetBoundGroupMeetBoundCount(KHE_MEET_BOUND_GROUP mbg);
KHE_MEET_BOUND KheMeetBoundGroupMeetBound(KHE_MEET_BOUND_GROUP mbg, int i);
```

To delete a meet bound group, including deleting all the meet bounds in it, call

```
bool KheMeetBoundGroupDelete(KHE_MEET_BOUND_GROUP mbg);
```

This function returns `true` when every call it makes to `KheMeetBoundDelete` returns `true`.

10.3.2. Exposing resource unavailability

If a meet contains a preassigned resource with some unavailable times, run times will be reduced if those times are removed from the meet's domain, since then futile time assignments will be ruled out quickly. This idea is implemented by

```
void KheMeetAddUnavailableBound(KHE_MEET meet, KHE_COST min_weight,
  KHE_MEET_BOUND_GROUP mbg);
```

This makes a meet bound based on the available times of the resources preassigned to `meet` and to meets with fixed assignments to `meet`, directly or indirectly. It adds this bound to `meet`, and to `mbg` if `mbg` is non-`NULL`.

The meet bound is an occupancy bound whose default time group is the full cycle minus `KheAvoidUnavailableTimesConstraintUnavailableTimes(c)` for each avoid unavailable

times constraint c for the relevant resources whose combined weight is at least min_weight. For example, setting min_weight to 0 includes all constraints; setting it to KheCost(1, 0) includes hard constraints only. Each time group is adjusted for the offset in meet of the meet containing the preassigned resource. If the resulting time group is the entire cycle, as it will be, for example, when meet's preassigned resources are always available, then no meet bound is made.

There is also

```
void KheSolnAddUnavailableBounds(KHE_SOLN soln, KHE_COST min_weight,
  KHE_MEET_BOUND_GROUP mbg);
```

which calls KheMeetAddUnavailableBound for each non-cycle meet in soln whose assignment is not fixed, taking care to visit the meets in a safe order (parents before children).

10.3.3. Preventing cluster busy times and limit idle times defects

This section presents a function which reduces the cost of cluster busy times and limit idle times monitors, by reducing heuristically the domains of the meets to which the monitors' resources are preassigned, before time assignment begins. For example, suppose teacher Jones is limited by a cluster busy times constraint to attend for at most three of the five days of the week. Choose any three days and reduce the time domains of the meets that Jones is preassigned to to those three days. Then those meets cannot cause a cluster busy times defect for Jones.

But first, we need to consider the alternatives. One is to do nothing special during the initial time assignment, and repair any defects later. But there are likely to be many defects then, casting doubt on the value of the initial assignment, since repairing cluster busy times defects is time-consuming and difficult. Repairing limit idle times defects is easier, but it still takes time.

A second alternative is to take these monitors into account as part of the usual method of constructing an initial assignment of times to meets. The usual method is to group the meets into layers (sets of meets which must be disjoint in time, because they share preassigned resources) and assign the layers in turn. Some monitors are handled during layer assignment, including demand and spread events monitors. Cluster busy times monitors can be too, as follows.

Suppose there is a cluster busy times monitor for resource *r* requiring that *r* be busy on at most four of the five days of the cycle. Create a meet with duration equal to the number of times in one day, whose domain is the set of first times on all days. Add a task preassigned *r* to this meet. Then, in the course of assigning *r*'s layer, this meet will be assigned a time, and if there are no clashes, the other meets preassigned *r* will be limited to at most four days as required. At the author's university, this method is used to give most students two half-days off.

There are a few detailed problems: a whole-day meet may not be assignable to any cycle meet, and the author's best method of assigning the meets of one layer (Section 10.6) works best when there are several meets of each duration, whereas here there may be only one whole-day meet. These problems can be surmounted by reducing the domains of the other meets instead of adding a new meet. But there are other problems—problems that may be called fundamental, because they arise from handling clustering one layer at a time.

A resource is *lightly loaded* when it is preassigned to meets whose total duration is much less than the cycle's duration. Cluster busy times monitors naturally apply to lightly loaded resources, because heavily loaded ones don't have the free time that makes clustering desirable.

In university problems, each layer is a set of meets preassigned just one resource: a lightly loaded student. The layers are fairly independent, being mutually constrained only by the capacities of class sections. Under these conditions, handling clustering one layer at a time works well.

But now consider the situation, common in high schools, where each meet contains two preassigned resources, one student group resource and one teacher resource. Suppose the student group resources are heavily loaded, and the teacher resources are lightly loaded and subject to cluster busy times constraints. It is best to timetable the meets one student group layer at a time, because the student group resources are heavily loaded, but this leaves no place to handle the teachers' cluster busy times monitors. Even if the meets were assigned in teacher layers, those layers are often not independent: electives, for example, have several simultaneous meets, requiring several teachers to have common available times.

This brings us to the third alternative, the subject of this section. Before time assignment begins, reduce the domains of meets subject to cluster busy times and limit idle times monitors to guarantee that the monitors have low (or zero) cost, whatever times are assigned later. Use the global tixel matching to avoid mistakes which would make meets unassignable. Function

```
void KheSolnClusterAndLimitMeetDomains(KHE_SOLN soln,
  KHE_COST min_cluster_weight, KHE_COST min_idle_weight,
  float slack, KHE_MEET_BOUND_GROUP mbg, KHE_OPTIONS options);
```

does this. It adds meet bounds to meets, and to `mbg` if `mbg` is non-`NULL`, based on cluster busy times monitors with combined weight at least `min_cluster_weight`, and on limit idle times monitors with combined weight at least `min_idle_weight`. `Minimum` limits are ignored. See below for precisely which monitors are included. If `KheOptionsDiversify(options)` is `true`, the result is diversified by varying the order in which domain reductions for limit idle times monitors are tried.

Carrying out all possible domain reductions is almost certainly too extreme; it gives other solvers no room to move. Parameter `slack` is offered to avoid this problem. For each resource r, function `KheSolnClusterAndLimitMeetDomains` keeps track of $p(r)$, the total duration of the events preassigned r, and $a(r)$, the total duration of the times available to these events, given the reductions made so far. Clearly, it is important for the function to ensure $a(r) \geq p(r)$, since otherwise these events will not have room to be assigned. But, letting s be the value of `slack`, the function actually ensures $a(r) \geq s \cdot p(r)$, or rather, it does not apply any reduction that makes this condition `false`. The minimum acceptable value of `slack` is `1.0`, which is almost certainly too small. A value around `1.5` seems more reasonable.

The remainder of this section describes the issues involved in reducing domains, and how `KheSolnClusterAndLimitMeetDomains` works in detail.

A set of resources may be *time-equivalent*: sure to be busy at the same times. There would be no change in cost if all the cluster busy times and limit idle times monitors of a set of time-equivalent resources applied to just one of them: their costs depend only on when their resource is busy. So although for simplicity the following discussion speaks of individual resources, in fact `KheSolnClusterAndLimitMeetDomains` deals with sets of time-equivalent resources, taken from the `structural_time_equiv` option of its `options` parameter. These must have been set previously by a call to `KheTimeEquivSolve` (Section 9.2).

A cluster busy times monitor for a resource `r` is included when its combined weight is at

least `min_cluster_weight`, its `Maximum` limit is less than its number of time groups, and each time group is either disjoint from or equal to each time group of each previously included monitor for `r`. A limit idle times monitor for a resource `r` of type `rt` is included when its combined weight is at least `min_idle_weight`, `rt` satisfies `KheResourceTypeDemandIsAllPreassigned(rt)`, its time groups are disjoint from each other, and each time group is either disjoint from or equal to each time group of each previously included monitor for that resource. The time groups are usually days, so the disjoint-or-equal requirement is usually no impediment.

An *exclusion operation*, or just *exclusion*, is the addition of an occupancy meet bound (Section 4.8.4) to each meet preassigned a given resource, ensuring that those meets do not overlap a given set of times. An exclusion is *successful* if its calls to `KheMeetAddMeetBound` succeed and do not increase the number of unmatched demand tixels in the global tixel matching. `KheSolnClusterAndLimitMeetDomains` keeps only successful exclusions; unsuccessful ones are tried, then undone. It repeatedly tries exclusions until for each monitor, either a guarantee of sufficiently low cost is obtained, or no further successful exclusions are available. Exclusions based on cluster busy times monitors are tried first, since they are most important. After they have all been tried, the algorithm switches to exclusions based on limit idle times monitors.

Build a graph with one vertex for each resource. For each resource, the aim is to exclude some of its cluster busy times monitors' time groups from its meets, enough to satisfy those monitors' `Maximum` limits. Thinking of each time group as a colour, the aim is to assign a given number of distinct colours from a given set to each vertex.

If some meet (or set of linked meets) has several preassigned resources, those resources should exclude some of the same time groups, to leave others available. Linked meets with preassigned teachers a, b, c, d, and e must not be excluded from Mondays by a, from Tuesdays by b, and so on. The global tixel matching test prevents this extreme example, but we also need to avoid even approaching it. So when two resources share meets, this evidence that they should have similar exclusions is recorded by connecting their vertices by a *positive edge* whose cost is the total duration of the meets they share.

Even when two resources share no meets, they may still influence each other's exclusions, when there is an intermediate resource which shares meets with both of them. Two teachers who teach the same student group are an example of this. If some time group is excluded by one of the teachers, it would be better if it was not excluded by the other, since that again limits choice. In this case the two resources' vertices are joined by a *negative edge* whose cost is the total duration of the meets they share with the intermediate resource. If there are several intermediate resources, the maximum of their costs is used.

Negative edges produce a soft graph colouring problem: a good result gives overlapping sets of colours to vertices connected by positive edges, and disjoint sets of colours to vertices connected by negative edges. This connection with graph colouring rules out finding an optimum solution quickly, but it also suggests a simple heuristic which is likely to work well, since it is based on the successful saturation degree heuristic for graph colouring.

A vertex is *open* when $a(v) > s \cdot p(v)$ (as explained above), and it has at least one untried exclusion with at least one cluster busy times monitor which would benefit from that exclusion. If there are no open vertices, the procedure ends. Otherwise an open vertex is chosen for colouring whose total cost of edges (positive and negative) going to partly or completely coloured vertices is maximum, with ties broken in favour of vertices of larger degree.

Once an open vertex is chosen, the cost of each of its untried colours is found, and the untried colours are tried in order of increasing cost until one of them succeeds or all have been tried. The cost of a colour c is the total cost of outgoing negative edges to vertices containing c, minus the total cost of outgoing positive edges to vertices containing c.

The numbers used by the heuristic are adjusted to take account of the idea that one vertex requiring several colours is similar to several vertices, each requiring one colour, and connected in a clique by strongly negative edges. In particular, being partly coloured increases a vertex's chance of being chosen for colouring, as does requiring more than one more colour.

Saturation degree heuristics are often initialized by finding and colouring a large clique, but nothing of that kind is attempted here. A time group which is a subset of the unavailable times of its resource should always be excluded. This is done, wherever applicable, at the start, after which there may be several partly coloured vertices.

When handling limit idle times monitors, individual times are excluded instead of entire time groups. The time groups of limit idle times monitors are compact, and the excluded times lie at the start or end of one of these time groups. Exclusions which remove a last unexcluded time are tried first, followed by exclusions which remove a first unexcluded time.

Whether an idle exclusion is needed depends on the following calculation. As above, let the *preassigned duration* $p(v)$ of a vertex v be the total duration of the meets that v's resource is preassigned to. Let the *availability* $a(v)$ of vertex v be the number of times that these same meets may occupy. Initially this is the number of times in the cycle, but as time groups are excluded during the cluster busy times phase it shrinks, and then as individual times are excluded during the limit idle times monitor phase it shrinks further.

As explained above, when an exclusion would cause $a(v) \geq s \cdot p(v)$ to become `false`, it is prevented. Assuming this obstacle is not present, consider limit idle times monitor m within v. A worst-case estimate of its number of deviations $d(m)$ can be found as follows.

Let $a(m)$, the *availability* of m, be the total number of unexcluded times in m's time groups. Since time groups are disjoint, $a(m) \leq a(v)$. The worst case for m occurs when as many meets as possible are assigned times outside its time groups, leaving many unassigned and potentially idle times inside. The maximum duration of meets that can be assigned outside m's time groups is $a(v) - a(m)$, leaving a minimum duration of

$$MD(m) = \max(0, p(v) - (a(v) - a(m)))$$

to be assigned within m's time groups. This assignment leaves $a(m) - MD(m)$ of m's available places unfilled. A little algebra shows that this difference is non-negative, given $a(v) \geq p(v)$.

Let $M(m)$ be m's `Maximum` attribute. The worst-case deviation $d(m)$ is the amount by which the number of unfilled places exceeds $M(m)$, that is,

$$d(m) = \max(0, a(m) - MD(m) - M(m))$$

If $d(m)$ is positive, an exclusion which reduces $a(m)$ further may be tried, and multiplying $d(m)$ by $w(m)$, the combined weight of m's constraint, gives a priority for trying such an exclusion.

Limit idle times monitors are tried in decreasing $d(m)w(m)$ order, updated dynamically, and modified by propagating exclusions across positive edges. Negative edges are not used.

10.4. Some basic time solvers

This section presents some basic time solvers. The simplest are

```
bool KheNodeSimpleAssignTimes(KHE_NODE parent_node, KHE_OPTIONS options);
bool KheLayerSimpleAssignTimes(KHE_LAYER layer, KHE_OPTIONS options);
```

They assign those meets of the child nodes of `parent_node` (or of the nodes of `layer`) that are not already assigned. For each such meet, in decreasing duration order, they try all offsets in all meets of the parent node. If `KheMeetAssignCheck` permits at least one of these, the best is made, measuring badness by calling `KheSolnCost`; otherwise the meet remains unassigned, and the result returned will be `false`. These functions do not use options or back pointers.

There is one wrinkle. When assigning a meet which is derived from an event `e`, these functions will not assign the meet to a meet which is already the target of an assignment of some other meet derived from `e`. This is because if two meets from the same event are assigned to the same meet, they are locked into being adjacent, or almost adjacent, in time, undermining the only possible motive for splitting them apart.

These functions are not intended for serious timetabling. They are useful for simple tasks: assigning nodes whose children are known to be trivially assignable, finding minimum runaround durations (Section 9.4.1), and so on.

The logical order to assign times to the nodes of a layer tree is postorder (from the bottom up), since until a node's children are assigned to it, its resource demands are not clear. Function

```
bool KheNodeRecursiveAssignTimes(KHE_NODE parent_node,
  KHE_NODE_TIME_SOLVER solver, KHE_OPTIONS options);
```

applies `solver` to all the nodes in the subtree rooted at `parent_node`, in postorder. It returns `true` when every call it makes on `solver` returns `true`. It uses options and back pointers if and only if `solver` uses them. For example,

```
KheNodeRecursiveAssignTimes(parent_node, &KheNodeSimpleAssignTimes, NULL);
```

carries out a simple assignment at each node, and

```
KheNodeRecursiveAssignTimes(parent_node, &KheNodeUnAssignTimes, NULL);
```

unassigns all meets in all proper descendants of `parent_node`.

Functions

```
bool KheNodeUnAssignTimes(KHE_NODE parent_node, KHE_OPTIONS options);
bool KheLayerUnAssignTimes(KHE_LAYER layer, KHE_OPTIONS options);
```

unassign any assigned meets of `parent_node`'s child nodes (or of `layer`'s nodes). They do not use options or back pointers. Also,

```
bool KheNodeAllChildMeetsAssigned(KHE_NODE parent_node);
bool KheLayerAllChildMeetsAssigned(KHE_LAYER layer);
```

return `true` when the meets of the child nodes of `parent_node` (or of `layer`) are all assigned.

Preassigned meets could be assigned separately first, then left out of nodes so that they are not visited by time assignment algorithms. The problem with this is that a few times may be preassigned to obtain various effects, such as Mathematics first in the day, and this should not affect the way that forms are coordinated. Accordingly, the author favours handling preassigned meets along with other meets, as far as possible.

However, when coordination is complete and real time assignment begins, it seems best to assign preassigned meets first, for two reasons. First, preassignments are special because they have effectively infinite weight. There is no point in searching for alternatives. Second, preassignments cannot be handled by algorithms that are guided by total cost, because they have no assign time constraints, so there is no reduction in cost when they are assigned. Functions

```
bool KheNodePreassignedAssignTimes(KHE_NODE root_node,
  KHE_OPTIONS options);
bool KheLayerPreassignedAssignTimes(KHE_LAYER layer,
  KHE_OPTIONS options);
```

search the child nodes of `root_node`, which must be the overall root node, or the nodes of `layer`, whose parent must be the overall root node, for unassigned meets whose time domains contain exactly one element. `KheMeetAssignTime` is called on each such meet to attempt to assign that one time to the meet. These functions do not use options or back pointers.

KHE's solvers assume that it is always a good thing to assign a time to a meet. However, occasionally there are cases where cost can be reduced by unassigning a meet, because the cost of the resulting assign time defect is less than the total cost of the defects introduced by the assignment. As some acknowledgement of these anomalous cases, KHE offers

```
bool KheSolnTryMeetUnAssignments(KHE_SOLN soln);
```

for use at the end. It tries unassigning each meet of `soln` in turn. If any unassignment reduces the cost of `soln`, it is not reassigned. The result is `true` if any unassignments were kept.

10.5. A time solver for runarounds

Time solver

```
bool KheRunaroundNodeAssignTimes(KHE_NODE parent_node,
  KHE_OPTIONS options);
```

assigns times to the unassigned meets of the child nodes of `parent_node`, using an algorithm specialized for runarounds. It tries to spread similar nodes out through `parent_node` as much as possible. By definition, some resources are scarce in runaround nodes, so it is good to spread demands for similar resources as widely as possible. It works well on symmetrical runarounds, but it can fail in more complex cases. If that happens, it undoes its work and makes a call to `KheNodeLayeredAssignTimes(parent_node, false)` from Section 10.8.2. This is not a very appropriate alternative, but any assignment is better than none.

`KheRunaroundNodeAssignTimes` begins by finding the child layers of `parent_node` using `KheNodeChildLayersMake` (Section 9.3.1), and placing similar nodes at corresponding indexes in the layers, using `KheLayerSimilar` (Section 5.3). It then assigns the unassigned meets of

these nodes. Its first priority is to not increase solution cost; its second is to avoid assigning two child meets to the same parent meet (this would prevent them from spreading out in time); and its third is to prevent corresponding meets in different layers from overlapping in time.

The algorithm is based on a procedure (let's call it `Solve`) which accepts a set of child layers, each accompanied by a set of triples of the form

```
(parent_meet, offset, duration)
```

meaning that `parent_meet` is open to assignment by a child meet of the layer, at the given offset and duration. The task of `Solve` is to assign all the unassigned meets of the nodes of its layers.

The initial call to `Solve` is passed all the child layers. Each layer's triples usually contain one triple for each parent meet, with offset 0 and the duration of the parent meet for duration, indicating that the parent meets are completely open for assignment. If any meets are assigned already, the triples are modified accordingly to record the smaller amount of open space.

`Solve` begins by finding the maximum duration, `md`, of an unassigned meet in any of its layers. It assigns all meets with this duration in all layers itself, and then makes recursive calls to assign the meets of smaller duration. For each layer, it takes the meets of duration `md` in the order they appear in the layer and its nodes. It assigns these meets to consecutive suitable positions through the layer, shifting the starting point of the search for suitable positions by one place in the parent layer as it begins each layer. It never makes an assignment which increases the cost of the solution, and it makes an assignment which causes two child meets to be assigned to the same parent meet only as a last resort. If some meet fails to assign, the whole algorithm fails and the problem is passed on to `KheNodeChildLayersAssignTimes` as described above.

As meets are assigned, the offsets and durations of the triples change to reflect the fact that the parent meets are more occupied. After all assignments of meets of duration `md` are complete, the layers are sorted to bring layers with equal triples together. Each set of layers with equal triples is then passed to a recursive call to `Solve`, which assigns its meets of smaller duration.

The purpose of handling sets of layers with equal triples together in this way can be seen in an example. Suppose the parent node has two doubles and each child node has one double. Then there are two ways to assign the child's double; half the child layers will get one of these ways, the other half will get the other way. The layers in each half have identical assignments so far, undesirably but inevitably. By bringing them together we maximize the chance that the recursive call which assigns the singles will find a way to vary the remaining assignments.

10.6. Extended layer matching with Elm

A good way to assign times to meets is to group the meets into nodes, group the nodes into layers, and assign times to the meets layer by layer. The advantage of doing it this way is that the meets of one layer strongly constrain each other, because they share preassigned resources so must be disjoint in time. Assigning times to the meets of one layer, then, is a key step.

Any initial assignment of times to the meets of one layer will probably require repair. But repair is time-consuming, and it will help if the initial assignment has few defects—as a first priority, few demand defects, but also few defects of other kinds. The method presented in this section, called *extended layer matching*, or *Elm* for short, is the author's best method of finding an initial assignment of times to the meets of one layer.

If all meets have duration 1 and minimizing ordinary demand defects is the sole aim, the problem can be solved efficiently using weighted bipartite matching. Make each meet a node and each time a node, and connect each meet to each time it may be assigned, by an edge whose cost is the number of demand defects that assignment causes. Among all matchings with the maximum number of edges, choose one of minimum cost and make the indicated assignments.

Elm is based on this kind of weighted bipartite matching, called *layer matching* by the author, making it good at minimizing demand defects. It is *extended* with ideas that heuristically reduce other defects. Layer matching was called *meta-matching* in the author's early work, because it operates above another matching, the global tixel matching.

Elm can be used without understanding it in detail, by calling

```
bool KheElmLayerAssign(KHE_LAYER layer,
  KHE_SPREAD_EVENTS_CONSTRAINT sec, KHE_OPTIONS options);
```

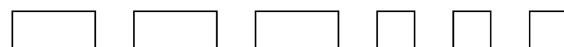
`KheElmLayerAssign` finds an initial assignment of the meets of the child nodes of `layer` to the meets of the parent node of `layer`, leaving any existing assignments unchanged, and returning `true` if every meet of `layer` is assigned afterwards. It works well with the reduced meet domains installed by solvers such as `KheSolnClusterAndLimitMeetDomains` (Section 10.3.3) for minimizing cluster busy times and limit idle times defects. It tries to minimize demand defects, and if `layer`'s parent node has zones, it also tries to make its assignments meet and node regular with those zones, which should help to minimize spread events defects. If the `diversify` option of `options` (Section 8.4) is `true`, it consults the solution's diversifier, and its results may vary with the diversifier. It does not repair its assignment, leaving that to other functions.

Parameter `sec` is optional (may be `NULL`); a simple choice for it would be any spread events constraint whose number of points of application is maximal. If `sec` is present, the algorithm tries to assign the same number of meets to each of `sec`'s time groups. To see why, consider an example of the opposite. Suppose the events are to spread through the days, and the Wednesday times are assigned eight singles, while the Friday times are assigned four doubles. It's likely that some events will end up meeting twice on Wednesdays and not at all on Fridays. The `sec` parameter acts only with low priority. It is mainly useful on the first layer, when there are no zones and the segmentation is more or less arbitrary.

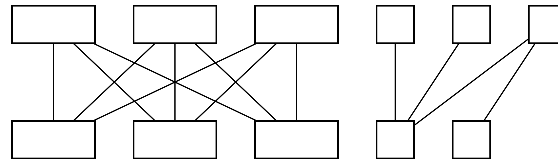
10.6.1. Introducing layer matching

This section introduces layer matching. Later sections describe the implementation. Suppose some layer has three meets of duration 2 and two meets of duration 1, like this:

These *child meets* have to be assigned to non-overlapping offsets in the meets of the parent node (the *parent meets*). Suppose there are three parent meets of duration 2 and three of duration 1:



and suppose (for the moment) that assignments are only possible between meets of the same duration. Then a bipartite graph can represent all the possibilities:



The child meets (the bottom row) are the demand nodes, and the parent meets (the top row) are the supply nodes. Each edge represents one potential assignment of one child meet. Not all edges are present: some are missing because of unequal durations, others because of preassignments and other domain restrictions. For example, the last child meet above appears to be preassigned.

When one of the potential assignments is made, there is a change in solution cost. Each edge may be labelled by this change in cost. Suppose that a matching of maximum size (number of edges) is found whose cost (total cost of selected edges) is minimum. There is a reasonably efficient algorithm for doing this. This matching is the *layer matching*; it defines a legal assignment for some (usually all) child meets, and its cost is a lower bound on the change in solution cost when these meets are assigned to parent meets without any overlapping, as is required since the child meets share a layer and thus presumably share preassigned resources.

The lower bound is only exact if each assignment changes the solution cost independently of the others. This is true for many kinds of monitors, but not all, and it is one reason why the lower bound produced by the matching is not exact. In fact, costs contributed by limit idle times, cluster busy times, and limit busy times monitors only confuse layer matching. So for each resource of the layer, any attached monitors of these kinds are detached at the beginning of `KheElmLayerAssign` and re-attached at the end.

Parent meets usually have larger durations than child meets, allowing choices in packing the children into the parents. The parent node typically represents the week, so it might have, say, 10 meets each of duration 4 (representing 5 mornings and 5 afternoons), whereas the child meets typically represent individual lessons, so they might have durations 1 and 2. A *segment* of parent meet `target_meet` is a triple

```
(target_meet, offset, durn)
```

such that it is legal to assign a child meet of duration `durn` to `target_meet` at `offset`. A *segmentation* of the parent meets is a set of non-overlapping segments that covers all offsets of all parent meets. It is the segments of a segmentation, not the parent meets themselves, that are used as supply nodes. There may be many segmentations, but the layer matching uses only one. This is the other reason why the lower bound is not exact.

A *layer matching graph* is a bipartite graph with one demand node for each meet of a given layer, and one supply node for each segment of some segmentation of the meets of the layer's parent node. For each unassigned child meet `meet`, there is one edge to each parent segment whose duration equals the duration of `meet` and to which `meet` is assignable according to `KheMeetAssignCheck`. The cost of the edge is the cost of the solution when the assignment is made, found by making the assignment, calling `KheSolnCost`, then unassigning again. (Using the solution cost rather than the change in cost ensures that edge costs are always non-negative, as required behind the scenes.) For each assigned child meet `meet`, a parent segment with `meet`'s target meet, offset, and duration is the only possible supply node that the meet can be connected to; if present, the edge cost is 0.

A *layer matching* is a set of edges from the graph such that no node is an endpoint of two or more of the selected edges. A *best matching* is a layer matching of minimum *cost* (sum of edge costs) among all matchings of maximum *size* (number of edges).

The layer giving rise to the demand nodes consists of nodes, each of which typically contains a set of meets for one course. This set of meets will typically want to be spread through the cycle, not bunched together. Each meet generates a demand node, and a set of demand nodes whose meets are related in this way is called a *demand node group*.

There is also a natural grouping of supply nodes, with each *supply node group* consisting of those supply nodes which originated from the same parent meet. Thus, the supply nodes of one group are adjacent in time.

It would be good to enforce the following rule: two demand nodes from the same demand node group may not match with two supply nodes from the same supply node group (because if they did, all chance of spreading out the demand nodes in time would be lost). There is no hope of guaranteeing this rule, because there are cases where it must be violated, and because minimizing cost while guaranteeing it appears to be an NP-complete problem. However, Elm encourages it. When finding a minimum-cost matching, it adds an artificial increment to the cost of each augmenting path that would violate it, thus making those paths relatively uncompetitive and unlikely to be applied. The approach is purely heuristic, but it usually works well.

The overall structure of the layer matching graph is now clear. There are demand nodes, each representing one meet of the layer, grouped into demand node groups representing courses. There are supply nodes, each representing one segment of one meet of the parent node, grouped into supply node groups representing the meets of the parent node. Edges between supply nodes and demand nodes are not defined explicitly; they are determined by the durations and assignability of the meets and segments.

10.6.2. The core module

This section describes the *core module*, which implements the layer matching graph, including maintaining a best matching. Elm also has *helper modules*, described in following sections. They have no behind-the-scenes access to the graph; they use only the operations described here.

The core module follows the previous description closely, except that it uses ‘demand’ for ‘demand node’, ‘demand group’ for ‘demand node group’, and so on—for brevity, and so that ‘node’ always means an object of type `KHE_NODE`. This Guide will do this too from now on.

Elm’s types and functions (apart from `KheElmLayerAssign`) are declared in a header file of their own, called `khe_elm.h`. So to access the functions described from here on,

```
#include "khe.h"
#include "khe_elm.h"
```

must be placed at the start of the source file.

We begin with the operations on type `KHE_ELM`, representing one elm. An elm for a given layer is created and deleted by functions

```
KHE_ELM KheElmMake(KHE_LAYER layer, KHE_OPTIONS options);
void KheElmDelete(KHE_ELM elm);
```

If the `diversify` option of `options` is `true`, then the layer's solution's diversifier is used to diversify the elm. In addition to the elm itself, `KheElmMake` creates one demand group for each child node of `layer`, containing one demand for each meet of the child node. It also creates one supply group for each meet of the layer's parent node, containing one supply representing the entire meet. `KheElmDelete` deletes all these objects along with the elm. The sets of meets in the parent and child nodes should not change during the elm's lifetime, although the state of one meet (its assignment, domain, etc.) may change.

The layer and options may be accessed by

```
KHE_LAYER KheElmLayer(KHE_ELM elm);
KHE_OPTIONS KheElmOptions(KHE_ELM elm);
```

To access the demand groups, call

```
int KheElmDemandGroupCount(KHE_ELM elm);
KHE_ELM_DEMAND_GROUP KheElmDemandGroup(KHE_ELM elm, int i);
```

in the usual way. To access the supply groups, call

```
int KheElmSupplyGroupCount(KHE_ELM elm);
KHE_ELM_SUPPLY_GROUP KheElmSupplyGroup(KHE_ELM elm, int i);
```

An elm also holds a best matching as defined above. The functions related to it are

```
int KheElmBestUnmatched(KHE_ELM elm);
KHE_COST KheElmBestCost(KHE_ELM elm);
bool KheElmBestAssignMeets(KHE_ELM elm);
```

`KheElmBestUnmatched` returns the number of unmatched demands in the best matching. `KheElmBestCost` returns its cost—not a solution cost, but a sum of edge costs, each of which is a solution cost. `KheElmDemandBestSupply`, defined below, reports which supply a given demand is matched with. To assign the unassigned meets of `elm`'s layer according to the best matching, call `KheElmBestAssignMeets`; it returns `true` if every meet is assigned afterwards. Elm updates the best matching only when one of these four functions is called, for efficiency.

Elm has a 'special node' which is begun and ended by calling

```
void KheElmSpecialModeBegin(KHE_ELM elm);
void KheElmSpecialModeEnd(KHE_ELM elm);
```

While the special mode is in effect, Elm assumes that edges can change their presence in the layer matching graph but not their cost. So when updating edges in special mode, Elm only needs to find whether each edge is present or not, which is much faster than finding costs as well.

To support splitting supplies so that their numbers in each time group of a spread events constraint are approximately equal, these functions are offered:

```
void KheElmUnevennessTimeGroupAdd(KHE_ELM elm, KHE_TIME_GROUP tg);
int KheElmUnevenness(KHE_ELM elm);
```

`KheElmUnevennessTimeGroupAdd` instructs `elm` to keep track of the number of supplies whose

starting times lie within tg. KheElmUnevenness returns the sum over all these time groups of a quantity related to the square of this number. For a given set of supplies, this will be smaller when they are distributed evenly among the time groups than when they are not.

Function

```
void KheElmDebug(KHE_ELM elm, int verbosity, int indent, FILE *fp);
```

produces a debug print of elm onto fp with the given verbosity and indent. Demands are represented by their meets, and supplies are represented by their meets, offsets, and durations. If verbosity >= 2, the print includes the best matching. Function

```
void KheElmDebugSegmentation(KHE_ELM elm, int verbosity,
  int indent, FILE *fp);
```

is similar except that it concentrates on elm's segmentation.

Demand groups have type KHE_ELM_DEMAND_GROUP. To access their attributes, call

```
KHE_ELM KheElmDemandGroupElm(KHE_ELM_DEMAND_GROUP dg);
KHE_NODE KheElmDemandGroupNode(KHE_ELM_DEMAND_GROUP dg);
int KheElmDemandGroupDemandCount(KHE_ELM_DEMAND_GROUP dg);
KHE_ELM_DEMAND KheElmDemandGroupDemand(KHE_ELM_DEMAND_GROUP dg, int i);
```

These return dg's enclosing elm, the child node of the original layer that gave rise to dg, dg's number of demands, and its ith demand.

Elm maintains edges between demands and supplies automatically. But if a demand's meet changes in some way (for example, if its domain changes), Elm has no way of knowing that this has occurred. When the meets of the demands of a demand group change, the user must call

```
void KheElmDemandGroupHasChanged(KHE_ELM_DEMAND_GROUP dg);
```

to inform Elm that the edges touching the demands of dg must be remade before being used.

A demand group may contain any number of zones. If there are none, then zones have no effect. If there is at least one zone, then the demand group's demands may match only with supplies that begin in one of its zones. The value NULL counts as a zone. Functions

```
void KheElmDemandGroupAddZone(KHE_ELM_DEMAND_GROUP dg, KHE_ZONE zone);
void KheElmDemandGroupDeleteZone(KHE_ELM_DEMAND_GROUP dg, KHE_ZONE zone);
```

add and delete a zone from dg, including calling KheElmDemandGroupHasChanged. The value of zone may be NULL. To check whether dg contains a given zone, call

```
bool KheElmDemandGroupContainsZone(KHE_ELM_DEMAND_GROUP dg, KHE_ZONE zone);
```

To visit the zones of a demand group, call

```
int KheElmDemandGroupZoneCount(KHE_ELM_DEMAND_GROUP dg);
KHE_ZONE KheElmDemandGroupZone(KHE_ELM_DEMAND_GROUP dg, int i);
```

Function

```
void KheElmDemandGroupDebug(KHE_ELM_DEMAND_GROUP dg,
  int verbosity, int indent, FILE *fp);
```

sends a debug print of dg with the given verbosity and indent to fp.

Demands have type KHE_ELM_DEMAND. To access their attributes, call

```
KHE_ELM_DEMAND_GROUP KheElmDemandDemandGroup(KHE_ELM_DEMAND d);
KHE_MEET KheElmDemandMeet(KHE_ELM_DEMAND d);
```

These return the enclosing demand group, and the meet that gave rise to the demand.

As explained above, when a demand's meet changes in some way that affects the demand's edges, Elm must be informed. For a single demand, this is done by calling

```
void KheElmDemandHasChanged(KHE_ELM_DEMAND d);
```

This is called by KheElmDemandGroupHasChanged for each demand in its demand group. To find out which supply d is matched with in the best matching, call

```
bool KheElmDemandBestSupply(KHE_ELM_DEMAND d,
  KHE_ELM_SUPPLY *s, KHE_COST *cost);
```

If d is matched with a supply in the best matching, KheElmDemandBestSupply sets *s to that supply and *cost to the cost of the edge, and returns true; otherwise it returns false. And

```
void KheElmDemandDebug(KHE_ELM_DEMAND d, int verbosity,
  int indent, FILE *fp);
```

sends a debug print of d with the given verbosity and indent to fp.

Supply groups have type KHE_ELM_SUPPLY_GROUP. To access their attributes, call

```
KHE_ELM KheElmSupplyGroupElm(KHE_ELM_SUPPLY_GROUP sg);
KHE_MEET KheElmSupplyGroupMeet(KHE_ELM_SUPPLY_GROUP sg);
int KheElmSupplyGroupSupplyCount(KHE_ELM_SUPPLY_GROUP sg);
KHE_ELM_SUPPLY KheElmSupplyGroupSupply(KHE_ELM_SUPPLY_GROUP sg, int i);
```

These return sg's enclosing elm, the meet of the layer's parent node that gave rise to it, its number of supplies (segments), and its ith supply. And

```
void KheElmSupplyGroupDebug(KHE_ELM_SUPPLY_GROUP sg,
  int verbosity, int indent, FILE *fp);
```

sends a debug print of sg with the given verbosity and indent to fp.

Supplies have type KHE_ELM_SUPPLY. To access their attributes, call

```
KHE_ELM_SUPPLY_GROUP KheElmSupplySupplyGroup(KHE_ELM_SUPPLY s);
KHE_MEET KheElmSupplyMeet(KHE_ELM_SUPPLY s);
int KheElmSupplyOffset(KHE_ELM_SUPPLY s);
int KheElmSupplyDuration(KHE_ELM_SUPPLY s);
```

KheElmSupplySupplyGroup is the enclosing supply group, KheElmSupplyMeet is the enclosing

supply group's meet, and KheElmSupplyOffset and KheElmSupplyDuration return an offset and duration within that meet, defining one segment.

To facilitate calculations with zones, each supply maintains the set of distinct zones that its offsets lie in. These may be accessed by calling

```
int KheElmSupplyZoneCount(KHE_ELM_SUPPLY s);
KHE_ZONE KheElmSupplyZone(KHE_ELM_SUPPLY s, int i);
```

A NULL zone counts as a zone, so KheElmSupplyZoneCount is always at least 1.

To facilitate the handling of preassigned and previously assigned demands, Elm offers

```
void KheElmSupplySetFixedDemand(KHE_ELM_SUPPLY s, KHE_ELM_DEMAND d);
KHE_ELM_DEMAND KheElmSupplyFixedDemand(KHE_ELM_SUPPLY s);
```

KheElmSupplySetFixedDemand informs elm that d is the only demand suitable for matching with s, or if d is NULL (the default), that there is no restriction of that kind. If d != NULL, d's duration must equal the duration of s. A call to KheElmDemandHasChanged(d) is included. KheElmSupplyFixedDemand returns s's current fixed demand, possibly NULL.

To facilitate the handling of irregular monitors, a supply can be temporarily removed from the graph (so that it does not match any demand) and subsequently restored:

```
void KheElmSupplyRemove(KHE_ELM_SUPPLY s);
void KheElmSupplyUnRemove(KHE_ELM_SUPPLY s);
```

KheElmSupplyRemove aborts if s has a fixed demand. A removed supply merely becomes unmatchabled, it does not get deleted from node lists and so on. Function

```
bool KheElmSupplyIsRemoved(KHE_ELM_SUPPLY s);
```

reports whether s is currently removed.

When KheElmMake returns, there is one demand group for each child node, one demand for each child meet, one supply group for each parent meet, and one supply for each supply group, with offset 0 and duration equal to the duration of the meet. All this is fixed except that supplies may be split and merged by calling

```
bool KheElmSupplySplitCheck(KHE_ELM_SUPPLY s, int offset, int durn,
  int *count);
bool KheElmSupplySplit(KHE_ELM_SUPPLY s, int offset, int durn,
  int *count, KHE_ELM_SUPPLY *ls, KHE_ELM_SUPPLY *rs);
void KheElmSupplyMerge(KHE_ELM_SUPPLY ls, KHE_ELM_SUPPLY s,
  KHE_ELM_SUPPLY rs);
```

KheElmSupplySplitCheck returns true when s may be split so that one of the fragments has the given offset and duration. If so, it sets *count to the total number of fragments that would be produced, either 1, 2, or 3. KheElmSupplySplit is the same except that it actually performs the split when possible, leaving s with the given offset and duration. Splitting is possible when

```
KheElmSupplyFixedDemand(s) == NULL &&
KheElmSupplyOffset(s) <= offset &&
offset + durn <= KheElmSupplyOffset(s) + KheElmSupplyDuration(s)
```

This says that `s` is not fixed to some demand, and that `offset` and `durn` define a set of offsets lying within the set of offsets currently covered by `s`. Otherwise it returns `false`.

If `KheElmSupplyOffset(s) < offset`, then a supply `*ls` is split off `s` at left, holding the offsets from `KheElmSupplyOffset(s)` inclusive to `offset` exclusive; otherwise `*ls` is set to `NULL`. If `offset + durn < KheElmSupplyOffset(s) + KheElmSupplyDuration(s)`, then a supply `*rs` is split off `s` at right, holding the offsets from `offset + durn` inclusive to `KheElmSupplyOffset(s) + KheElmSupplyDuration(s)` exclusive; otherwise `*rs` is set to `NULL`. The original `s` is left with offsets from `offset` inclusive to `offset + durn` exclusive.

`KheElmSupplyMerge` undoes the corresponding `KheElmSupplySplit`. Either or both of `ls` and `rs` may be `NULL`. No meet splitting or merging is carried out by these operations.

Finally,

```
void KheElmSupplyDebug(KHE_ELM_SUPPLY s, int verbosity,
  int indent, FILE *fp);
```

sends a debug print of `s` with the given verbosity and indent to `fp`.

10.6.3. Splitting supplies

The elm returned by `KheElmMake` has only a trivial segmentation, with one segment per parent meet. Few or no demands will match with these supplies, because only demands and supplies of equal duration match. So the initial supplies have to be split using `KheElmSupplySplit`.

Elm has a helper module which splits supplies heuristically. It offers just one function:

```
void KheElmSplitSupplies(KHE_ELM elm, KHE_SPREAD_EVENTS_CONSTRAINT sec);
```

If the `diversify` option of `elm`'s `options` attribute is `true`, its result varies depending on the layer's solution's diversifier. The `sec` parameter of `KheElmSplitSupplies` may be `NULL`. If non-`NULL`, `KheElmSplitSupplies` tries to find a segmentation in which each time group of `sec` covers the same number of segments, as explained for `KheElmLayerAssign` above.

`KheElmSplitSupplies` works as follows. Begin by handling demands whose meets are preassigned or already assigned. For each such demand, split a supply to ensure that exactly the right segment is present, and use `KheElmSupplySetFixedDemand` to fix the supply to the demand. If the required split cannot be made, the demand remains permanently unmatched.

Sort the remaining demands by increasing size of their meets' domains (in practice this also sorts by decreasing duration), breaking ties by decreasing demand. Use `KheMeetAssignFix` to ensure that these meets cannot be assigned. This removes them from the matching to begin with (strictly speaking, it prevents them from having any outgoing edges in the matching graph).

For each demand in turn, unfix its meet and observe the effect of this on the best matching. If the size of the best matching increases by one, proceed to the next demand. Otherwise, the demand has failed to match, and this must be corrected (if possible) by splitting segments of larger duration into smaller segments that it can match with. For each supply whose duration

is larger than the duration of the demand, try splitting the supply in all possible ways into two or three smaller segments such that at least one of the fragments has the same duration as the demand. If there was at least one successful split, redo the best of them.

The best split is determined by an evaluation with five components:

1. The split must be *successful*: it must increase the size of the best matching by one. Only successful splits are eligible for use; if there are none, the demand remains unmatched.

2. It is better to split a segment into two fragments than into three. For example, when splitting a double from a meet of duration 4, it is better to take the first two times or the last two, rather than the middle two, since the latter leaves fewer choices for future splits.

3. If the parent node has zones, it is desirable to use a segment overlapping only one zone, to produce meet regularity (Section 5.4) with the layer used to create the zones.

4. The split should produce a best matching whose cost is as small as possible.

5. If `sec != NULL`, the split should encourage the evenness that `sec`'s presence requests.

These are combined lexicographically: later criteria only apply when earlier ones are equal. Meet regularity has higher priority than cost because cost can often be improved later, whereas meet regularity once lost is lost forever.

After all demands are processed, if any supplies have duration larger than the duration of all demands, split them into smaller pieces, preferably supplies regular with the zones, if any. This adds more edges, and so may reduce the cost of the best matching, at no risk to its size. It is important when timetabling layers of small duration, such as layers containing staff meetings.

10.6.4. Improving node regularity

When the parent node has zones, `KheElmSplitSupplies` produces good meet regularity but does nothing to promote node regularity. This can be done by following it with a call to

```
void KheElmImproveNodeRegularity(KHE_ELM elm);
```

implemented by another Elm helper module. It does nothing when there are no zones. When there are, it removes edges from the matching graph to improve the node regularity of the edges with respect to the zones. If requested by the `diversify` option of `elm`'s `options` attribute, it consults the solution's diversifier, and the edges it removes vary with the diversifier.

The problem of removing edges from a layer matching graph to maximize node regularity with zones while keeping the matching cost low may seem obscure, but it is one of the keys to effective time assignment in high school timetabling. Bin packing is reducible to this problem, so it is NP-complete. Even the small instances (up to ten nodes in each layer, say) that occur in practice seem hard to solve to optimality. The author tried a tree search which would have produced an optimal result, but could not make it efficient, even with several pruning rules. So `KheElmImproveNodeRegularity` is heuristic.

Although many kinds of defects contribute to the edge costs that make up the matching cost, in practice the cost is dominated by demand cost (including the cost of avoid clashes and

avoid unavailable times defects). Every unit of demand cost incurred when assigning a time represents an unassignable resource at that time, implying that either the final solution will have a significant defect, or else that the time assignment will have to be changed later.

However, not all demand costs are equally important. When the cost is incurred by a child node with no children, all of the meets of that node at that time will have to be moved later, which is very disruptive. An assignment scarcely deserves to be called node-regular if that is going to happen. But when the cost is incurred by a child node with children, after flattening it is often possible to remove the defect by moving just one meet, disrupting node regularity only slightly. So it is important to give priority to nodes with no children.

This is done in two ways. First, the cost of edges leading out of meets whose nodes have no children is multiplied by 10. Second, when evaluating alternatives while improving node regularity, the cost of the best matching is divided into two parts: the total cost of edges leading out of meets in nodes with no children (the *without-children cost*) and the total cost of the remaining edges (the *with-children cost*), and without-children cost takes priority.

The heuristic sorts the child nodes by decreasing duration. Nodes with equal duration are sorted by increasing number of children. Although it is important to minimize without-children cost, even at the expense of with-children cost, it would be wrong to maximize without-children node regularity at the expense of with-children node regularity. Node regularity is generally harder to achieve for nodes of longer duration, so they are handled first.

For each child node in sorted order, the heuristic generates a sequence of sets of zones. For each set of zones, it reduces the matching edges leading out of the meets of the child node so that they go only to segments whose times overlap with the times of the zones. A best set of zones is chosen, the limitation of the child node's meets to those nodes is fixed, and the heuristic proceeds to the next child node.

The best set is the first one with a lexicographically minimum value of the triple

```
(without_children_cost, zones_cost, with_children_cost)
```

The `without_children_cost` and `with_children_cost` components are as defined above. The `zones_cost` component measures the badness of the set of zones. It is the number of zones in the set (we are trying to minimize this number, after all), adjusted to favour zones of smaller duration and zones already present in sets fixed on previously, to encourage the algorithm to use up zones completely wherever possible.

The algorithm for generating sets of zones generates all sets of cardinality 1, then all sets of cardinality 2, then one set containing every zone that the current best matching touches. This last set is included to ensure that at least one set leading to a reasonable matching cost is tried. A few optimizations are implemented, including skipping sets of insufficient duration, and skipping zones known to be fully utilized already.

10.6.5. Handling irregular monitors

Each edge of the layer matching graph is assigned a cost by making one meet assignment and measuring the solution cost afterwards. This amounts to assuming that the cost of each edge is independent of which other edges are present in the best matching. Costs come from monitors, and the truth of this assumption varies with the monitor type, as follows.

Assign time and prefer times costs. Independent when the cost function is `Linear`, which it always is in practice for these kinds of monitors.

Split events and distribute split events costs. Not changed by meet assignments.

Spread events costs. Non-independent. Previous sections have addressed this problem, by varying path costs to discourage two demands from one demand group from matching with two supplies from one supply group, and by improving node regularity.

Link events costs. Not changed by meet assignments when handled structurally, which they always are in practice.

Order events costs. Non-independent when both events lie in the current layer.

Assign resource, prefer resources, and avoid split assignments costs. Not changed by meet assignments.

Avoid clashes costs. Independent, because clashes are never introduced within one layer.

Avoid unavailable times costs. Independent when the cost function is `Linear`.

Limit idle times, cluster busy times, and limit busy times costs. Non-independent when present (when resources subject to them are preassigned in the layer's meets).

Limit workload costs. Not changed by meet assignments.

Demand costs. Independent when they monitor clashes and unavailable times. More subtle interactions can be non-independent, but most layer matchings are built when the timetable is incomplete and subtle demand overloads are unlikely.

Order events, limit idle times, cluster busy times, and limit busy times monitors stand out as needing attention. These will be called *irregular monitors*.

At present, the author has no experience with order events monitors, so Elm does nothing with them. The irregular monitors handled by Elm are those limit idle times, cluster busy times, and limit busy times monitors of the resources of the layer match's layer which are attached at the time the elm is created. The Elm core module stores these monitors in an array, accessible via

```
int KheElmIrregularMonitorCount(KHE_ELM elm);
KHE_MONITOR KheElmIrregularMonitor(KHE_ELM elm, int i);
void KheElmSortIrregularMonitors(KHE_ELM elm,
  int(*compar)(const void *, const void *));
```

`KheElmIrregularMonitorCount` and `KheElmIrregularMonitor` visit them in the usual way. `KheElmSortIrregularMonitors` sorts them; `compar` is a function suited to passing to `qsort` when sorting an array of monitors. Core function

```
bool KheElmIrregularMonitorsAttached(KHE_ELM elm);
```

returns `true` if all irregular monitors are currently attached. By definition, this is true initially.

As a first step in handling the irregular monitors of its layer, Elm offers functions

```
void KheElmDetachIrregularMonitors(KHE_ELM elm);
void KheElmAttachIrregularMonitors(KHE_ELM elm);
```

to detach any irregular monitors that are not already detached, and attach any that are not already attached. `KheElmLayerAssign` uses them to detach irregular monitors at the start and reattach them at the end. This ensures that the best matching never takes them into account. It would only cause confusion if it did.

For improving its performance when irregular monitors are present, Elm offers

```
void KheElmReduceIrregularMonitors(KHE_ELM elm);
```

If irregular monitors are attached, it detaches them. It installs the best matching's assignments, attaches irregular monitors, and remembers the solution cost. Then for each supply s, it detaches irregular monitors, removes s from the graph, installs the best matching's assignments, attaches irregular monitors, remembers the solution cost, and restores s to the graph. If none of the removals improves cost, it returns irregular monitors to their original state of attachment and terminates. Otherwise, it permanently removes the supply that produced the best cost and repeats from the start.

Some optimizations avoid futile work. If removing s would reduce the total duration of supply nodes to below the total duration of demand nodes, or reduce the number of supplies of s's duration to below the number of demands of s's duration, the removal of s is not tried. And the function returns immediately if the layer has no irregular monitors.

`KheElmReduceIrregularMonitors` is a plausible way to attack limit idle times and limit busy times defects, but it is not radical enough for cluster busy times defects. These are better handled by other means, such as `KheSolnClusterAndLimitMeetDomains` (Section 10.3.3).

10.7. Time repair

This section presents the time solvers packaged with KHE that take an existing time assignment and repair it (that is, attempt to improve it). However carefully an initial time assignment is made, it must proceed in steps, and it can never incorporate enough forward-looking information to ensure that each step does not create problems for later steps. So a repair phase after the initial assignment is complete seems to be a practical necessity.

10.7.1. Node-regular time repair using layer node matching

Suppose we have a time assignment with good node regularity, but with some spread and demand defects. Repairs that move meets arbitrarily might fix some defects, but the resulting loss of node regularity might have serious consequences later, during resource assignment. This section offers one idea for repairing time assignments without sacrificing node regularity.

One useful idea is to make repairs which are *node swaps*: swaps of the assignments of (the meets of) entire nodes. The available swaps are quite limited, because the nodes concerned must lie in the same layers and have the same number of meets with the same durations.

For any parent node, take any set of child nodes lying in the same layers whose meets are all assigned. Build a bipartite graph in which each of these child nodes is one demand node, and the set of assignments of its meets is one supply node. An assignment is a triple of the form

```
(target_meet, offset, durn)
```

as for layer matchings (Section 10.6), but here a supply node is a set of triples, not one triple.

For each case where a child node can be assigned to a set of triples, because the number of triples and their durations match the node's number of meets and durations, add an edge to the graph labelled by the change in solution cost when the corresponding set of assignments is made. Find a maximum matching of minimum cost in this graph and reassign the child nodes in accordance with it. The existing assignment is one maximum matching, so this will either reproduce that or find something which has a good chance of being better. Function

```
bool KheLayerNodeMatchingNodeRepairTimes(KHE_NODE parent_node,
  KHE_OPTIONS options);
```

applies these ideas to the child nodes of `parent_node`, returning `true` if it considers its work to have been useful, as is usual for time repair solvers. First, if `parent_node` has no child layers it calls `KheNodeChildLayersMake` to build them. Then it partitions the child nodes so that the nodes of each partition lie in the same set of layers. Then, for each partition in turn, it builds the weighted bipartite graph and carries out the corresponding reassignments. If the solution cost does not decrease, the reassignments are undone. It continues cycling around the partitions until *n* reassignments have occurred without a cost decrease, where *n* is the number of partitions. Finally, if it made layers to begin with it removes them. A related function is

```
bool KheLayerNodeMatchingLayerRepairTimes(KHE_LAYER layer,
  KHE_OPTIONS options);
```

It starts with the child nodes of `layer` rather than all the child nodes of its parent.

On a real instance, `KheLayerNodeMatchingNodeRepairTimes` found no improvements at all after all layers were assigned. Applied after each layer after the first was assigned, it found one improvement, which reduced the number of unassignable tixels by 1 or 2. This improvement was carried through to the final solution: the median number of unassigned tixels when solving 16 instances was reduced from about 9 to about 7, and there were modest reductions in spread defects and split assignment defects as well. The extra run time was about 0.6 seconds.

10.7.2. Ejection chain time repair

Time solvers

```
bool KheEjectionChainNodeRepairTimes(KHE_NODE parent_node,
  KHE_OPTIONS options);
bool KheEjectionChainLayerRepairTimes(KHE_LAYER layer,
  KHE_OPTIONS options);
```

use ejection chains (Chapter 12) to repair the assignments of the meets of the descendants of the child nodes of `parent_node`, or the assignments of the meets of the descendants of the child nodes of `layer`. For full details, consult Section 12.7.

10.7.3. Tree search layer time repair

Very large-scale neighbourhood (VLSN) search [1, 10] deassigns a relatively large chunk of the solution, then reassigns it in a hopefully better way. If the chunk is chosen carefully, it may be

possible to find an optimal reassignment in a moderate amount of time.

One well-known VLSN neighbourhood is the set of meets of one layer (a set of meets which must be disjoint in time, usually because they have a resource in common). For example, finding a timetable for one university student is a kind of layer reassignment, with the choices of times for the meets determined by when sections of the student's courses are running. Function

```
bool KheTreeSearchLayerRepairTimes(KHE_SOLN soln, KHE_RESOURCE r);
```

reassigns the meets of `soln` currently assigned resource `r`, using a tree search. Once the number of nodes explored reaches a fixed limit, it switches to a simple heuristic, giving up the guarantee of optimality to ensure that running time remains moderate. Function

```
bool KheTreeSearchRepairTimes(KHE_SOLN soln, KHE_RESOURCE_TYPE rt,
  bool with_defects);
```

calls `KheTreeSearchLayerRepairTimes` for each resource in `soln`'s instance (or each of type `rt`, if `rt` is non-`NULL`). If `with_defects` is `true`, these calls are only made for resources with at least one resource defect, otherwise they are made for all resources. The rest of this section describes `KheTreeSearchLayerRepairTimes` in detail.

If a tree search is given a high standard to reach, it will run quickly because many paths will fail the standard and get pruned, and so it is quite likely to run to completion and reach that high standard if it is reachable at all. If it is given a low standard, it will run more slowly and quite possibly not run to completion. Either approach is legitimate, but a choice has to be made.

Because VLSN search is relatively slow, it seems best to use it near the end of a solve, when there are few defects left to target. `KheTreeSearchLayerRepairTimes` is intended to be used as a last resort in this way, when there is likely to be just one or two defects related to the layer being targeted. Accordingly, it aims high, for an assignment with no defects at all. It prunes paths whenever it can see that there is a defect that cannot be corrected by further assignments.

The meets are first sorted into decreasing duration order and unassigned. Each is given a *current domain*, which is initially its usual domain minus any starting times that would cause the meet to overlap a time when any of its resources are unavailable. Then a traditional tree search is carried out, which at each node of level i assigns a time from its current domain to the ith meet in the sorted list. The best leaf is remembered and replaces the original set of assignments if its solution cost is smaller. Three rules are used for pruning the tree.

First, any assignment which returns `false` or causes the number of unmatched demand tixels to exceed its value in the initial solution is rejected.

Second, after a fixed number of nodes is reached, new nodes are still explored, but only the first assignment that does not increase the number of unmatched demand tixels is tried therein.

Third, a form of forward checking is used. Let m_1 and m_2 be meets of the layer, and let t_1 and t_2 be times. At the start, a set of *exclusions* is built, each of the form

$$(m_1, t_1) \Rightarrow \neg(m_2, t_2)$$

This means that if m_1 is assigned starting time t_1, then m_2 may not be assigned starting time t_2. While the search is running, when m_1 is assigned t_1 this exclusion is applied, removing t_2 from the domain of m_2. When m_1 is unassigned later, the exclusion is removed (m_2 must come later in the

list of meets to be assigned than m_1, so that at the moment m_1 is assigned, m_2 is not assigned).

Following is a list of true statements about various situations:

- Since the meets all share a resource, no two of the meets may overlap in time.
- Two meets linked by a spread events constraint cannot be assigned within the same time group of that constraint, when that time group has a `Maximum` attribute of 1.
- Two meets linked by an order events constraint must be assigned in a certain chronological order, possibly with a given separation.
- Given two meets with the same duration and the same resources, and monitored by the same event monitors, it is safe (and useful for avoiding symmetrical searches) to arbitrarily insist that the first one in the assignment list should appear earlier in the cycle than the second.

Each statement gives rise to exclusions, and all these exclusions are added, except that at present a couple of shortcuts are being used: order events constraints are not currently taken into account, and the symmetry breaking idea of the last point is being applied to a different set of pairs of meets, namely those which are linked by a spread events constraint and have the same duration.

Exclusions are used in two ways. First, when a meet's turn comes to be assigned, only times in its current domain (its initial domain minus any exclusions) are tried. Second, each meet keeps a count of the number of times in its current domain. If this number ever drops to 0, the assignment that caused that to happen is rejected immediately.

On instance IT-I4-96, with limit 10000, this method improved the timetables of four resources, reducing final cost from 0.00397 to 0.00390, and adding about 2 seconds to total run time. There was wide variation in the completeness of the search: for some resources, every possible timetable was tried; for others, there was only time to try timetables that assigned the first meet to the first time. It did not reduce the 0.00067 cost of the best of 8 solutions, nor find any improvements when solving instance AU-BG-98. A run with limit 1000000 improved a fifth resource in IT-I4-96, and showed that many searches do reach even this quite large limit.

10.7.4. Meet set time repair and the fuzzy meet move

Another VLSN idea is to use a tree search to repair the assignments of an arbitrary (but small) set of meets. Given a set of meets, build the set of all target meets they are assigned to, and for each target meet, the set of offsets within it that they are running. The aim is to reassign the meets optimally within these same target meets and offsets. The only pruning rule is that the number of unmatched demand tixels may not exceed its initial value.

The functions that implement this idea are

```
KHE_MEET_SET_SOLVER KheMeetSetSolveBegin(KHE_SOLN soln, int max_meets);
void KheMeetSetSolveAddMeet(KHE_MEET_SET_SOLVER mss, KHE_MEET meet);
bool KheMeetSetSolveEnd(KHE_MEET_SET_SOLVER mss);
```

`KheMeetSetSolveBegin` makes a meet-set solver object which coordinates the operation. `KheMeetSetSolveAddMeet` adds one meet to the solver, and may be called any number of times, building up a set of meets. If the number of meets added reaches the `max_meets` parameter of

KheMeetSetSolveBegin, further calls to KheMeetSetSolveAddMeet are allowed but ignored. Finally, KheMeetSetSolveEnd uses a tree search to find an optimal reassignment of the meets to (collectively) their original target meets and offsets, returning true if it reduced the cost of the solution, and frees the memory used by the solver object. If the number of nodes in the search tree exceeds a given fixed limit, the search switches to a simple linear heuristic at each remaining tree node, losing the guarantee of optimality but ensuring that run times remain moderate.

As a first application of these functions, KHE offers

```
bool KheFuzzyMeetMove(KHE_MEET meet, KHE_MEET target_meet, int offset,
  int width, int depth, int max_meets);
```

This may move meet to target_meet at offset, but not necessarily. Instead, it selects a set of meets likely to be affected by that move, including meet, and passes them all to the meet set solver above for (hopefully) optimal reassignment. It returns true if and only if it changed the solution, which will be if and only if it reduced the cost of the solution.

The point of KheFuzzyMeetMove is that if the caller has identified this move as likely to be useful, but with some uncertainty about its consequences, it allows the move to be tried, but with adjustments in the neighbourhood to get the most out of it. These adjustments are not unlike those made by Kempe meet moves, only more general and more costly in run time.

Two sets of meets are selected. To be in the first set, a meet has to be assigned to the same target meet as meet, at an offset lying between meet's current offset minus width, and meet's current offset plus width. Furthermore, if depth is 1 (the smallest reasonable value), a selected meet has to share a resource (assigned or preassigned) with meet. If depth is 2, a selected meet has to share a resource with a meet that would be selected when the depth is 1, and so on: the depth signifies the maximum length of a chain of shared resources that must connect a selected meet to meet. The second set of meets is the same as the first, only defined using target_meet and offset instead of meet's current target meet and offset.

As for meet set time repair, at most max_meets meets will be selected. If width and depth are small, it is reasonable for max_meets to be INT_MAX.

10.8. Layered time assignment

The heart of time assignment when layer trees are used is to assign the meets of the child nodes of a given parent node to the meets of the parent node. A *layered time assignment* is one which groups the child nodes into layers and assigns them layer by layer. This is a good way to do it, since the nodes of each layer strongly constrain each other (they must be disjoint in time).

KheElmLayerAssign (Section 10.6) is KHE's main solver for assigning the meets of the child nodes of one layer. But there is work to be done to prepare the way for calling this function, beyond the structural work of building the layer tree. This section presents KHE's functions for carrying out this preparatory work and calling KheElmLayerAssign.

10.8.1. Layer assignments

When assigning layers it is useful to be able to record an assignment of the meets of a layer, for undoing and redoing later. Marks and paths could do this, but they record every step. A layer

assignment algorithm could be very long and wandering, so it is better to record just its result.

Accordingly, KHE offers the *layer assignment* object, with type `KHE_LAYER_ASST`:

```
KHE_LAYER_ASST KheLayerAsstMake(void);
void KheLayerAsstDelete(KHE_LAYER_ASST layer_asst);
void KheLayerAsstBegin(KHE_LAYER_ASST layer_asst, KHE_LAYER layer);
void KheLayerAsstEnd(KHE_LAYER_ASST layer_asst);
void KheLayerAsstUndo(KHE_LAYER_ASST layer_asst);
void KheLayerAsstRedo(KHE_LAYER_ASST layer_asst);
void KheLayerAsstDebug(KHE_LAYER_ASST layer_asst, int verbosity,
  int indent, FILE *fp);
```

`KheLayerAsstMake` and `KheLayerAsstDelete` make and delete one. `KheLayerAsstBegin` is called before some algorithm for assigning `layer` is run. It records which of `layer`'s meets are unassigned then. `KheLayerAsstEnd` is called after the algorithm ends. For each meet recorded by `KheLayerAsstBegin`, it records the assignment of that meet. `KheLayerAsstUndo` undoes the recorded assignments, and `KheLayerAsstRedo` redoes them. `KheLayerAsstDebug` produces a debug print of `layer_asst` onto `fp`.

10.8.2. A solver for layered time assignment

Time solver

```
bool KheNodeLayeredAssignTimes(KHE_NODE parent_node, KHE_OPTIONS options);
```

assigns the meets of the child nodes of `parent_node` to the meets of `parent_node`, calling `KheElmLayerAssign` (Section 10.6) to assign them layer by layer. Existing assignments of the meets affected may change. The implementation is described at the end of this section.

If `parent_node` is the cycle node, `KheNodePreassignedAssignTimes` should be called first, to give priority to demands made by preassigned meets.

`KheNodeLayeredAssignTimes` is affected by three options. If the `time_node_regularity` option of `options` is `true`, it tries to make the assignments node-regular (Section 5.4). This will usually be appropriate for the cycle node, but not for other nodes, since in practice they are runaround nodes, and irregularity is wanted in them rather than regularity.

`KheNodeLayeredAssignTimes` usually assigns each layer in turn, in a heuristically chosen order. But if the `time_layer_swap` option is `true`, it does something more interesting. For each layer i other than the first and last, it (a) tries assigning and repairing layer i followed by layer $i + 1$, then (b) tries assigning and repairing layer $i + 1$ followed by layer i. If the solution cost after (a) is less than after (b), it leaves (a)'s assignment of layer i in place and proceeds to the next layer; otherwise it leaves (b)'s assignment of layer $i + 1$ in place and proceeds to the next layer. So one layer is assigned on each iteration, as usual, but it could be either the usual one or the next one.

The `time_layer_repair` option determines how `KheNodeLayeredAssignTimes` repairs each layer after assigning it. Its type is `KHE_OPTIONS_TIME_LAYER_REPAIR`, defined by

```
typedef enum {
  KHE_OPTIONS_TIME_LAYER_REPAIR_NONE,
  KHE_OPTIONS_TIME_LAYER_REPAIR_LAYER,
  KHE_OPTIONS_TIME_LAYER_REPAIR_NODE,
  KHE_OPTIONS_TIME_LAYER_REPAIR_LAYER_BACKOFF,
  KHE_OPTIONS_TIME_LAYER_REPAIR_NODE_BACKOFF,
} KHE_OPTIONS_TIME_LAYER_REPAIR;
```

The first three values request no repair, repair using `KheEjectionChainLayerRepairTimes` (Section 10.7.2), and repair using `KheEjectionChainNodeRepairTimes` on the layer's parent. The last two values add to the previous two the use of exponential backoff (Section 8.6) to ration the number of layers repaired. The default value is `KHE_OPTIONS_TIME_LAYER_REPAIR_LAYER`.

The rest of this section describes the implementation of `KheNodeLayeredAssignTimes`.

If `parent_node` has no layers, `KheNodeLayeredAssignTimes` first makes them, by calling `KheNodeChildLayersMake` (Section 9.3.1). It then sorts the layers, assigns and optionally repairs them, and ends with `KheNodeChildLayersDelete` if it called `KheNodeChildLayersMake`.

When sorting the layers, the first priority is to ensure that already assigned layers come first. These are marked by assigning visit number 1 to them. Among unvisited layers, a heuristic rule is used: decreasing value of the sum of the duration and the duration of meets that have already been assigned, minus the number of meets. The reasoning here is that layers with larger durations are harder to assign, and they become even harder when many of their meets' assignments are already decided on (since the algorithm does not change them); but, on the other hand, the more meets there are, the smaller their durations must be for a given overall duration, making assignment easier. Here is the layer comparison function; it may be called separately:

```
int KheNodeLayeredLayerCmp(const void *t1, const void *t2)
{
  KHE_LAYER layer1 = * (KHE_LAYER *) t1;
  KHE_LAYER layer2 = * (KHE_LAYER *) t2;
  int value1, value2, demand1, demand2;
  if( KheLayerVisitNum(layer1) != KheLayerVisitNum(layer2) )
    return KheLayerVisitNum(layer2) - KheLayerVisitNum(layer1);
  value1 = KheLayerDuration(layer1) - KheLayerMeetCount(layer1) +
    KheLayerAssignedDuration(layer1);
  value2 = KheLayerDuration(layer2) - KheLayerMeetCount(layer2) +
    KheLayerAssignedDuration(layer2);
  if( value1 != value2 )
    return value2 - value1;
  demand1 = KheLayerDemand(layer1);
  demand2 = KheLayerDemand(layer2);
  if( demand1 != demand2 )
    return demand2 - demand1;
  return KheLayerParentNodeIndex(layer1) -
    KheLayerParentNodeIndex(layer2);
}
```

As a last resort it compares total demand, then layer indexes, to give a non-zero result in all cases: `qsort`'s specification is non-deterministic, which is best avoided, if the result is zero.

`KheNodeLayeredAssignTimes` sets the `time_vizier_node` option to `false` before making the call that repairs the first layer, and resets it to its original value afterwards. It's a small point, but a vizier node would be redundant when repairing the first layer.

Let the *whole-timetable monitors* be the limit idle times, cluster busy times, and limit busy times monitors. These depend on the whole timetable of their resource, or large parts of it. The other resource monitors either depend on local parts of the timetable (avoid clashes and avoid unavailable times monitors) or are independent of the timetable (limit workload monitors).

In practice, evaluating a whole-timetable monitor before its resource's layer is assigned is problematical, since it depends on the whole timetable, which does not exist then. For example, a partial timetable may have idle times which could well be filled later when its resource's other meets are assigned times. Accordingly, `KheNodeLayeredAssignTimes` begins by detaching all whole-timetable monitors of all resources in all its layers. Just before assigning each layer, it attaches the whole-timetable monitors of the resources of the layer.

This detachment of whole-timetable monitors is similar to the detachment of irregular monitors during the assignment of one layer by Elm (Section 10.6.5). Both detachments are done because the monitors in question would not produce useful cost information if attached. However, in the case of Elm that is because of the particular algorithm employed, whereas here it is because of something more fundamental: the fact that only a partial timetable is present.

The remainder of this section describes the three extra things that are done when the `time_node_regularity` option of `options` is `true`.

First, when a meet from another layer is already assigned (because it is preassigned, usually), it is good to make that same assignment to a meet of the same duration in the first layer, for regularity between the two meets. Such an assignment to a meet of the first layer is called a *parallel assignment*. If there is a node from another layer containing two or more assigned meets, then it is good to make the corresponding parallel assignments within one node of the first layer, for regularity between the nodes; and if two nodes from one layer contain assigned meets, it is good to make the corresponding parallel assignments to distinct nodes of the first layer. The layer solver that makes these parallel assignments to the meets of the first layer is called only when `time_node_regularity` is `true`, but it is also available separately:

```
bool KheLayerParallelAssignTimes(KHE_LAYER layer, KHE_OPTIONS options);
```

It makes parallel assignments to `layer` heuristically, returning `true` if every assigned meet in every sibling layer of `layer` has a parallel assignment afterwards. It uses no options.

Second, `KheElmLayerAssign` takes a spread events constraint as an optional parameter. When `time_node_regularity` is `true`, `KheNodeLayeredAssignTimes` searches the instance for a spread events constraint with as many points of application as possible, and passes this constraint (if any) to `KheElmLayerAssign`.

Third, and most important, when `time_node_regularity` is `true`, after the first layer has been assigned and optionally repaired, `KheNodeLayeredAssignTimes` uses the first layer's assignments to define zones in the parent node, by calling `KheLayerInstallZonesInParent` (Section 5.4) and `KheNodeExtendZones` (Section 9.6). These zones encourage the following

calls to KheElmLayerAssign and KheEjectionChainLayerRepairTimes to find and preserve zone-regular assignments.

10.8.3. A complete time solver

Time solver

```
bool KheCycleNodeAssignTimes(KHE_NODE cycle_node, KHE_OPTIONS options);
```

combines the ideas of this chapter into one solver that assigns the meets in the proper descendants of cycle_node, assumed to be the cycle node.

After first assigning preassigned meets, KheCycleNodeAssignTimes assigns times layer by layer using KheNodeLayeredAssignTimes (Section 10.8.2). Then it removes any regularity features (zones and interior nodes) installed earlier and returns.

KheCycleNodeAssignTimes is influenced by three options: time_cluster_meet_domains, which causes meet domains to be clustered using KheSolnClusterAndLimitMeetDomains (Section 10.3.3) at the start and loosened when the regularity features are loosened; time_tighten_domains, which causes resource domains to be tightened (Section 11.3.4) at the start and loosened at the end; and time_node_repair, which causes KheEjectionChainNodeRepairTimes (Section 10.7.2) to be called twice at the end, once before and once after regularity features are removed. Other options influence the calls to KheNodeLayeredAssignTimes.

Chapter 11. Resource Solvers

A *resource solver* assigns resources to tasks, or changes existing resource assignments. This chapter presents the resource solvers packaged with KHE.

11.1. Specification

The recommended interface for resource solvers, defined in `khe.h`, is

```
typedef bool (*KHE_TASKING_SOLVER)(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

It assigns resources to some of the tasks of `tasking`, influenced by `options`, returning `true` if it changed, or at least usually changes, the solution. Taskings were defined in Section 5.5.

Except for preassignments, there is no reason to assign resources, at least in large numbers, before times are assigned. Accordingly, a resource solver may choose to assume that all meets have been assigned times. It may alter time assignments in its quest for resource assignments.

A *split assignment* is an assignment of two or more distinct resources to the tasks monitored by an avoid split assignments monitor. A *partial assignment* is an assignment of resources to some of these same tasks, but not all. An assignment can be both split and partial.

11.2. The resource assignment invariant

If all tasks have duration 1, then the matching defines an assignment of resources to tasks which maximizes the number of assignments. Although larger durations are common, and maximizing the number of assignments is not the only objective, still it is clear from this fact that the matching deserves a central place in resource assignment.

Accordingly, the author's work in resource assignment [9] emphasizes algorithms that preserve the following condition, called the *resource assignment invariant*:

The number of unmatchable demand tixels equals its initial value.

Assignments are permitted only when the number of unmatchable demand tixels does not increase. This keeps the algorithms on a path that cannot lead to new violations of required avoid clashes constraints, avoid unavailable times constraints, limit busy times constraints, and limit workload constraints. In practice, most tasks can be assigned while preserving this invariant.

The invariant is not usually checked after each individual operation. Rather, a sequence of related operations is carried out, and then the number of unmatchable demand tixels at the end of the sequence is compared with the number at the start. If it has increased, the sequence of operations needs to be undone. Such sequences were called *atomic sequences* in Section 4.10, where the following code (using a mark object) was recommended for obtaining them:

```
mark = KheMarkBegin(soln);
success = SomeSequenceOfOperations(...);
KheMarkEnd(mark, !success);
```

When preserving the resource invariant, this needs to be changed to

```
mark = KheMarkBegin(soln);
init_count = KheSolnMatchingDefectCount(soln);
success = SomeSequenceOfOperations(...);
if( KheSolnMatchingDefectCount(soln) > init_count )
  success = false;
KheMarkEnd(mark, !success);
```

This works even without the matching, since then `KheSolnMatchingDefectCount` returns 0.

As a simple but effective aid to getting this right, this code is encapsulated in functions

```
void KheAtomicOperationBegin(KHE_SOLN soln, KHE_MARK *mark,
  int *init_count, bool resource_invariant);
bool KheAtomicOperationEnd(KHE_SOLN soln, KHE_MARK *mark,
  int *init_count, bool resource_invariant, bool success);
```

which may be placed before and after a sequence of operations, like this:

```
KheAtomicOperationBegin(soln, &mark, &init_count, resource_invariant);
success = SomeSequenceOfOperations(...);
KheAtomicOperationEnd(soln, &mark, &init_count, resource_invariant,
  success);
```

Here `mark` and `init_count` are variables of type `KHE_MARK` and `int`, not used for anything else, `resource_invariant` is `true` if the operations must preserve the resource invariant to be considered successful, and `success` is their diagnosis of their own success, not including checking the resource invariant. `KheAtomicOperationEnd` returns `true` if `success` is `true` and (if `resource_invariant` is `true`) the number of unmatchable demand tixels did not increase:

```
void KheAtomicOperationBegin(KHE_SOLN soln, KHE_MARK *mark,
  int *init_count, bool resource_invariant)
{
  *mark = KheMarkBegin(soln);
  *init_count = KheSolnMatchingDefectCount(soln);
}

bool KheAtomicOperationEnd(KHE_SOLN soln, KHE_MARK *mark,
  int *init_count, bool resource_invariant, bool success)
{
  if( resource_invariant &&
      KheSolnMatchingDefectCount(soln) > *init_count )
    success = false;
  KheMarkEnd(*mark, !success);
  return success;
}
```

The code is trivial, but useful because it encapsulates a common but slightly confusing pattern.

If the resource invariant is being enforced, there may be no need to include the cost of demand monitors in the solution cost, since their cost cannot increase. They must continue to monitor the solution, however, so detaching is not appropriate. Function

```
void KheDisconnectAllDemandMonitors(KHE_SOLN soln, KHE_RESOURCE_TYPE rt);
```

disconnects all demand monitors (or all demand monitors which monitor entities of type `rt`, if `rt` is non-`NULL`) from all their parents, including the solution object if it is a parent. Thus, as required, they continue to monitor the solution, but the costs they compute are not added to the cost of any group monitor. `KheSolnMatchingDefectCount` still works, however, and there is nothing to prevent them from being made children of other group monitors later.

11.3. Resource-structural solvers

A *resource-structural solver* is a solver that changes how tasks are organized, rather than actually assigning resources. Arguably, the solvers presented in this section really belong in Chapter 9, but the structural solvers presented there are basically about time, not resources.

11.3.1. Task bound groups

Task domains are reduced by adding task bound objects to tasks (Section 4.9.3). Frequently, task bound objects need to be stored somewhere where they can be found and deleted later. The required data structure is trivial—just an array of task bounds—but it is convenient to have a standard for it, so KHE defines a type `KHE_TASK_BOUND_GROUP` with suitable operations.

To create a task bound group, call

```
KHE_TASK_BOUND_GROUP KheTaskBoundGroupMake(void);
```

To add a task bound to a task bound group, call

```
void KheTaskBoundGroupAddTaskBound(KHE_TASK_BOUND_GROUP tbg,
  KHE_TASK_BOUND tb);
```

To visit the task bounds of a task bound group, call

```
int KheTaskBoundGroupTaskBoundCount(KHE_TASK_BOUND_GROUP tbg);
KHE_TASK_BOUND KheTaskBoundGroupTaskBound(KHE_TASK_BOUND_GROUP tbg, int i);
```

To delete a task bound group, including deleting all the task bounds in it, call

```
bool KheTaskBoundGroupDelete(KHE_TASK_BOUND_GROUP tbg);
```

This function returns `true` when every call it makes to `KheTaskBoundDelete` returns `true`.

11.3.2. Task trees

What meets do for time, tasks do for resources. A meet has a time domain and assignment; a task has a resource domain and assignment. Link events constraints cause meets to be assigned

to other meets; avoid split assignments constraints cause tasks to be assigned to other tasks.

There are differences. Tasks lie in meets, but meets do not lie in tasks. Task assignments do not have offsets, because there is no ordering of resources like chronological order for times.

Since the layer tree is successful in structuring meets for time assignment, let us see what an analogous tree for structuring tasks for resource assignment would look like. A layer tree is a tree, whose nodes each contain a set of meets. The root node contains the cycle meets. A meet's assignment, if present, lies in the parent of its node. By convention, meets lying outside nodes have fixed assignments to meets lying inside nodes, and those assignments do not change.

A *task tree*, then, is a tree whose nodes each contain a set of tasks. The root node contains the cycle tasks (or there might be several root nodes, one for each resource type). A task's assignment, if present, lies in the parent of its node. By convention, tasks lying outside nodes have fixed assignments to tasks lying inside nodes, and those assignments do not change.

Type `KHE_TASKING` is KHE's nearest equivalent to a task tree node. It holds an arbitrary set of tasks, but there is no support for organizing taskings into a tree structure, since that does not seem to be needed. It is useful, however, to look at how tasks are structured in practice, and to relate this to task trees, even though they are not explicitly supported by KHE.

A task is assigned to a non-cycle task and fixed, to implement an avoid split assignments constraint. Such tasks would therefore lie outside nodes (if there were any). When a solver assigns a task to a cycle task, the task would have to lie in a child node of a node containing the cycle tasks (again, if there were any). So there are three levels: a first level of nodes containing the cycle tasks; a second level of nodes containing unfixed tasks wanting to be assigned resources; and a third level of fixed, assigned tasks that do not lie in nodes.

This shows that the three-way classification of tasks presented in Section 4.9.1, into cycle tasks, unfixed tasks, and fixed tasks, is a proxy for the missing task tree structure. Cycle tasks are first-level tasks, unfixed tasks are second-level tasks, and fixed tasks are third-level tasks. `KHE_TASKING` is only needed for representing second-level nodes, since tasks at the other levels do not require assignment. By convention, then, taskings will contain only unfixed tasks.

11.3.3. Task tree construction

KHE offers a solver for building a task tree holding the tasks of a given solution:

```
void KheTaskTreeMake(KHE_SOLN soln, KHE_TASK_JOB_TYPE tjt,
  KHE_OPTIONS options);
```

Like any good solver, this function has no special access to data behind the scenes. Instead, it works by calling basic operations and helper functions:

- It calls `KheTaskingMake` to make one tasking for each resource type of `soln`'s instance, and it calls `KheTaskingAddTask` to add the unfixed tasks of each type to the tasking it made for that type. These taskings may be accessed by calling `KheSolnTaskingCount` and `KheSolnTasking` as usual, and they are returned in an order suited to resource assignment, as follows. Taskings for which `KheResourceTypeDemandIsAllPreassigned(rt)` is `true` come first. Their tasks will be assigned already if `KheSolnAssignPreassignedResources` has been called, as it usually has been. The remaining taskings are sorted by decreasing order of `KheResourceTypeAvoidSplitAssignmentsCount(rt)`. These functions are

described in Section 3.5.1. Of course, the user is not obliged to follow this ordering. It is a precondition of `KheTaskTreeMake` that `soln` must have no taskings when it is called.

- It calls `KheTaskAssign` to convert resource preassignments into resource assignments, and to satisfy avoid split assignments constraints, as far as possible. Existing assignments are preserved (no calls to `KheTaskUnAssign` are made).

- It calls `KheTaskAssignFix` to fix the assignments it makes. These may be removed later.

- It calls `KheTaskSetDomain` to set the domains of tasks to satisfy preassigned resources, prefer resources constraints, and other influences on task domains, as far as possible. `KheTaskTreeMake` never adds a resource to any domain, however; it either leaves a domain unchanged, or reduces it to a subset of its initial value.

These elements interact in ways that make them impossible to separate. For example, a prefer resources constraint that applies to one task effectively applies to all the tasks that are linked to it, directly or indirectly, by avoid split assignments constraints. The two parameters not yet mentioned, `tjt` and `options`, are explained below.

The implementation of `KheTaskTreeMake` has two stages. The first creates one tasking for each resource type of `soln`'s instance, in the order described, and adds to each the unfixed tasks of its type. This stage can be carried out separately by repeated calls to

```
KHE_TASKING KheTaskingMakeFromResourceType(KHE_SOLN soln,
  KHE_RESOURCE_TYPE rt);
```

which makes a tasking containing the unfixed tasks of `soln` of type `rt`, or of all types if `rt` is `NULL`. It aborts if any of these unfixed tasks already lies in a tasking.

The second stage is more complex. It applies public function

```
bool KheTaskingMakeTaskTree(KHE_TASKING tasking, KHE_TASK_JOB_TYPE tjt,
  KHE_TASK_BOUND_GROUP tbg, KHE_OPTIONS options);
```

to each tasking made by the first stage. When `KheTaskingMakeTaskTree` is called from within `KheTaskTreeMake`, its parameters other than `tasking` are inherited from `KheTaskTreeMake`.

As specified for `KheTaskTreeMake`, `KheTaskingMakeTaskTree` assigns tasks and tightens domains; it does not unassign tasks or loosen domains. If `tbg` is non-`NULL`, any task bounds created while tightening domains are added to `tbg`. If the `resource_invariant` option of `options` is `true`, only assignments and tightenings that preserve the resource assignment invariant (Section 11.2) are kept. Tasks assigned to non-cycle tasks have their assignments fixed, and cease to be unfixed tasks, so are deleted from `tasking`.

The implementation of `KheTaskingMakeTaskTree` imitates the layer tree construction algorithm: it applies *jobs* in decreasing priority order. There are fewer kinds of jobs, but the situation is more complex in another way: sometimes, some kinds of jobs are wanted but not others. The three kinds of jobs of highest priority install existing domains and task assignments, and assign resources to unassigned tasks derived from preassigned event resources. These jobs are always included; the first two always succeed, and so does the third unless the user has made peculiar task or domain assignments earlier. The other kinds of jobs are optional, and parameter

`tjt` of `KheTaskingMakeTaskTree` says which of them are wanted. Its type is

```
typedef enum {
  KHE_TASK_JOB_HARD_PRC = 1,
  KHE_TASK_JOB_SOFT_PRC = 2,
  KHE_TASK_JOB_HARD_ASAC = 4,
  KHE_TASK_JOB_SOFT_ASAC = 8,
  KHE_TASK_JOB_PARTITION = 16
} KHE_TASK_JOB_TYPE;
```

As the reader has probably guessed, `tjt` is actually a set.

If `KHE_TASK_JOB_HARD_PRC` is included in `tjt`, a job is made for each point of application of each required prefer resources constraint of non-zero weight. The priority of the job is the combined weight of its constraint, and it attempts to reduce the domains of the tasks of `tasking` monitored by the constraint's monitors so that they are subsets of the constraint's domain. `KHE_TASK_JOB_SOFT_PRC` is the same, except that it requests jobs for non-required constraints.

If `KHE_TASK_JOB_HARD_ASAC` is included in `tjt`, a job is made for each point of application of each hard avoid split assignments constraint of non-zero weight. Its priority is the combined weight of its constraint, and it attempts to assign tasks to each other so that all the tasks of the job's point of application of the constraint are assigned, directly or indirectly, to the same root task. Again, only tasks lying in `tasking` are affected. `KHE_TASK_JOB_SOFT_ASAC` is the same, except that it requests jobs for soft constraints.

If `KHE_TASK_JOB_PARTITION` is included in `tjt`, a very peculiar job, of minimal priority, is included. The remainder of this section is devoted to explaining it.

We begin by explaining the circumstances in which it is useful. For definiteness, suppose we are dealing with teachers, and that they have partitions (Section 3.5.1) which are their faculties (English, Mathematics, Science, and so on). Some partitions may be heavily loaded (that is, required to supply teachers for tasks whose total workload approaches the total available workload of their resources) while others are lightly loaded.

Some tasks may be taught by teachers from more than one partition. These *multi-partition tasks* should be assigned to teachers from lightly loaded partitions, and so should not overlap in time with other tasks from these partitions. `KHE_TASK_JOB_PARTITION` tightens the domain of each multi-partition task to one partition; the choice of partition is explained below. It is best to do this after preassigned meets have been assigned, but before general time assignment. The tightened domains encourage time assignment to avoid the undesirable overlaps.

After time assignment, the changes should be removed, since otherwise they constrain resource assignment unnecessarily. A task bound group can be used to do this:

```
tighten_tbg = KheTaskBoundGroupMake(soln);
for( i = 0;  i < KheSolnTaskingCount(soln);  i++ )
  KheTaskingTightenToPartition(KheSolnTasking(soln, i),
    tighten_tbg, options);
... assign times ...
KheTaskBoundGroupDelete(tighten_tbg);
```

`KheTaskingTightenToPartition`, defined below, wraps a call to `KheTaskingMakeTaskTree`.

This job does nothing when the tasking has no resource type, or the tasks of its resource type are all preassigned according to `KheResourceTypeDemandIsAllPreassigned` (Section 3.5.1), or the resource type has no partitions, or its number of partitions is less than four or more than one-third of its number of resources. Nothing useful can be done in these cases.

Tasks whose domains lie entirely within one partition are not touched. The remaining multi-partition tasks are sorted by decreasing combined weight then duration, except that tasks with a *dominant partition* come first. A task with an assigned resource has a dominant partition, namely the partition that its assigned resource lies in. An unassigned task has a dominant partition when at least three-quarters of the resources of its domain come from that partition.

For each task in turn, an attempt is made to tighten its domain so that it is a subset of one partition. If the task has a dominant partition, only that partition is tried. Otherwise, the partitions that the task's domain intersects with are tried one by one, stopping at the first success, after sorting them by decreasing average available workload (defined next).

Define the *workload supply* of a partition to be the sum, over the resources r of the partition, of the number of times in the cycle minus the number of workload demand monitors for r in the matching. Define the *workload demand* of a partition to be the sum, over all tasks t whose domain is a subset of the partition, of the workload of t. Then the *average available workload* of a partition is its workload supply minus its workload demand, divided by its number of resources. Evidently, if this is large, the partition is lightly loaded.

Each successful tightening increases the workload demand of its partition. This ensures that equally lightly loaded partitions share multi-partition tasks equally.

In a task with an assigned resource, the dominant partition is the only one compatible with the assignment. In a task without an assigned resource, preference is given to a dominant partition, if there is one, for the following reason. Schools often have a few *generalist teachers* who are capable of teaching junior subjects from several faculties. These teachers are useful for fixing occasional problems, smoothing out workload imbalances, and so on. But the workload that they can give to faculties other than their own is limited and should not be relied on. For example, suppose there are five Science teachers plus one generalist teacher who can teach junior Science. That should not be taken by time assignment as a licence to routinely schedule six Science meets simultaneously. Domain tightening to a dominant partition avoids this trap.

Tightening by partition works best when the `resource_invariant` option of `options` is `true`. For example, in a case like Sport where there are many simultaneous multi-partition tasks, it will then not tighten more of them to a lightly loaded partition than there are teachers in that partition. Assigning preassigned meets beforehand improves the effectiveness of this check.

11.3.4. Other task tree solvers

This section documents some miscellaneous functions that reorganize task trees, represented by taskings. They assume that only unfixed tasks lie in taskings, and they preserve this condition. Some merely call `KheTaskingMakeTaskTree`, passing certain combinations of parameters; others are separate algorithms.

The operation of tightening domains to a partition was discussed at some length above. For convenience, this operation is packaged as function

```
bool KheTaskingTightenToPartition(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

It tightens the domains of some tasks, without any wholesale reconstruction of the task tree:

```
bool KheTaskingTightenToPartition(KHE_TASKING tasking,
  KHE_OPTIONS options)
{
  return KheTaskingMakeTaskTree(tasking, KHE_TASK_JOB_PARTITION,
    options);
}
```

It is best if the `resource_invariant` option of `options` is `true` here.

A good way to minimize split assignments is to prohibit them at first but allow them later. To change a tasking from the first state to the second, call

```
void KheTaskingAllowSplitAssignments(KHE_TASKING tasking,
  bool unassigned_only);
```

It unfixes and unassigns all tasks assigned to the tasks of `tasking` and adds them to `tasking`. If one of the original unfixed tasks is assigned (to a cycle task), the tasks assigned to it are assigned to that task, so that existing resource assignments are not forgotten. If `unassigned_only` is `true`, these actions are only applied to the unassigned tasks of `tasking`. (This option is included for completeness, but it is not recommended, since it leaves few choices open.) `KheTaskingAllowSplitAssignments` preserves the resource assignment invariant.

If any room or any teacher is better than none, then it will be worth assigning any resource to tasks that remain unassigned at the end of resource assignment. Function

```
void KheTaskingEnlargeDomains(KHE_TASKING tasking, bool unassigned_only);
```

permits this by enlarging the domains of the tasks of `tasking` and any tasks assigned to them (and so on recursively) to the full set of resources of their resource types. If `unassigned_only` is true, only the unassigned tasks of `tasking` participate in these changes. The tasks are visited in postorder—that is, a task's domain is enlarged only after the domains of the tasks assigned to it have been enlarged—ensuring that the operation cannot fail.

11.3.5. Task groups

There are cases where two tasks are interchangeable as far as resource assignment is concerned, because they demand the same kinds of resources at the same times. The *task group* embodies KHE's approach to taking advantage of interchangeable tasks.

The *full task set* of an unfixed task is the task itself and all the tasks assigned to it, directly or indirectly (all its followers), omitting tasks that do not lie in a meet. An unfixed task is *time-complete* if each task of its full task set lies in a meet that has been assigned a time. Two time-complete tasks are *time-equal* if their full task sets have equal cardinality, and the two sets can be sorted so that corresponding tasks have equal starting times, durations, and workloads. Two unfixed tasks are *interchangeable* if they are time-complete and time-equal, and their domains are equal. When two resources are assigned to two interchangeable tasks, either resource can be

assigned to either task and it does not matter which is assigned to which.

A *task group* is a set of pairwise interchangeable tasks. Task groups occur naturally when there are linked events, or when time assignments are regular. Virtually any resource assignment algorithm can benefit from task groups. Assigning to a task group rather than to a task eliminates symmetries that can slow down searching. A given resource can only be assigned to one task of a task group, since its tasks overlap in time, so task groups help with estimating realistically how many resources are available, and how much workload is open to a resource.

Objects of type `KHE_TASK_GROUP` hold one set of interchangeable tasks, and objects of type `KHE_TASK_GROUPS` hold a set of task groups. Such a set can be created by calling

```
KHE_TASK_GROUPS KheTaskGroupsMakeFromTasking(KHE_TASKING tasking);
```

It places every task of `tasking` into one task group. The task groups are maximal.

To remove a set of task groups (but not their tasks), call

```
void KheTaskGroupsDelete(KHE_TASK_GROUPS task_groups);
```

To access the task groups, call

```
int KheTaskGroupsTaskGroupCount(KHE_TASK_GROUPS task_groups);
KHE_TASK_GROUP KheTaskGroupsTaskGroup(KHE_TASK_GROUPS task_groups, int i);
```

To access the tasks of a task group, call

```
int KheTaskGroupTaskCount(KHE_TASK_GROUP task_group);
TASK KheTaskGroupTask(KHE_TASK_GROUP task, int i);
```

There must be at least one task in a task group, otherwise the task group would not have been made. Task groups are not kept up to date as the solution changes, so if time assignments are being altered the affected tasks cannot be relied upon to remain interchangeable.

The tasks of a task group have the same total duration, total workload, and domain, and these common values are returned by

```
int KheTaskGroupTotalDuration(KHE_TASK_GROUP task_group);
float KheTaskGroupTotalWorkload(KHE_TASK_GROUP task_group);
KHE_RESOURCE_GROUP KheTaskGroupDomain(KHE_TASK_GROUP task_group);
```

`KheTaskGroupTotalDuration` is the value of `KheTaskTotalDuration` shared by the tasks, not the sum of their durations; and similarly for `KheTaskGroupTotalWorkload`.

For the convenience of algorithms that use task groups, function

```
int KheTaskGroupDecreasingDurationCmp(KHE_TASK_GROUP tg1,
  KHE_TASK_GROUP tg2);
```

is a comparison function that may be used to sort task groups by decreasing duration.

Because the tasks of a task group are interchangeable, it does not matter which of them is assigned when assigning resources to them. This makes the following functions possible:

```
int KheTaskGroupUnassignedTaskCount(KHE_TASK_GROUP task_group);
bool KheTaskGroupAssignCheck(KHE_TASK_GROUP task_group, KHE_RESOURCE r);
bool KheTaskGroupAssign(KHE_TASK_GROUP task_group, KHE_RESOURCE r);
void KheTaskGroupUnAssign(KHE_TASK_GROUP task_group, KHE_RESOURCE r);
```

`KheTaskGroupUnassignedTaskCount` returns the number of unassigned tasks in `task_group`; `KheTaskGroupAssignCheck` checks whether `r` can be assigned to a task of `task_group` (by finding the first unassigned task and checking there); `KheTaskGroupAssign` is the same, only it actually makes the assignment, using `KheTaskAssign`, if it can; and `KheTaskGroupUnAssign` finds a task of `task_group` currently assigned `r`, and unassigns that task.

The tasks of a task group may have different constraints, in which case assigning one may change the solution cost differently from assigning another. This is handled heuristically as follows. The first time `KheTaskGroupAssign` returns `true`, it tries assigning `r` to each task of the task group, notes the solution cost after each, and sorts the tasks into increasing order of this cost. Then it and all later calls assign the first unassigned task in this order.

The usual debug functions are available:

```
void KheTaskGroupDebug(KHE_TASK_GROUP task_group, int verbosity,
  int indent, FILE *fp);
void KheTaskGroupsDebug(KHE_TASK_GROUPS task_groups, int verbosity,
  int indent, FILE *fp);
```

print `task_group` and `task_groups` onto `fp` with the given verbosity and indent.

11.4. Most-constrained-first assignment

When each unfixed task has no followers, so that each demands a resource for a single interval of time, as is usual with room assignment, a simple ‘most constrained first’ heuristic assignment algorithm that maintains the resource assignment invariant is usually sufficient to obtain a virtually optimal assignment. Function

```
bool KheMostConstrainedFirstAssignResources(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

implements this algorithm. It attempts to assign each unassigned unfixed task of `tasking`, leaving assigned ones untouched. For each such task, it maintains the set of resources that can currently be assigned to the task without increasing the number of unmatchable demand tixels. It repeatedly selects a task with the fewest number of such resources, assigns it if possible, and repeats until all tasks have been handled.

The chosen assignment must preserve the resource assignment invariant. If no resources satisfy that condition, the task remains unassigned. Among all resources that satisfy it, as a first priority a resource whose assignment minimizes `KheSolnCost` is chosen, and as a second priority, resources that have already been assigned to other tasks of the event resources of the task and the tasks assigned to it are preferred. In this way, even when an avoid split assignments constraint is not present, the algorithm favours assigning the same resource to all the tasks of a given event resource, for regularity.

In fact, `KheMostConstrainedFirstAssignResources` assigns task groups (Section 11.3.5), not individual tasks. Each task of a task group is assignable by the same resources, so one list of suitable resources is kept per task group. At each step, a task group is selected for assignment for which the number of suitable resources minus the number of unassigned tasks is minimal.

When a resource is assigned to a task, it becomes less available, so its suitability for assignment to its other task groups is rechecked. If it proves to be no longer assignable to some of them, their priorities are changed. The task groups are held in a priority queue (Appendix A.3), which allows their queue positions to be updated efficiently when their priorities change.

11.5. Resource packing

To *pack* a resource means to find assignments of tasks to the resource that make the solution cost as small as possible, while preserving the resource assignment invariant, in effect utilizing the resource as much as possible [9]. Function

```
bool KheResourcePackAssignResources(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

assigns resources to the unassigned tasks of `tasking` using resource packing, as follows.

The tasks are clustered into task groups (Section 11.3.5). Two numbers help to estimate the difficulty of utilizing a resource effectively: the *demand duration* and the *supply duration*. A resource's demand duration is the total duration of the task groups it is assignable to. Its supply duration is the number of times it is available for assignment: the cycle length, minus the number of its workload demand monitors, minus the total duration of any tasks it is already assigned to.

The resources are placed in a priority queue, ordered by increasing demand duration minus supply duration. That is, the less demand there is for the resource, or the more supply, the more important it is to pack it sooner rather than later. In practice, part-time teachers come first in this order, which is good, because they are difficult to utilize effectively.

The main loop of the algorithm removes a resource of minimum priority from the priority queue and packs it. If this causes any task groups to become completely assigned, they are unlinked from the resources assignable to them, reducing those resources' demand durations and thus altering their position in the priority queue. This is repeated until the queue is empty.

Each resource `r` is packed using a binary tree search: at each tree node, one available task group is either assigned to `r`, or not. The task groups are taken in decreasing order of the maximum, over all tasks `t` of the task group, of `KheMeetDemand(m)`, where `m` is the first unfixed meet on the chain of assignments out of the meet containing `t`. This gives preference to tasks whose meets are hard to move, reasoning that the leftovers will be given split assignments, and repairing them may require moving their meets. The search tree has a moderate depth limit. At the limit, the algorithm switches to a simple heuristic which assigns as many tasks as it can.

11.6. Split assignments

After solver functions such as `KheMostConstrainedFirstAssignResources` (Section 11.4)

and `KheEjectionChainRepairResources` (Section 11.8) have assigned resources to most tasks, some tasks may remain unassigned. These will have to receive split assignments. Function

```
bool KheFindSplitResourceAssignments(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

reduces the cost of the solution as much as it can, by making split assignments to the unassigned tasks of `tasking` while maintaining the resource assignment invariant. Any tasks which were unassigned to begin with are replaced in `tasking` by their child tasks.

At the core of `KheFindSplitResourceAssignments` is a procedure which takes every pair of resources capable of constituting a split assignment to some task and tries to assign them greedily to the task, keeping the assignment that produces the lowest solution cost. However, before entering on that, `KheFindSplitResourceAssignments` eliminates resources that cannot be assigned even to one child task, makes assignments that are forced because there is only one available resource (not forgetting that one forced assignment might lead to another, or that once a resource has been assigned to one child task it makes sense to assign it to as many others as possible), and divides each task into independent components (in the sense that no resource is assignable to two components). In practice, much of what it does is more or less forced.

11.7. Kempe and ejecting task moves

KHE offers several functions in the area of Kempe and ejecting meet moves (Section 10.2.2), but at present there is only one function in the area of Kempe and ejecting task moves:

```
bool KheTaskEjectingMoveResource(KHE_TASK task, KHE_RESOURCE r);
```

This attempts to move `task` to `r`, unassigning `r` from all clashing tasks, and returns `true` if it succeeds. Unlike the functions for ejecting meet moves, it does not consult the matching, nor does it require the presence of any group monitor. Instead, it works as follows.

It calls `KheResourceHardUnavailableTimeGroup(r)` (Section 3.5.3) to determine when `r` is unavailable, returning `false` if `task` is running at any of those times. Next, by consulting `r`'s timetable monitor at the times of `task`, it finds the tasks assigned `r` that clash with `task` and unassigns `r` from them. If any cannot be unassigned (because they are fixed or preassigned), it returns `false`. Finally, it calls `KheTaskMoveResource(task, r)` and returns what it returns.

Failed ejecting task moves leave the solution in its state at the point of failure, so need to be used with marks. Ejecting task moves do not attempt to preserve the resource assignment invariant, leaving that to higher-level solvers.

11.8. Ejection chain repair

Function

```
bool KheEjectionChainRepairResources(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

uses ejection chains (Chapter 12) to improve the solution by changing the assignments of the tasks of `tasking`. For full details, consult Section 12.7.

11.9. Resource pair repair

One idea for repairing resource assignments is to unassign all tasks assigned to two resources, then try to reassign those tasks to the same two resources in a better way—an example of very large-scale neighbourhood (VLSN) search [1, 10]. The search space, although formally exponential in size, is often small enough to search completely, giving an optimal result.

11.9.1. The basic function

The basic function for carrying out this kind of repair is

```
bool KheResourcePairReassign(KHE_SOLN soln, KHE_RESOURCE r1,
  KHE_RESOURCE r2, bool resource_invariant, bool fix_splits);
```

It knows that when one task is assigned to another, the two tasks must be assigned the same resource; and it believes that tasks that overlap in time must be assigned different resources. It does not change task domains, fixed assignments, or assignments of tasks to non-cycle tasks. If it can find a reassignment to `r1` and `r2` of the tasks currently assigned to `r1` and `r2` which satisfies these conditions and gives `soln` a lower cost, it makes it and returns `true`; otherwise it changes nothing and returns `false`. If `resource_invariant` is `true`, only changes that preserve the resource assignment invariant are allowed. `KheResourcePairReassign` accepts any resources, but it is most likely to succeed on resources with similar capabilities that are involved in defects.

If `fix_splits` is `true`, the algorithm focuses on repairing split assignments, by forcing tasks unassigned by the algorithm which are linked by avoid split assignments constraints of non-zero cost to be assigned the same resource in the reassignment. This runs faster, because it has fewer choices to try, but it may overlook other kinds of improvements.

Within the set of tasks assigned to `r1` and `r2` originally, there may be subsets which are not assignable to two resources without introducing clashes. Clashes in the original assignments can cause this, as can split assignments when `fix_splits` is set. Such subsets are ignored by `KheResourcePairReassign`; their original assignments are left unchanged.

11.9.2. A resource pair solver

Resource solver

```
bool KheResourcePairRepair(KHE_TASKING tasking, KHE_OPTIONS options);
```

calls `KheResourcePairReassign` for many pairs of resources. The `resource_invariant` arguments of all these calls are set to the `resource_invariant` option of `options`. Precisely what `KheResourcePairRepair` does depends on the `resource_pair` option of `options`, which has type `KHE_OPTIONS_RESOURCE_PAIR`:

```
typedef enum {
  KHE_OPTIONS_RESOURCE_PAIR_NONE,
  KHE_OPTIONS_RESOURCE_PAIR_SPLITS,
  KHE_OPTIONS_RESOURCE_PAIR_PARTITIONS,
  KHE_OPTIONS_RESOURCE_PAIR_ALL
} KHE_OPTIONS_RESOURCE_PAIR;
```

If resource_pair is KHE_OPTIONS_RESOURCE_PAIR_NONE, it does nothing.

If resource_pair is KHE_OPTIONS_RESOURCE_PAIR_SPLITS, then for all pairs of distinct resources involved in all split assignments of tasking, KheResourcePairRepair calls KheResourcePairReassign for those two resources, with the fix_splits parameter set to true. These choices focus the solver on repairing split assignments.

If the value of resource_pair is KHE_OPTIONS_RESOURCE_PAIR_PARTITIONS, then KheResourcePairReassign calls KheResourcePairRepair for each pair of resources in each partition of the resource type of tasking, or in all resource types if tasking has no resource type, with the fix_splits parameter set to false. Each resource type with no partitions is treated as though all resources lie in a single shared partitition. These choices focus the solver on improving resources' assignments generally. However the search space is often larger, increasing the chance that the search will be cut short, losing optimality.

If resource_pair is KHE_OPTIONS_RESOURCE_PAIR_ALL, the behaviour is the same as for KHE_OPTIONS_RESOURCE_PAIR_PARTITIONS except that partitions are ignored, so that there is a call on KheResourcePairReassign for every pair of distinct resources of the types involved.

KheResourcePairRepair collects statistics about its calls to KheResourcePairReassign, held in the resource_pair_calls, resource_pair_successes, and resource_pair_truncs attributes of options. Each time KheResourcePairReassign is called, resource_pair_calls is incremented. Each time it returns true, resource_pair_successes is incremented. And each time it truncates an overlong search (at most once per call), resource_pair_truncs is incremented. It is up to the caller to make sure these options are initialized and retrieved at the right moments, using the usual functions for retrieving and setting options (Section 8.4.4).

11.9.3. Partition graphs

Resource pair repair is essentially about two-colouring a clash graph whose nodes are tasks and whose edges are pairs of tasks that overlap in time. Although the basic idea is simple enough, the details become quite complicated, especially when optimizing by removing symmetries in the search. It has proved convenient to build on a separate *partition graph* module, which is the subject of this section. It finds the connected components of a graph (called *components* here), and, if requested, partitions components into two *parts* by two-colouring them.

The module stores a graph whose nodes are represented by values of type void *. There are operations for creating and deleting a graph, adding nodes to it, and visiting those nodes:

```
KHE_PART_GRAPH KhePartGraphMake(KHE_PART_GRAPH_REL_FN rel_fn);
void KhePartGraphDelete(KHE_PART_GRAPH graph);
void KhePartGraphAddNode(KHE_PART_GRAPH graph, void *node);
int KhePartGraphNodeCount(KHE_PART_GRAPH graph);
void *KhePartGraphNode(KHE_PART_GRAPH graph, int i);
```

Deleting a graph includes deleting all its components and parts, but not its nodes. These functions and the others in this section are declared in include file khe_part_graph.h.

To define the edges, the user passes in a *relation function* of type KHE_PART_GRAPH_REL_FN which the module calls back whenever it needs to know whether two nodes are connected by an edge. As the user would define it, this function looks like this:

```
KHE_PART_GRAPH_REL RelationFn(void *node1, void *node2)
{
  ...
}
```

where type KHE_PART_GRAPH_REL is

```
typedef enum {
  KHE_PART_GRAPH_UNRELATED,
  KHE_PART_GRAPH_DIFFERENT,
  KHE_PART_GRAPH_SAME
} KHE_PART_GRAPH_REL;
```

Values KHE_PART_GRAPH_UNRELATED and KHE_PART_GRAPH_DIFFERENT are the usual options for clash graphs, the first saying that there is no edge between the two nodes, the second that there is an edge which requires the two nodes to be coloured with different colours. The third value, KHE_PART_GRAPH_SAME, says that the two nodes must be coloured the same colour. It is used, for example, when the two nodes represent tasks which are linked by an avoid split assignments constraint, and the fix_splits option is in force.

After all nodes have been added, the user may call

```
void KhePartGraphFindConnectedComponents(KHE_PART_GRAPH graph);
```

to find the connected components, which may then be visited by

```
int KhePartGraphComponentCount(KHE_PART_GRAPH graph);
KHE_PART_GRAPH_COMPONENT KhePartGraphComponent(KHE_PART_GRAPH graph, int i);
```

The graph that a component is a component of may be found by

```
KHE_PART_GRAPH KhePartGraphComponentGraph(KHE_PART_GRAPH_COMPONENT comp);
```

and the nodes of a component may be visited by

```
int KhePartGraphComponentNodeCount(KHE_PART_GRAPH_COMPONENT comp);
void *KhePartGraphComponentNode(KHE_PART_GRAPH_COMPONENT comp, int i);
```

KhePartGraphFindConnectedComponents considers two nodes to be connected when rel_fn returns KHE_PART_GRAPH_SAME or KHE_PART_GRAPH_DIFFERENT when passed those nodes.

If requested, the module will partition the nodes of a component into two sets, such that two-colouring the component will give the nodes in one set one colour, and the nodes in the other set the other colour. This gives exactly two ways to two-colour the component, which is all there are, since once a colour is assigned to one node, its neighbours must be assigned the other colour, their neighbours must be assigned the first colour, and so on. To carry out this partitioning, call

```
void KhePartGraphComponentFindParts(KHE_PART_GRAPH_COMPONENT comp);
```

After that, to retrieve the two parts, call

```
bool KhePartGraphComponentParts(KHE_PART_GRAPH_COMPONENT comp,
  KHE_PART_GRAPH_PART *part1, KHE_PART_GRAPH_PART *part2);
```

If `KhePartGraphComponentFindParts` was able to partition the component into two parts, `KhePartGraphComponentParts` returns `true` and sets `*part1` and `*part2` to non-`NULL` values; otherwise it returns `false` and sets them to `NULL`. To find a part's enclosing component, call

```
KHE_PART_GRAPH_COMPONENT KhePartGraphPartComponent(
  KHE_PART_GRAPH_PART part);
```

The nodes of a part may be visited by

```
int KhePartGraphPartNodeCount(KHE_PART_GRAPH_PART part);
void *KhePartGraphPartNode(KHE_PART_GRAPH_PART part, int i);
```

as usual.

11.9.4. The implementation of resource pair reassignment

This section describes the implementation of `KheResourcePairReassign`. It builds two partition graphs altogether, a *first graph* which does the basic analysis, and a *second graph* which is used to find and remove symmetries in the first graph.

The same node type is used in both graphs. A node holds a set of tasks. A resource is *assignable to a node* when it is assignable to each task of the node. A resource is assignable to a fixed task when it is assigned to that task (fixed tasks are never unassigned). A resource is assignable to an unfixed task when it lies in the domain of that task. It is possible for neither, one, or both resources to be assignable to a node. If neither is assignable, the node is *unassignable*, otherwise it is *assignable.*

When a resource is assignable to a node, there are operations for assigning and unassigning it. To assign it, assign it to each unfixed task of the node. To unassign it, unassign it from each unfixed task of the node.

The first graph contains one node for each task initially assigned `r1` or `r2`, containing just that task. Thus, in the first graph there are no unassignable nodes. Given two nodes, the first graph's relation function first checks which resources are assignable to each. If there is no way to assign the same resource to both nodes, it returns `KHE_PART_GRAPH_DIFFERENT`. Otherwise, if there is no way to assign different resources to the nodes, it returns `KHE_PART_GRAPH_SAME`. Otherwise, if `fix_splits` is `true` and the two nodes share an avoid split assignments monitor of non-zero cost, it returns `KHE_PART_GRAPH_SAME`. Otherwise, if the two nodes overlap in time, it returns `KHE_PART_GRAPH_DIFFERENT`. Otherwise it returns `KHE_PART_GRAPH_UNRELATED`.

Next, the graph's connected components are found and partitioned. It is easy to see, referring to the relation function, that if a component was successfully partitioned there must be at least one way (and possibly two ways) to assign `r1` to the nodes of one part and `r2` to the nodes of the other part. So a component of the first graph is called *assignable* if it was successfully partitioned, and *unassignable* otherwise.

For each assignable component, the nodes of one part are merged into one node, and the nodes of the other are merged into a second node. These two nodes are assignable to different resources in one or two ways. For each unassignable component, all the nodes are merged into

a single node. It does not matter whether this node is assignable or not; it is never assigned.

Next, the assignable components are sorted into increasing order of number of possible assignments. Each of the C assignable components has 1 or 2 possible assignments. A tree search is carried out which tries each of these on each component in turn. The total search space size is at most 2^C. This is often small enough to search completely. For safety, the search only explores both assignments until 512 tree nodes have been visited; after that it tries only one assignment for each component. In the usual way, each time the tree search reaches a leaf it compares its solution cost with the best so far, and if it is better (and if the resource assignment invariant is preserved, if required) it takes a copy of its decisions. At the end, the cost of the best solution found is compared with the initial solution cost, and if the best solution is better it is installed; otherwise the initial solution is restored.

The search space often has symmetries which would waste time and cause the node limit to be reached often enough to compromise optimality in practice if they were not removed. The rest of this section describes them and how `KheResourcePairReassign` removes them.

Suppose `r1` and `r2` are Mathematics teachers assigned to two Mathematics courses from the same form, each split into 4 meets of the same durations, running simultaneously. This gives 4 components and a search space of size 2^4, yet clearly this could be reduced safely to 1. If two of the simultaneous meets are made not simultaneous, the search space size can still be reduced safely, to 2. If `fix_splits` is `true`, each set of 4 meets is related, making 1 component and a search space of size 2—still unnecessarily large when the meets are simultaneous.

A component is *symmetrical* if it makes no difference which of its two assignments is chosen. In that case, its assignment choices can be reduced from 2 to 1 by arbitrarily removing one, halving the search space size. But note the complicating factor in the Mathematics example: one cannot arbitrarily remove one choice from each component, because some combinations of choices lead to split assignments and others do not. Instead, a way must be found to first merge the four components into one, which can then be assigned arbitrarily.

Symmetry arises when the two assignment choices of a component affect monitors in the same way. They need to have the same effect on the state of monitors, so that no difference arises when the monitors change state again later in response to changes outside the component.

The two choices always have the same effect on the state of event monitors (no effect at all), and on the state of assign resources monitors, which care only whether tasks are assigned resources, not which resources. As far as these kinds of monitors are concerned, all components are symmetrical. Classify the remaining monitors into three groups: resource monitors, prefer resources monitors, and avoid split assignments monitors.

A component is *r-symmetrical*, *p-symmetrical*, or *s-symmetrical* when it is assignable both ways and they affect in the same way all resource, prefer resources, or avoid split assignments monitors that monitor tasks of the component. (In particular, if there are no monitors of some type, the component is vacuously symmetrical in that type.) Combinations of prefixes denote conjunctions of these conditions. For example, *symmetrical* is shorthand for *rps-symmetrical.*

Although these definitions are clear in principle, they are rather abstract. An algorithm needs concrete, easily computable conditions that imply the abstract ones and are likely to hold in practice. Here are the concrete conditions used by `KheResourcePairReassign`, assuming that the component is assignable both ways.

Suppose that some component's two parts run at the same times and have the same total workload. Then the component is r-symmetrical, because only these things affect resource monitors, except clashes—but component assignments have no clashes in themselves, and since the two parts run at the same times, they have the same clashes with tasks outside the component.

Suppose that, for every prefer resources monitor of non-zero cost which monitors any task of some component, either `r1` and `r2` are both preferred by the monitor's constraint, or they are both not preferred. Then the component is p-symmetrical.

Suppose that, for each task in some component c which is monitored by an avoid split assignments monitor of non-zero cost, every task monitored by that monitor either was not assigned `r1` or `r2` originally, or else it lies in c. Then the component is s-symmetrical.

To prove this, take one avoid split assignments monitor, and partition the set of tasks monitored by it into those that were not assigned `r1` or `r2` originally, and so are beyond the scope of the reassignment (call them S_1), and those that were (call them S_2). If the tasks of S_2 lie within two or more components, then which way those components are assigned does matter. But if they lie within one component, then the cost of the monitor will be the same whichever assignment is chosen. This is because `r1` and `r2` do not appear among the resources assigned to the tasks of S_1 (if they did, those tasks would be in S_2), so the assignments to S_2 introduce fresh resources to the monitor. If all the tasks of S_2 lie in one part of the component, one fresh resource is introduced by both assignments; if some lie in one part and the others in the other, two fresh resources are introduced by both assignments. Either way, the effect on the monitor is the same.

When `fix_splits` is `true`, all tasks which share an avoid split assignments monitor lie in the same part, so in the same component. So every component is s-symmetrical in that case.

It is easy to check whether a component is rp-symmetrical. This is done as each component is partitioned. Merely checking for s-symmetry is not enough: as illustrated by the Mathematics example, several components may need to be merged (by merging their parts) to produce one s-symmetrical component. This is done using the second partitioning graph, as follows.

The second-graph nodes are the merged nodes from the first-graph components. When two nodes come from the same first-graph component, `KHE_PART_GRAPH_DIFFERENT` is returned by the relation function. Otherwise, if they share an avoid split assignments monitor of non-zero cost, it returns `KHE_PART_GRAPH_SAME`. Otherwise it returns `KHE_PART_GRAPH_UNRELATED`.

Two nodes representing the two parts of a first-graph component must lie in the same second-graph component, because there is an edge between them. So each second-graph component is a set of first-graph components linked by avoid split assignments constraints.

For each second-graph component, its first-graph components may be merged if it does not contain an unassignable first-graph component, at most one of its first-graph components is not rp-symmetrical, and it is partitionable. The two nodes of the merged component are built by merging the nodes of each part of the second-graph component. If all the first-graph components being merged are rp-symmetrical, the resulting component is rps-symmetrical, so either one of its assignments may be removed. But component merges are valuable even without rps-symmetry.

11.10. Resource rematching

Function

```
bool KheResourceRematch(KHE_TASKING tasking, KHE_OPTIONS options);
```

repairs the assignments of the tasks of `tasking` as follows.

Take each task t_k of `tasking` which is either unassigned, or assigned an unpreferred resource, or part of a split assignment, or involved in a clash. Let s_k be the set of times that t_k is running. Build the set S of all distinct (not necessarily disjoint) such sets of times s_k.

The algorithm works for any set of times s_k, but it is only tried on the sets of times s_k in S, because only those offer any realistic prospect of improvement. For each such set of times s_k, then, the algorithm proceeds as follows.

Build a bipartite graph as follows. There is one supply node for each resource r_i (or each resource r_i of type `KheTaskingResourceType(tasking)`, if that is non-`NULL`). There is one demand node for each of these resources r_i, containing the set of tasks T_i which lie in `tasking`, overlap s_k in time, and are assigned to r_i. In addition, for each unassigned individual task which lies in `tasking` and overlaps s_k in time there is one demand node containing that task.

Unassign all tasks in all demand nodes. Join a supply node s to a demand node d when s's resource can be assigned to all of the tasks of d, and weight the edge by the cost of the solution when that is done. Find a matching in this graph of maximum size and minimum total weight, and use it to reassign all the tasks. If the resulting solution has smaller cost than the original, use it; otherwise return to the original solution.

Rematching methods are often inexact [12], but the reassignment found by this rematching is optimal, assuming that assigning is always better than not assigning. This can be seen by a careful examination of the 16 constraint types: each is either unaffected by the reassignment, or else its effect is independent for each resource/task pair, proving that the weights of the edges of any matching are valid in combination as well as individually.

11.11. Trying unassignments

KHE's solvers assume that it is always a good thing to assign a resource to a task. However, occasionally there are cases where cost can be reduced by unassigning a task, because the cost of the resulting assign resource defect is less than the cost of the defects introduced by the assignment. As some acknowledgement of these anomalous cases, KHE offers

```
bool KheSolnTryTaskUnAssignments(KHE_SOLN soln);
```

for use at the end. It tries unassigning each task of `soln` in turn. If any unassignment reduces the cost of `soln`, it is not reassigned. The result is `true` if any unassignments were kept.

11.12. Putting it all together

Three decisions face the designer of a resource solver. Should the solver work with split assignments, or with unsplit ones? Should it preserve the resource assignment invariant, or not? Should it respect the domains of tasks, or not? Fortunately, KHE makes it easy to write solvers

that can be used with any combination of these three decisions, as follows.

Get unsplit assignments by building a task tree with avoid split assignments jobs. Allow split assignments by calling `KheTaskingAllowSplitAssignments` (Section 11.3.4). Either way, the solver assigns resources to unfixed tasks, without knowing or caring if they have followers.

By enclosing each attempt to change the solution in `KheAtomicTransactionBegin` and `KheAtomicTransactionEnd` (Section 11.2), a solver can preserve the resource assignment invariant, or not, depending on the value of a Boolean parameter.

If domains are to be respected, do nothing; if not, then before running the solver, call `KheTaskingEnlargeDomains` (Section 11.3.4) to enlarge them to the full set of resources.

A sequence of three functions,

```
bool KheTaskingAssignResourcesStage1(KHE_TASKING tasking,
  KHE_OPTIONS options);
bool KheTaskingAssignResourcesStage2(KHE_TASKING tasking,
  KHE_OPTIONS options);
bool KheTaskingAssignResourcesStage3(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

packages this chapter's ideas into a three-stage solver which assigns resources to the tasks of `tasking`. Called in order, they take a 'progressive corruption' approach to the decisions just described: they are spotless at first, but they slide into the gutter towards the end.

`KheTaskingAssignResourcesStage1` first ensures that domains and assignments take account of all constraints:

```
KheOptionsSetResourceInvariant(options, true);
tjt = KHE_TASK_JOB_HARD_PRC | KHE_TASK_JOB_SOFT_PRC |
      KHE_TASK_JOB_HARD_ASAC | KHE_TASK_JOB_SOFT_ASAC;
KheTaskingMakeTaskTree(tasking, tjt, NULL, options);
```

Then it assigns resources to the unassigned unfixed tasks of `tasking`, using resource packing if there are avoid split assignments constraints, and the simpler most constrained first algorithm otherwise. This is followed by an ejection chain repair algorithm:

```
rt = KheTaskingResourceType(tasking);
if( rt == NULL || KheResourceTypeAvoidSplitAssignmentsCount(rt) > 0 )
  KheResourcePackAssignResources(tasking, options);
else
  KheMostConstrainedFirstAssignResources(tasking, options);
KheEjectionChainRepairResources(tasking, options);
```

So far, there are no split assignments, the resource assignment invariant is preserved, and domains are respected. The great majority of the tasks, probably, have been assigned resources.

`KheTaskingAssignResourcesStage2` calls `KheFindSplitResourceAssignments` to build split assignments, and `KheTaskingAllowSplitAssignments` to permit all tasks, assigned or not, to be split. It then carries out ejection chain, rematching, and resource pair repairs:

```
rt = KheTaskingResourceType(tasking);
if( KheResourceTypeAvoidSplitAssignmentsCount(rt) > 0 )
{
  KheFindSplitResourceAssignments(tasking, options);
  KheTaskingAllowSplitAssignments(tasking, false);
  KheEjectionChainRepairResources(tasking, options);
  if( KheOptionsResourceRematch(options) )
    KheResourceRematch(tasking, options);
  KheResourcePairRepair(tasking, options);
}
```

Since there are split assignments now, the ejection chain algorithm will try to unsplit them (it has always had an augment function for this, but there have been no split assignments to trigger it until now). It also tries to assign unassigned tasks, even at the cost of splitting assignments that were previously unsplit. The calls to `KheResourceRematch` and `KheResourcePairRepair` only act when permitted by the relevant options (Section 8.4).

`KheTaskingAssignResourcesStage3` is very corrupt indeed:

```
KheOptionsSetResourceInvariant(options, false);
KheTaskingEnlargeDomains(tasking, true);
KheEjectionChainRepairResources(tasking, options);
```

The domains of the remaining unassigned tasks are enlarged to the full resource type using `KheTaskingEnlargeDomains`, and the ejection chain algorithm is run yet again, this time without preserving the resource assignment invariant. Enlarging domains makes sense only at the very end, and will make a difference only if any room or any teacher is better than none. Because of the removal of the invariant, this stage should be run only after the first two stages have been run for each resource type.

Chapter 12. Ejection Chains

Ejection chains are sequences of repairs that generalize the augmenting paths from bipartite matching. They are due to Glover [3], who applied them to the travelling salesman problem.

12.1. Introduction

An ejection chain algorithm targets one defect and tries a set of alternative *repairs* on it. A repair could be a simple move or swap, or something arbitrarily complex. It removes the defect, but may introduce new defects. If no new defects of significant cost appear, that is success. If just one significant new defect appears, the method calls itself recursively to try to remove it; in this way a chain of coordinated repairs is built up. If several significant new defects appear, or the recursive call fails to remove the new defect, it undoes the repair and continues with alternative repairs. It can also try to remove all the new defects, although that is not often useful.

Corresponding to the well-known function for finding an augmenting path in a bipartite graph, starting from a given node, is this function, formulated by the author, for 'augmenting' (improving) a solution, starting from a given defect:

```
bool Augment(Solution s, Cost c, Defect d);
```

(KHE's interface is somewhat different to this.) `Augment` has precondition

```
cost(s) >= c && cost(s) - cost(d) < c
```

If it can change `s` to reduce its cost to less than `c`, it does so and returns `true`; if not, it leaves `s` unchanged and returns `false`. The precondition implies that removing `d` without adding new defects would be one way to succeed. Here is an abstract implementation of `Augment`:

```
bool Augment(Solution s, Cost c, Defect d)
{
  repair_set = RepairsOf(d);
  for( each repair r in repair_set )
  {
    new_defect_set = Apply(s, r);
    if( cost(s) < c )
      return true;
    for( each e in new_defect_set )
      if( cost(s) - cost(e) < c && Augment(s, c, e) )
        return true;
    UnApply(s, r);
  }
  return false;
}
```

It begins by finding a set of ways that `d` could be repaired. For each repair, it applies it and

receives the set of new defects introduced by that repair, looks for success in two ways, then if neither of those works out it unapplies the repair and continues by trying the next repair, returning `false` when all repairs have been tried without success.

Success could come in two ways. Either a repair reduces `cost(s)` to below `c`, or some new defect `e` has cost large enough to ensure that removing it alone would constitute success, and a recursive call targeted at `e` succeeds. Notice that `cost(s)` may grow without limit as the chain deepens, while there is a defect `e` whose removal would reduce the solution's cost to below `c`.

The key observation that justifies the whole approach is this: the new defects targeted by the recursive calls are not known to have resisted attack before. It might be possible to repair one of them without introducing any new defects of significant cost.

The algorithm stops at the first successful chain. An option for finding the best successful chain rather than the first has been withdrawn, because of design problems in combining it with ejection trees (Section 12.5.3). It is no great loss: it produced nothing remarkable, and ran slowly. Another option, for limiting the disruption caused by the repairs, has also been withdrawn. It too was not very useful. It can be approximated by limiting depth, as described next.

The tree searched by `Augment` as presented may easily grow to exponential size, which is not the intention. The author has tried two methods of limiting its size, both of which seem useful. They may be used separately or together.

The first method is to limit the depth of recursion to a fixed constant, perhaps 3 or 4. The maximum depth is passed as an extra parameter to `Augment`, and reduced on each recursive call, with value 0 preventing further recursion. Not only is this attractive in itself, it also supports *iterative deepening*, in which `Augment` is called several times on the same defect, with the depth parameter increased each time. Another idea is to use a small depth on the first iteration of the main loop (see below), and increase it on later iterations.

The second method is the one used by augmenting paths in bipartite matching. Just before each call on `Augment` from the main loop, the entire solution is marked unvisited (by incrementing a global visit number, not by traversing the entire solution). When a repair changes some part of the solution, that part is marked visited. Repairs that change parts of the solution that are already marked visited are tabu. This limits the size of the tree to the size of the solution.

Given a solution and a list of its defects, the main loop cycles through the list repeatedly, calling `Augment` on each defect in turn, with `c` set to `cost(s)`. When the main loop exits, every defect has been tried at least once without success since the most recent success, so no further successful augments are possible, unless there is a random element within `Augment`. Under reasonable assumptions, this very clear-cut stopping criterion ensures that the whole algorithm runs in polynomial time, for the same reason that hill-climbing does.

When there are several defect types, several `Augment` algorithms are needed, one for each defect type, dynamically dispatched on the type. The repairs are usually applied directly, rather than indirectly via objects built to represent them.

Careful work is needed to maximize the effectiveness of ejection chains. Grouping together monitors that measure the same thing is important, because it reduces the number of defects and increases their cost, increasing the chance that a chain will be continued. Individual repair operations should actually remove the defects that they are called to repair (the framework does not check this), and should do whatever seems most likely to avoid introducing new defects.

12.2. Ejector construction

KHE offers *ejector* objects which provide a framework for ejection chain algorithms, reducing the implementation burden to writing just the augment functions. The framework uses visit numbers (Section 4.6), in the conventional way.

To support statistics gathering (Section 12.6), a single ejector object may be re-used many times, even on different instances (although not in parallel—an ejector object is highly mutable and cannot be shared by two or more threads). This makes it important to distinguish between those parts of the ejector which are constant throughout its lifetime, and those parts which vary from solve to solve. This section is concerned with the constant parts.

An ejector is constructed by a sequence of calls beginning with

```
KHE_EJECTOR KheEjectorMakeBegin(void);
```

and ending with

```
void KheEjectorMakeEnd(KHE_EJECTOR ej);
```

Its constant parts are set by calling the functions described in this section in between these two calls. After `KheEjectorMakeEnd` is called the constant parts are fixed and cannot be changed.

It is also possible to construct an ejector by copying an existing ejector:

```
KHE_EJECTOR KheEjectorCopy(KHE_EJECTOR ej);
```

This returns a new ejector whose constant parts are the same as `ej`'s. It is like a sequence of calls beginning with `KheEjectorMakeBegin` and ending with `KheEjectorMakeEnd`, with calls in between which install into the new ejector the same constant parts as `ej` has. Function

```
void KheEjectorDelete(KHE_EJECTOR ej);
```

deletes an ejector when it is no longer needed.

The constant parts of an ejector object are a sequence of *major schedules* and a set of *augment functions*. The major schedules control the detailed behaviour of each solve, while the augment functions are callback functions passed in by the user, containing the repairs.

A major schedule is represented by an object of type `KHE_EJECTOR_MAJOR_SCHEDULE`. It contains a sequence of *minor schedules*. A minor schedule is represented by an object of type `KHE_EJECTOR_MINOR_SCHEDULE`. It contains two attributes: *maximum depth* and *may-revisit*. The entire main loop of the algorithm, which repeatedly tries to augment out of each defect until no further improvements can be found, is repeated once for each major schedule in order. An ejector with no major schedules does nothing. Within each main loop, the augment for one defect is tried once for each minor schedule of the current major schedule, until an augment succeeds in reducing the cost of the solution or all minor schedules have been tried. A major schedule with no minor schedules does nothing.

An ejector's major schedules may be accessed at any time by

```
int KheEjectorMajorScheduleCount(KHE_EJECTOR ej);
KHE_EJECTOR_MAJOR_SCHEDULE KheEjectorMajorSchedule(KHE_EJECTOR ej, int i);
```

in the usual way. To begin and end adding one major schedule, call

```
void KheEjectorMajorScheduleBegin(KHE_EJECTOR ej);
void KheEjectorMajorScheduleEnd(KHE_EJECTOR ej);
```

Its minor schedules are added by calls to `KheEjectorMinorScheduleAdd` (described below) in between these calls, and may then be accessed by calling

```
int KheEjectorMajorScheduleMinorScheduleCount(KHE_EJECTOR_MAJOR_SCHEDULE ejm);
KHE_EJECTOR_MINOR_SCHEDULE KheEjectorMajorScheduleMinorSchedule(
  KHE_EJECTOR_MAJOR_SCHEDULE ejm, int i);
```

in the usual way.

Minor schedules are added to a major schedule by calls to

```
void KheEjectorMinorScheduleAdd(KHE_EJECTOR ej,
  int max_depth, bool may_revisit);
```

in between the calls to `KheEjectorMajorScheduleBegin` and `KheEjectorMajorScheduleEnd`. The attributes of a minor schedule may be retrieved by

```
int KheEjectorMinorScheduleMaxDepth(KHE_EJECTOR_MINOR_SCHEDULE ejms);
bool KheEjectorMinorScheduleMayRevisit(KHE_EJECTOR_MINOR_SCHEDULE ejms);
```

The `max_depth` attribute determines the maximum depth of recursion (the maximum number of repairs allowed on one chain). Value 0 allows no repairs at all and is forbidden. Value 1 allows augment calls from the main loop, but prevents them from making recursive calls, producing a kind of hill climbing. Value 2 allows the calls made from the main loop to make recursive calls, but prevents those calls from recursing. And so on.

Each *tree* (rooted at one augment call in the main loop) gets a new global visit number, making it free to change any part of the solution. When `may_revisit` is `false`, each part of the solution may be changed by at most one of the recursive calls within one tree; when it is `true`, each part may be changed by any number of them, although only once along any one chain.

Here are some examples. To allow up to two repairs on any chain, with revisiting:

```
KheEjectorMajorScheduleBegin(ej);
KheEjectorMinorScheduleAdd(ej, 2, true);
KheEjectorMajorScheduleEnd(ej);
```

To allow arbitrary-depth recursion, but no revisiting:

```
KheEjectorMajorScheduleBegin(ej);
KheEjectorMinorScheduleAdd(ej, INT_MAX, false);
KheEjectorMajorScheduleEnd(ej);
```

It is a bad idea to set `max_depth` to a large value and `may_revisit` to `true` in the same schedule, because the algorithm will then usually take exponential time. But setting `max_depth` to a small constant, or setting `may_revisit` to `false`, or both, guarantees polynomial time. Another interesting idea is *iterative deepening*, in which several depths are tried. For example,

```
KheEjectorMajorScheduleBegin(ej);
KheEjectorMinorScheduleAdd(ej, 1, true);
KheEjectorMinorScheduleAdd(ej, 2, true);
KheEjectorMinorScheduleAdd(ej, 3, true);
KheEjectorMinorScheduleAdd(ej, INT_MAX, false);
KheEjectorMajorScheduleEnd(ej);
```

tries maximum depth 1, then 2, then 3, and finishes with arbitrary depth.

Here are two faster ways to add schedules:

```
void KheEjectorAddDefaultSchedules(KHE_EJECTOR ej);
void KheEjectorSetSchedulesFromString(KHE_EJECTOR ej,
  char *ejector_schedules_string);
```

`KheEjectorAddDefaultSchedules` adds some major and minor schedules, chosen by the author as reasonable defaults. At present it does this:

```
KheEjectorMajorScheduleBegin(ej);
KheEjectorMinorScheduleAdd(ej, 1, true);
KheEjectorMajorScheduleEnd(ej);
KheEjectorMajorScheduleBegin(ej);
KheEjectorMinorScheduleAdd(ej, INT_MAX, false);
KheEjectorMajorScheduleEnd(ej);
```

`KheEjectorSetSchedulesFromString` interprets its string as a sequence of instructions for adding schedules to `ej`, and follows the instructions. It supports tests that compare schedules.

The string contains a sequence of one or more major schedules separated by commas. Each major schedule consists of a sequence of minor schedules, each represented by two characters. The first character of each minor schedule is a digit or `u`, and defines the depth limit; `u` means unlimited. The second is either `+`, meaning with revisiting, or `-`, meaning without it. For example, `"1+,u-"` defines two major schedules. The first has one minor schedule with depth limit 1 and revisiting; the second has one minor schedule with unlimited depth and no revisiting.

The other constant part of an ejector object is the set of augment functions, one function for each kind of defect that the user wants the ejector to repair. These augment functions are written by the user, as described in Section 12.4, and passed to the ejector by calls to

```
void KheEjectorAddAugment(KHE_EJECTOR ej, KHE_MONITOR_TAG tag,
  KHE_EJECTOR_AUGMENT_FN augment_fn, int augment_type);
void KheEjectorAddGroupAugment(KHE_EJECTOR ej, int sub_tag,
  KHE_EJECTOR_AUGMENT_FN augment_fn, int augment_type);
```

The first says that defects which are non-group monitors with tag `tag` should be handled by `augment_fn`; the second says that defects which are group monitors with sub-tag `sub_tag` should be handled by `augment_fn`. Here `sub_tag` must be between 0 and 29 inclusive. Any values not set are handled by doing nothing, as though an unsuccessful attempt was made to repair them. Ejectors handle the polymorphic dispatch by defect type. The `augment_type` parameter is used by statistics gathering (Section 12.6), and may be 0 if statistics are not wanted.

12.3. Ejector solving

Once an ejector has been set up, the ejection chain algorithm may be run by calling

```
bool KheEjectorSolve(KHE_EJECTOR ej, KHE_GROUP_MONITOR start_gm,
  KHE_GROUP_MONITOR continue_gm, KHE_OPTIONS options);
```

This runs the main loop of the ejection chain algorithm once for each major schedule, returning `true` if it reduces the cost of the solution monitored by `start_gm` and `continue_gm`.

The main loop repairs only the defective child monitors of `start_gm`, and the recursive calls repair only the defective child monitors of `continue_gm`. These two group monitors could be equal, and either or both could be an upcast solution. Although it is not required, in practice every child monitor of `start_gm` is also a child monitor of `continue_gm`.

Just as an ejector is constructed by a sequence of calls enclosed in `KheEjectorMakeBegin` and `KheEjectorMakeEnd`, so a solve is carried out by a sequence of calls beginning with

```
void KheEjectorSolveBegin(KHE_EJECTOR ej, KHE_GROUP_MONITOR start_gm,
  KHE_GROUP_MONITOR continue_gm, KHE_OPTIONS options);
```

and ending with

```
bool KheEjectorSolveEnd(KHE_EJECTOR ej);
```

`KheEjectorSolveEnd` does the actual solving. Function `KheEjectorSolve` above just calls `KheEjectorSolveBegin` and `KheEjectorSolveEnd` with nothing in between.

The only functions callable between `KheEjectorSolveBegin` and `KheEjectorSolveEnd` (at least, the only ones that change anything) are

```
void KheEjectorAddMonitorCostLimit(KHE_EJECTOR ej,
  KHE_MONITOR m, KHE_COST cost_limit);
void KheEjectorAddMonitorCostLimitReducing(KHE_EJECTOR ej,
  KHE_MONITOR m);
```

The call `KheEjectorAddMonitorCostLimit(ej, m, cost_limit)` says that for a chain to end successfully, not only must the solution cost be less than the initial cost, but `KheMonitorCost(m)` must be no larger than `cost_limit`. `KheEjectorAddMonitorCostLimitReducing(ej, m)` is the same except that the cost limit is initialized to `KheMonitorCost(m)`, and if a successful chain is found and applied which reduces `KheMonitorCost(m)` to below its current limit, that limit is reduced to the new `KheMonitorCost(m)` for subsequent chains.

To visit these *limit monitors*, call

```
int KheEjectorMonitorCostLimitCount(KHE_EJECTOR ej);
void KheEjectorMonitorCostLimit(KHE_EJECTOR ej, int i,
  KHE_MONITOR *m, KHE_COST *cost_limit, bool *reducing);
```

The returned values are the monitor, its current cost limit, and whether that limit may be reduced. Any number of limit monitors may be added, but large numbers will not be efficient.

Each time the ejector enters the main loop, it makes a copy of `start_gm`'s list of defects and

sorts the copy by decreasing cost. If the `diversify` option (Section 8.4.1) is `true`, ties are broken differently depending on the value of the solution's diversifier. If the `ejector_limit_defects` option is set to some integer other than its default value, `INT_MAX`, defects are dropped from the end of the sorted list to ensure that there are no more than `ejector_limit_defects` of them.

Consider a defect *d* that the main loop of the ejection chain solver is just about to attempt to repair. Suppose that the most recent change either to the solution or to the major schedule occurred before the most recent previous attempt to repair *d*. Then, if the repair is deterministic, the current attempt to repair *d* is certain to fail like the previous attempt did. Accordingly, it is skipped. The implementation of this optimization uses visit numbers stored in monitors.

In practice, repairs are not deterministic, since, for diversity, KHE's augment functions vary the starting points of their traversals of lists of repairs between calls. However, the author carried out an experiment on a large instance (NL-KP-03), in which this optimization was turned off but a check was made to see whether there were any cases where repairs which it would have caused to be skipped were successful. Over 8 diversified solves there were 15 cases.

The following functions may be called while `KheEjectorSolve` is running (that is, from within augment functions):

```
KHE_GROUP_MONITOR KheEjectorStartGroupMonitor(KHE_EJECTOR ej);
KHE_GROUP_MONITOR KheEjectorContinueGroupMonitor(KHE_EJECTOR ej);
KHE_OPTIONS KheEjectorOptions(KHE_EJECTOR ej);
KHE_SOLN KheEjectorSoln(KHE_EJECTOR ej);
KHE_COST KheEjectorTargetCost(KHE_EJECTOR ej);
KHE_EJECTOR_MAJOR_SCHEDULE KheEjectorCurrMajorSchedule(KHE_EJECTOR ej);
KHE_EJECTOR_MINOR_SCHEDULE KheEjectorCurrMinorSchedule(KHE_EJECTOR ej);
bool KheEjectorCurrMayRevisit(KHE_EJECTOR ej);
int KheEjectorCurrDepth(KHE_EJECTOR ej);
int KheEjectorCurrAugmentCount(KHE_EJECTOR ej);
```

`KheEjectorStartGroupMonitor`, `KheEjectorContinueGroupMonitor`, and `KheEjectorOptions` are `start_gm`, `continue_gm`, and `options` from `KheEjectorSolve`. `KheEjectorSoln` is `start_gm`'s (and also `continue_gm`'s) solution. `KheEjectorTargetCost` is the cost that the chain must improve on in order to succeed (`c` in the abstract presentation above): the cost that the solution had when `Augment` was most recently called from the main loop, except when ejection trees are in use, as explained in Section 12.5.3. `KheEjectorCurrMajorSchedule` is the current major schedule. `KheEjectorCurrMinorSchedule` is the current minor schedule, and `KheEjectorCurrMayRevisit` is its `may_revisit` attribute—frequently needed within augment functions, as will be seen. `KheEjectorCurrDepth` is the current depth (1 when the augment function was called from the main loop, 2 when called from an augment function called from the main loop, etc.). `KheEjectorCurrAugmentCount` is the number of augments since this solve began.

12.4. How to write an augment function

An augment function has type

```
typedef void (*KHE_EJECTOR_AUGMENT_FN)(KHE_EJECTOR ej, KHE_MONITOR d);
```

The parameters are the ejector `ej` passed to `KheEjectorSolve`, and the defect `d` that the augment function is supposed to repair. It is a precondition that `d` have non-zero cost and removing that cost would make for a successful augment.

Augment functions often look like this, although not necessarily exactly:

```
void ExampleAugment(KHE_EJECTOR ej, KHE_MONITOR d)
{
  KHE_ENTITY e;  bool success;  REPAIR r;
  e = SomeSolnEntityRelatedTo(d);
  if( !KheEntityVisited(e) )
  {
    KheEntityVisit(e);
    for( each r in RepairsOf(e) )
    {
      KheEjectorRepairBegin(ej);
      success = Apply(r);
      if( KheEjectorRepairEnd(ej, 0, success) )
        return;
    }
    if( KheEjectorCurrMayRevisit(ej) )
      KheEntityUnVisit(e);
  }
}
```

Function `SomeSolnEntityRelatedTo` uses `d` to identify some entity (node, meet, task, etc.) that will be changed by the repairs, but that should only be changed if it has not already been visited (tested by calling `KheMeetVisited` etc. from Section 4.6). Function `RepairsOf` builds a set of alternative repairs `r` of `e`, and `Apply(r)` stands for the code that applies repair `r`. In practice, repairs just need to be iterated over and applied; an explicit set of them is not needed.

Functions `KheEjectorRepairBegin` and `KheEjectorRepairEnd` are supplied by KHE:

```
void KheEjectorRepairBegin(KHE_EJECTOR ej);
bool KheEjectorRepairEndLong(KHE_EJECTOR ej, int repair_type,
  bool success, int max_sub_chains, bool save_and_sort,
  void (*on_success_fn)(void *on_success_val), void *on_success_val);
bool KheEjectorRepairEnd(KHE_EJECTOR ej, int repair_type, bool success);
```

`KheEjectorRepairEnd` and `KheEjectorRepairEndLong` are the same except that `KheEjectorRepairEnd`, the best choice in most circumstances, relieves the user of the burden of supplying the usual values for the last four parameters, namely `1`, `false`, `NULL`, and `NULL`. If other values are wanted for any of these parameters, then `KheEjectorRepairEndLong` must be called. For `max_sub_chains` see Section 12.5.3; for `save_and_sort` see Section 12.5.4; and for `on_success_fn` and `on_success_val` see Section 12.5.5. '`KheEjectorRepairEnd`' means '`KheEjectorRepairEnd` or `KheEjectorRepairEndLong`' from here on.

Calls to `KheEjectorRepairBegin` and `KheEjectorRepairEnd` must occur in matching pairs. A call to `KheEjectorRepairBegin` informs `ej` that a repair is about to begin, and the following call to `KheEjectorRepairEnd` informs it that that repair has just ended. The repair is

undone and redone (if required) behind the scenes by `KheEjectorRepairEnd`, using marks and paths, so undoing is not the user's concern.

The `repair_type` parameter of `KheEjectorRepairEnd` is used to gather statistics about the solve (Section 12.6). It may be 0 if statistics are not wanted.

The `success` parameter tells the ejector whether the caller thinks the current repair was successful. If it is `false`, the ejector undoes the repair and forgets that it ever happened. The other parameters are ignored in that case. If it is `true`, the ejector checks whether the repair reduced the cost of the solution, whether there is a single new defect worth recursing on, and so on. The writer of an augment function can forget that all this is happening behind the scenes.

If `KheEjectorRepairEnd` returns `true`, the ejector has decided that there is no point in trying more repairs for the current defect. The reason for this decision is not the business of the augment function; it must return without trying any more repairs (it is an error not to do this). In that case it does not matter whether the entity is marked unvisited or not before exit.

12.5. Variants of the ejection chains idea

This section presents some variants of the basic ejection chains idea.

12.5.1. Defect promotion

Successful chains begin by repairing a defect which is one of `start_gm`'s children, and continue by repairing defects which are children of `continue_gm`. The intention is that `start_gm` should monitor some region of the solution that has only just been assigned, so that there has been no chance yet to repair its defects, while `continue_gm` monitors the entire solution so far, or the part of it that is relevant to repairing the defects of `start_gm`. These two regions may be the same, which is why `start_gm` and `continue_gm` may be the same group monitor; but when they are different, the difference is important, as the following argument shows.

Suppose only `start_gm` is used. Then the ejector sets out to repair the right defects, but is unable to follow chains of repairs into parts of the solution that have been assigned previously. Or suppose only `continue_gm` is used. If the children of `continue_gm` are a superset of the children of `start_gm`, as is always the case in practice, this does allow a full search, but at the cost of trying again to repair many defects for which a previous repair attempt failed (those in `continue_gm` which are not also in `start_gm`). This can waste a lot of running time.

At this point, however, an unexpected issue enters. Suppose a successful chain is found which causes some child `d` of `continue_gm` to become defective, but which nevertheless terminates without repairing `d` because it improves the overall solution cost. Here is a new defect, a child of `continue_gm` not known to have been repaired previously, and thus worthy of being targeted for repair; but if it is not also a child of `start_gm`, it won't be.

Defect promotion addresses this issue. When an ejection chain is declared successful, the ejector examines the defects created by that chain's last repair that are children of `continue_gm`. These come from the trace object in the usual way. It makes any of these that are not children of `start_gm` into children of `start_gm`: they get dynamically added to the set of defects targeted by the current solve. Of course, when `start_gm` and `continue_gm` are the same, it does nothing.

Defect promotion is optional, controlled by option `ejector_promote_defects`, whose

default value is `true`. On one run it reduced the final solution cost from 0.04571 to 0.03743, while increasing running time from 286.84 seconds to 490.21 seconds—a substantial amount, but nothing like what would have occurred if `start_gm` had been replaced by `continue_gm`.

12.5.2. Fresh visit numbers for sub-defects

It is common for a monitor to monitor several points in the solution. For example, a prefer times monitor monitors several meets, all those derived from one point of application of the corresponding prefer times constraint (one event). Arguably, having one monitor for each meet would make more sense; but there is a problem with this, at least when the cost function is not `Linear`, because then there is no well-defined value of the cost of such a monitor. A cost is only defined after all the deviations of the *sub-defects* at all the monitored points are added up.

The usual way to repair a defective monitor which monitors several points is to visit each point, determine whether that point is a sub-defect, and try some repairs if so. When the repair is at depth 1, it makes sense for the augment function to give a fresh visit number to each sub-defect, so that the repair at each sub-defect is free to search the whole solution, as in this template:

```
for( i = 0;  i < KheMonitorPointCount(m);  i++ )
{
  p = KheMonitorPoint(m, i);
  if( KheMonitorPointIsDefective(p) )
  {
    if( KheEjectorCurrDepth(ej) == 1 )
      KheSolnNewGlobalVisit(soln);
    if( KheMonitorPointTryRepairs(p) )
      return;
  }
}
```

Calling `KheSolnNewGlobalVisit` opens up the whole solution for visiting. This is what would happen if the monitor was broken into smaller monitors, one for each point. It is important, however, not to call `KheSolnNewGlobalVisit` at deeper levels, since that amounts to allowing revisiting, so it leads to exponential time searches.

Fresh visit numbers are *not* assigned in this way within the augment functions supplied with KHE. Instead, a more radical version of the idea is offered by the `ejector_fresh_visits` option. When set to `true`, it causes

```
if( KheEjectorCurrDepth(ej) == 1 )
  KheSolnNewGlobalVisit(soln);
```

to be executed within each call to `KheEjectorRepairBegin`, opening up the entire solution, not just to each sub-defect at depth 1, but to each repair of each sub-defect at depth 1.

12.5.3. Ejection trees

An *ejection tree* is like an ejection chain except that at each level below the first, instead of repairing one newly introduced defect, it tries to repair several (or all) of the newly introduced

defects, producing a tree of repairs rather than a chain.

Ejection trees are not likely to be useful often. It is true that the run time of an ejection tree is limited as usual by the size of the solution, but its chance of success is lower than usual, because it must repair several defects at the lower level to succeed at the higher level. If repairing the first defect produces two new defects, repairing each of those produces two more, and so on, then the result is a huge number of defects that must all be repaired successfully. And to make a repair which introduces a defect and then repair that defect using an ejection tree is to spend a lot of time on a defect that can be removed much more easily by undoing the initial repair.

However, when the original solution has a very awkward defect, the best option may be a complex repair which usually introduces several new defects. For example, the best way to repair a cluster busy times overload defect may be to unassign every meet on one of the problem days. In that case, it makes sense to use an ejection tree at that level alone: that is, to try a repair that introduces several defects, then try to repair them by finding an ejection chain for each.

The `max_sub_chains` parameter of `KheEjectorRepairEndLong` allows for ejection trees, by specifying the maximum number of defects introduced by that repair that are to be repaired. Different repairs may have different values of `max_sub_chains`. For example, the complex cluster busy times repair could be tried only when `KheEjectorCurrDepth(ej)` is 1, with `max_sub_chains` set to `INT_MAX`. All other repairs could be given value 1 for `max_sub_chains`, producing ordinary chains elsewhere.

A set of defects now has to be repaired, not necessarily just one. One option would have been to change the interface of `Augment` to pass this set to the user. This was not done, because it would be a major change from the targeted repairs used by ejection chains. Instead, just as the framework handles the dynamic dispatch by defect type, so it also accepts a whole set of defects for repair and passes them one by one to conventional `Augment` calls.

The remainder of this section explains the implementation of ejection trees (and indeed ejection chains) by presenting a more detailed version of the `Augment` function than the one given at the start of this chapter. One detail concerns the influence of monitor lower bounds. Define

```
Potential(d) = KheMonitorCost(d) - KheMonitorLowerBound(d)
```

The potential is the maximum improvement obtainable by repairing `d`, a quantity that turns out to be more relevant here than cost alone.

Another detail concerns monitor cost limits, which require that the solution not change so as to cause the cost of some given monitors to exceed given limits (Section 12.3). To handle them, the interface of `Augment` is changed to

```
bool Augment(Solution s, Cost c, Limits x, Defect d);
```

where `x` is a set of monitor cost limits. `Augment` returns `true` if the value of `s` afterwards is such that `s`'s cost is less than `c` and the limits `x` are all satisfied. This condition is evaluated by

```
bool Success(Solution s, Cost c, Limits x)
{
  return cost(s) < c && LimitsAllSatisfied(s, x);
}
```

The precondition of `Augment(s, c, x, d)` is changed to

```
!Success(s, c, x) && cost(s) - Potential(d) < c
```

Its postcondition is `Success(s, c, x)` if `true` is returned, and ‘`s` is unchanged’ otherwise.

The new defects chosen for repair must be *open defects*: defects whose cost increased during the previous repair, as reported by the trace of that repair. The `max_sub_chains` open defects of largest potential, or all open defects if fewer than `max_sub_chains` open defects are reported by the trace, are selected. In the code below, this selection is made by line

```
{d1, ..., dn} = SelectOpenDefects(new_defect_set, MaxSubChains(r));
```

where `0 <= n <= MaxSubChains(r)`.

Here is the more detailed implementation of `Augment`:

```
bool Augment(Solution s, Cost c, Limits x, Defect d)
{
  repair_set = RepairsOf(d);
  for( each repair r in repair_set )
  {
    new_defect_set = Apply(s, r);
    if( Success(s, c, x) )
      return true;
    if( NotAtDepthLimit() )
    {
      {d1, ..., dn} = SelectOpenDefects(new_defect_set, MaxSubChains(r));
      for( i = 1;  i <= n;  i++ )
      {
        sub_c = c + Potential(d(i+1)) + ... + Potential(dn);
        sub_x = (i < n ? {} : x);  /* empty limit set except at end */
        if( Success(s, sub_c, sub_x) )
          continue;
        if( cost(s) - Potential(di) >= sub_c )
          break;
        if( !Augment(s, sub_c, sub_x, di) )
          break;
        if( Success(s, c, x) )
          return true;
      }
    }
    reset s to its state just before Apply(s, r);
  }
  return false;
}
```

As before, all of this except the loop that iterates over and applies repairs is hidden in calls to `KheEjectorRepairBegin` and `KheEjectorRepairEnd`. It is easy to verify that this satisfies the revised postcondition. The reset near the end is carried out by a call to `KheMarkUndo`.

After the usual test for success immediately after the repair, if the depth limit has not been

reached the new code selects n open defects for repair, then calls Augment recursively on each in turn. The complicating factor is the choice of a target cost and set of limits for each recursive call, denoted sub_c and sub_x above. Using the original c and x, as is done with ejection chains, would wrongly place the entire burden of improving the solution onto the first recursive call.

When repairing d1, the right cost target to shoot for is

```
sub_c = c + Potential(d2) + ... + Potential(dn);
```

The best that can be hoped for from repairing d2 is Potential(d2), the best that can be hoped for from repairing d3 is Potential(d3), etc. So if the first recursive Augment cannot reduce cost(s) below the given value of sub_c, there is little hope that after all the recursive augments it will be reduced below c. The same idea is applied for each of the di.

When a recursive call to Augment changes the solution, some Potential(di) values may change. So this code re-evaluates sub_c from scratch on each iteration of the inner loop, rather than attempting to save time by adjusting the previous value of sub_c.

The choice of sub_x causes limits to be ignored except when carrying out the last augment. This is in accord with the intention of monitor cost limits, which is to only check them at the end. It would be a mistake to check them earlier. For example, the repair of the cluster busy times defects described above is likely to violate a monitor cost limit when it deassigns meets. These do need to be reassigned by the end, but they will not all be reassigned earlier.

After defining sub_c and sub_x but before the call to Augment, the code executes

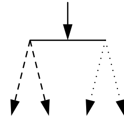
```
if( Success(s, sub_c, sub_x) )
  continue;
if( cost(s) - Potential(di) >= sub_c )
  break;
```

These lines ensure that the precondition of the recursive Augment call holds at the time it is made. If Success(s, sub_c, sub_x) holds, then the aim of that call has already been achieved, so the algorithm moves on to the next one. It does not matter that it skips the Success(s, c, x) test further on, because there has been such a test since the last time the solution changed. If cost(s) - Potential(di) >= sub_c holds, then the algorithm has no real hope of beating sub_c by repairing di, and so no real hope of success at all, so it abandons the current repair.

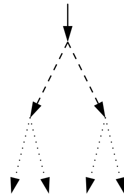
Success(s, c, x) implies Success(s, sub_c, sub_x) throughout Augment, because sub_c >= c and sub_x is a subset of x. This cannot be used to simplify Augment, but it does have one or two interesting consequences. For example, it applies transitively down through all active calls to Augment, so while Success(s, sub_c, sub_x) is false at any level of recursion, the original aim of the ejection tree cannot be satisfied.

When repairing dn, sub_c == c and sub_x == x. This gives confidence that Augment could succeed, and shows that it reduces to the original Augment when MaxSubChains(r) == 1, except for the different expression of how one open defect is selected.

The method described here finds the first chain that repairs d1, fixes it, and moves on to d2. Representing the higher path by a solid arrow, the chains (successful or not) that repair d1 by dashed arrows, and the chains (successful or not) that repair d2 by dotted arrows, the picture is



Another possibility is to find the first chain that repairs `d1`, then try to find chains for `d2`, but if that fails, to continue searching for other chains for `d1`:



This approach is implementable within the current framework, but it has not been tried. There is no reason to think that it would be better.

12.5.4. Sorting repairs

Each repair is usually followed immediately by recursive calls which extend the chain, where applicable. Setting the `save_and_sort` parameter of `KheEjectorRepairEndLong` to `true` invokes a different arrangement. Paths representing the repairs are saved in the ejector without recursion. After the last repair they are sorted into increasing order of the cost of the solutions they produce, and each is tried in turn, just as though they had occurred in that sorted order [5].

In practice, `save_and_sort` would be given the same value for every repair of a given defect. However, it is legal to use a mixture of values. Those given value `true` will be saved, those given value `false` will be recursed on immediately in the usual way. If any of those lead to success, that chain is accepted and any saved repairs are forgotten.

Only repairs with some hope of success are saved: those for which

```
Success(s, c, x) || (NotAtDepthLimit() &&
  cost(s) - (Potential(d1) + ... + Potential(dn)) < c)
```

holds after the repair, in the terminology of Section 12.5.3.

The author's experience with `save_and_sort` has been disappointing. Chains can end successfully anywhere in the search tree, and low solution cost at an intermediate point is not a good predictor of a successful end. Every saved repair is executed once before sorting to establish the solution cost after it, then undone. If the repair is tried later, it is executed again (by a path redo). The significant benefit needed to justify this extra work does not seem to be there.

12.5.5. Adjustment on success

Suppose that, in order to encourage ejection chains to remove a cluster busy times defect, some days when the resource will be busy are chosen, all meets assigned the resource outside those days are unassigned, and repairs are tried which move those meets to the chosen days.

While the repairs are underway, it is desired to limit the domains of the resource's meets to the chosen days, to keep the repairs on track. So the repair altogether consists of unassigning

some of the resource's meets and adding a meet bound to each of the resource's meets.

Whether the repair is successful or not, after it and the chains below it are finished, the meet bound must be removed, since the domains of the meets are not supposed to be restricted permanently. If the repair is unsuccessful, the meet bound is removed by the ejector as part of undoing the repair. But if the repair is successful there is a problem, because it is not undone.

`KheEjectorRepairEnd` returns `true` to tell the user's augment function to not generate any more repairs. Although this is often because the repair just ended was successful, it is not so always. So it would be a mistake to use this `true` result as the signal to do this kind of work.

Instead, cases like this may be handled by passing non-`NULL` values for the `on_success_fn` and `on_success_val` parameters of `KheEjectorRepairEndLong`. If the ejector subsequently decides to not undo that repair, it will then call

```
on_success_fn(on_success_val)
```

and the user can ensure that this removes meet bounds or whatever is wanted.

The call to `on_success_fn` should not change the cost of the solution; in practice it is limited to enlarging domains, unfixing, and so on. It works with all kinds of repairs and options, including `save_and_sort`. It is made at a time when the associated repair has been done or redone and not undone, but by no means directly after it is done or redone, since there may be a long chain to execute after that before success can be established. It is an error to assume that the state of the solution when `on_success_fn` is called is its state at the end of the repair.

12.6. Gathering statistics

Ejectors gather statistics about their performance. This takes a negligible amount of time, as the author has verified by comparing run times with preprocessor flag `KHE_EJECTOR_WITH_STATS` in the ejector source file set to 0 (no statistics) and 1 (all statistics). On two typical instances, the increase in overall run time caused by gathering statistics was less than 0.1 seconds.

12.6.1. Options for choosing ejectors and schedules

Each ejector holds its own statistics, independently of other ejectors. Some statistics accumulate across the entire lifetime of an ejector; they are never reset. This makes it possible, for example, to measure the performance of time repair ejection chains and resource repair ejection chains over an entire set of instances, by carrying out all time repairs in all instances using one ejector and all resource repairs in all instances using another.

To facilitate marshalling multiple ejectors for these purposes, `options` objects (Section 8.4) contain an array of ejectors, which may be set and retrieved as usual:

```
KHE_EJECTOR KheOptionsEjector(KHE_OPTIONS options, int index);
void KheOptionsSetEjector(KHE_OPTIONS options, int index,
  KHE_EJECTOR ej);
```

A value is always defined for each non-negative `index`; each value has default value `NULL`. `KheEjectionChainNodeRepairTimes` uses ejector 0, `KheEjectionChainLayerRepairTimes` uses ejector 1, and `KheEjectionChainRepairResources` uses ejector 2. Some or all of these

may be the same ejector. In each case, if the ejector turns out to be `NULL`, the function makes a new ejector with the default schedules, but does not add it to `options`.

It is not safe to pass a single value of `options` to multiple threads when it contains (or will contain) ejectors, since that would expose those ejectors to the risk of being used by multiple threads in parallel, which they cannot handle.

It is up to the user to create ejectors and call `KheOptionsSetEjector` to add them to the `options` object. This raises the question of what schedules to give to these ejectors. A set of schedules is an option, so the `options` object has an option for it, whose value is a string:

```
char *KheOptionsEjectorSchedulesString(KHE_OPTIONS options);
void KheOptionsSetEjectorSchedulesString(KHE_OPTIONS options,
  char *ejector_schedules_string);
```

Its default value is `"1+,u-"`, for the meaning of which see Section 12.2.

`KheOptionsSetEjectorSchedulesString` does not set any ejector schedules, it merely sets one option of `options`, to a fresh copy of the string it is given. User code must set the actual schedules, using helper function `KheEjectorSetSchedulesFromString` (Section 12.2).

12.6.2. Statistics for analysing Kempe meet moves

The ejector itself does not maintain statistics for analysing Kempe meet moves. These are stored in `kempe_stats` objects (Section 10.2.2), one of which is conveniently available from option `time_kempe_stats` (Section 8.4.3). This object is passed to the calls to `KempeMeetMove` made by the augment functions described in this chapter. Only Kempe meet moves which are complete repairs on their own are passed this object, not Kempe meet moves combined with other operations (meet splits and merges, for example). So by the end of an ejction chain run, statistics about these Kempe meet moves will have been accumulated in the `time_kempe_stats` option of the `options` object passed to the ejection chain repairs.

12.6.3. Statistics describing a single solve

The statistics presented in this section make sense only for one call to `KheEjectorSolveEnd`. So they are available only until the next call to `KheEjectorSolveEnd`, when they are reset.

An *improvement* is an ejection chain or tree, rooted in a defect examined by the main loop, which is applied to the solution and reduces its cost. Each time an improvement is applied, four facts about it are recorded. The number of improvements applied is returned by

```
int KheEjectorImprovementCount(KHE_EJECTOR ej);
```

and the four facts about the `i`th improvement (counting from 0 as usual) are returned by

```
int KheEjectorImprovementNumberOfRepairs(KHE_EJECTOR ej, int i);
float KheEjectorImprovementTime(KHE_EJECTOR ej, int i);
KHE_COST KheEjectorImprovementCost(KHE_EJECTOR ej, int i);
int KheEjectorImprovementDefects(KHE_EJECTOR ej, int i);
```

These return the number of repairs in the `i`th improvement (this tends to increase with `i`), the time

from the moment when `KheEjectorSolveEnd` was called to the moment after the improvement was applied, the solution cost afterwards, and the number of defects of `start_gm` afterwards. Times are measured in seconds, to a precision much better than one second. There are also

```
KHE_COST KheEjectorInitCost(KHE_EJECTOR ej);
int KheEjectorInitDefects(KHE_EJECTOR ej);
```

which return the cost and number of defects when `KheEjectorSolve` began.

12.6.4. Statistics describing multiple solves

The statistics presented in this section make sense across multiple calls to `KheEjectorSolveEnd`. They are initialized when the ejector is created and never reset.

It is interesting to see how many repairs make up one improvement. Each time an improvement occurs on any solve during the lifetime of the ejector, one entry in a histogram of numbers of repairs is incremented. This histogram can be accessed at any time by calling

```
int KheEjectorImprovementRepairHistoMax(KHE_EJECTOR ej);
int KheEjectorImprovementRepairHistoFrequency(KHE_EJECTOR ej,
  int repair_count);
```

`KheEjectorImprovementRepairHistoMax` returns the maximum, over all improvements x, of the number of repairs that make up x, or 0 if there have been no improvements. `KheEjectorImprovementRepairHistoFrequency` returns the number of improvements with the given number of repairs. Also, functions

```
int KheEjectorImprovementRepairHistoTotal(KHE_EJECTOR ej);
float KheEjectorImprovementRepairHistoAverage(KHE_EJECTOR ej);
```

use this same basic information to find the total number of improvements, and the average number of repairs per improvement when there is at least one improvement.

Another histogram, again with one element for each improvement, records the number of calls to `Augment` since the most recent on in the main loop:

```
int KheEjectorImprovementAugmentHistoMax(KHE_EJECTOR ej);
int KheEjectorImprovementAugmentHistoFrequency(KHE_EJECTOR ej,
  int augment_count);
int KheEjectorImprovementAugmentHistoTotal(KHE_EJECTOR ej);
float KheEjectorImprovementAugmentHistoAverage(KHE_EJECTOR ej);
```

This is helpful, for example, in deciding whether it would be useful to terminate a search after some number of augments has failed to find an improvement. A method of doing this is build into ejectors, but not offered as an official option at the moment.

Another interesting question is how successful the various augment functions and repairs are. There are methodological issues here, however. For example, if one kind of repair is tried before another, it has more opportunities to both succeed and fail than the other. If there are several alternatives to choose from, the best test would be to compare the results of several complete runs, one for each alternative. No statistical support is needed for that. But even after

the best alternatives are chosen, there remains the question of whether each component is pulling its weight. The statistics to be described now attempt to answer this question.

An *augment type* is a small integer representing one kind of augment function. A *repair type* is a small integer representing one kind of repair. Functions `KheEjectorAddAugment` and `KheEjectorAddGroupAugment` assign an augment type to each augment function, and thus to each call on an augment function. Each repair is followed by a call to `KheEjectorRepairEnd` (Section 12.4), and its `repair_type` parameter assigns a repair type to that repair. Based on this information, the ejector records the following statistics:

1. For each distinct `augment_type`, the number of repairs made by calls on augment functions with that augment type;

2. For each distinct `(augment_type, repair_type)` pair, the number of repairs of that repair type made by calls on augment functions with that augment type;

3. For each distinct `augment_type`, the number of successful repairs made by calls on augment functions with that augment type;

4. For each distinct `(augment_type, repair_type)` pair, the number of successful repairs of that repair type made by calls on augment functions with that augment type.

Only repairs with a `true` value for the `success` parameter of `KheEjectorRepairEndLong` are counted. When the `save_and_sort` option is in use, not all saved repairs are counted, only those redone after sorting. For the purposes of statistics gathering, a repair is considered successful if it causes its enclosing `Augment` function to return `true`, whether this happens immediately, or after recursion, or after saving and sorting. The statistics may be retrieved at any time by calling

```
int KheEjectorTotalRepairs(KHE_EJECTOR ej, int augment_type);
int KheEjectorTotalRepairsByType(KHE_EJECTOR ej, int augment_type,
  int repair_type);
int KheEjectorSuccessfulRepairs(KHE_EJECTOR ej, int augment_type);
int KheEjectorSuccessfulRepairsByType(KHE_EJECTOR ej, int augment_type,
  int repair_type);
```

where `augment_type` and `repair_type` are arbitrary non-negative integers. Based on these numbers, a reasonable analysis of the effectiveness of the augment functions and their repairs can be made. For example, the effectiveness of an augment function can be measured by the ratio of the third number to the first. Adding up the result of `KheEjectorTotalRepairsByType` over all values of `repair_type` produces the result of `KheEjectorTotalRepairs`, and adding up the result of `KheEjectorSuccessfulRepairsByType` over all values of `repair_type` produces the result of `KheEjectorSuccessfulRepairs`.

12.6.5. Organizing augment and repair types

`KheEjectorAddAugment` and `KheEjectorAddGroupAugment` accept any `augment_type` values. The user should define these values using an enumerated type. The following function may be called any number of times during the ejector's setup phase, to tell it what values to expect:

```
void KheEjectorAddAugmentType(KHE_EJECTOR ej, int augment_type,
  char *augment_label);
```

This tells `ej` to expect calls to `KheEjectorAddAugment` and `KheEjectorAddGroupAugment` with the given value of `augment_type`, and associates a label with that augment type. Labels must be non-`NULL`; copies are stored, not originals. No checks are made that the values passed via `KheEjectorAddAugment` and `KheEjectorAddGroupAugment` match those declared by `KheEjectorAddAugmentType`. But if they do, then making tables of statistics is simplified by calling the following functions afterwards.

To visit all the augment types declared by calls to `KheEjectorAddAugmentType`, call

```
int KheEjectorAugmentTypeCount(KHE_EJECTOR ej);
int KheEjectorAugmentType(KHE_EJECTOR ej, int i);
```

To retrieve the label corresponding to an augment type, call

```
char *KheEjectorAugmentTypeLabel(KHE_EJECTOR ej, int augment_type);
```

In this way, suitable values for passing to `KheEjectorTotalRepairs` and the other statistics functions above can be generated, along with labels to identify the statistics.

The same functionality is offered for repair types. `KheEjectorRepairBegin` may be passed any values for `repair_type`, but the user knows which values will be passed, and the following function may be called any number of times during the ejector's setup phase to tell it this:

```
void KheEjectorAddRepairType(KHE_EJECTOR ej, int repair_type,
  char *repair_label);
```

`KheEjectorAddRepairType` declares that `ej` can expect calls to `KheEjectorRepairBegin` with the given value of `repair_type`, and associates a label with that repair type. Labels must be non-`NULL`; copies are stored, not originals. No checks are made that the values passed via `KheEjectorRepairBegin` match those declared by `KheEjectorAddRepairType`. But if they do, then making tables of statistics is simplified by calling the following functions afterwards.

To visit all the repair types declared by calls to `KheEjectorAddRepairType`, call

```
int KheEjectorRepairTypeCount(KHE_EJECTOR ej);
int KheEjectorRepairType(KHE_EJECTOR ej, int i);
```

To retrieve the label corresponding to a repair type, call

```
char *KheEjectorRepairTypeLabel(KHE_EJECTOR ej, int repair_type);
```

There is no way to declare which combinations of augment type and repair type to expect. The author handles this by ignoring cases where `KheEjectorTotalRepairsByType` returns 0.

12.7. Ejection chain time and resource repair functions

In this section, the ejector objects described above are used to build three ejection chain time and resource repair functions:

```
bool KheEjectionChainNodeRepairTimes(KHE_NODE parent_node,
  KHE_OPTIONS options);
bool KheEjectionChainLayerRepairTimes(KHE_LAYER layer,
  KHE_OPTIONS options);
bool KheEjectionChainRepairResources(KHE_TASKING tasking,
  KHE_OPTIONS options);
```

`KheEjectionChainNodeRepairTimes` repairs the assignments of the meets of the descendants of the child nodes of `parent_node`, and `KheEjectionChainLayerRepairTimes` repairs the assignments of the meets of the descendants of the child nodes of `layer`. This is useful for repairing the time assignments of a layer immediately after they are made, without wasting time on earlier layers where repairs have already been tried and are very unlikely to succeed. `KheEjectionChainRepairResources` repairs the assignments of the tasks of `tasking`.

All three functions make assignments as well as change them, so may be used to construct solutions as well as repair them. However, there are better ways to construct solutions.

All three functions require certain monitor groupings, but they set them up and remove them themselves. It is reasonable to worry about the time it takes to set up these group monitors. To investigate this question, the author ran just the group monitor setup and removal parts of function `KheEjectionChainNodeRepairTimes` 10000 times on a typical instance (BGHS98) and measured the time taken. This was 31.35 seconds, or about 0.003 seconds per setup/remove. This is not significant if it is done infrequently.

Although these functions target different parts of the solution, they share much of their implementation. In particular, they call the same augment functions, although the detailed behaviour of those functions depends on several options, some of which tailor them specifically to time or resource repair. Option `ejector_vizier_node` determines whether a vizier node (Section 9.5.4) is inserted at the top of the layer tree during time repair. Options which affect the augment functions directly are described at the beginning of Section 12.7.5 below. The following subsections describe the implementation in detail.

12.7.1. Limiting the scope of changes

Ejection chains work best when they are free to follow chains into any part of a solution, and make any repairs that help. This freedom can conflict with the caller's desire to limit the scope of the changes they make, typically because initial assignments have not yet been made to some parts of the solution, and an ejection chain repair should not anticipate them.

For example, suppose resource *r* is preassigned to some tasks, but there are others that it could be assigned to. The preassigned tasks go into *r*'s timetable when their meets are assigned times, and could then create resource defects, for example avoid unavailable times defects, that an ejection chain time repair algorithm needs to know about. Suppose a limit busy times underload defect is created (this is quite likely when the workload on any day first becomes non-zero), and its augment function tries (among other things) to find unassigned tasks that it can assign to *r* to increase its workload on that day. This is not done at present, but it is plausible. Then there will be an unexpected burst of resource assignment in the middle of time assignment.

One romantic possibility is to 'let a thousand flowers bloom' and just accept such repairs. The problem with this is that a carefully constructed initial assignment can be much better than

the result of a set of uncoordinated individual repairs.

Another possibility is to fix the assignments of all variables beyond the scope of the current phase of the solve to their current values, often null values. This is a very reliable approach, and arguably the most truthful, because it says to the ejection chain algorithm, in effect, 'for reasons beyond your comprehension, you are not permitted to change these variables.' But it suffers from a potentially severe efficiency problem: a large amount of time could be spent in discovering a large number of repairs, which all fail through trying to change fixed variables.

Yet another possibility is to have one ejector object for each kind of call (one for repairing time assignments, another for repairing resource assignments, and so on), with different augment functions. The augment functions for time repair would never assign a task, for example. This was the author's original approach, but as the code grew it became very hard to maintain.

At present the author is using the following somewhat ad-hoc ideas to limit the scope of changes. They do the job at very little cost in code and run time.

The start group monitor is one obvious aid to restricting the scope of a call. For example, time repair calls do not include event resource monitors in their start group monitors.

Many repairs move meets and tasks, but do not assign them. It seems that once a meet or task has been assigned, it is always reasonable to move it during repair. So the danger areas are augment functions that assign meets and tasks, not augment functions that merely move them.

Augment functions for assign time and assign resource defects must contain 'dangerous' assignments. But suppose that the assign time or assign resource monitor for some meet or task is not in the start group monitor. Then a repair of that monitor cannot occur first on any chain; and if the meet or task is unassigned to begin with, it cannot occur later either, since the monitor starts off with maximum cost, so its cost cannot increase, and only monitors whose cost has increased are repaired after the first repair on a chain. So assign time and assign resource augment functions can be included without risk of the resulting time and resource assignments being out of scope. This is just as well, since they are needed after ejecting meet and task moves.

If it can be shown, as was just done, that certain events will remain unassigned, then they can have no other event defects, since those require the events involved to be assigned. Similarly, unassigned event resources will never give rise to other event resource defects.

Another idea is to add options to the options object that control which repairs are tried. This is as general as different ejector objects with different augment functions are, but, if the options are few and clearly defined, it avoids the maintenance problems. If many calls on augment functions achieve nothing because options prevent them from trying things, that would be an efficiency problem, but there is no problem of that kind in practice.

The options object contains an `ejector_repair_times` option, which when `true` allows repairs that assign and move meets, and an `ejector_repair_resources` option, which when `true` allows repairs that assign and move tasks. It takes virtually no code or time to consult these options; often, just one test at the start of an augment function is enough.

When moving a meet, its chain of assignments is followed upwards, trying moves at each level. But if the aim is to repair only a small area (one runaround, say), then even if a repair starts within scope, it can leave it as it moves up the chain. This has happened, and has caused problems. So the options object contains an `ejector_limit_node` option, of type `KHE_NODE`. If it is non-`NULL`, meet assignments and moves are not permitted outside its proper descendants.

Functions `KheEjectionChainNodeRepairTimes` and `KheEjectionChainLayerRepairTimes` set option `ejector_repair_times` to `true`, `ejector_repair_resources` to `false`, and `ejector_limit_node` to the parent node, or to `NULL` if it is the cycle node. The `false` value for `ejector_repair_resources` solves the hypothetical problem, given as an example at the start of this section, of limit busy times repairs assigning resources during time assignment.

Function `KheEjectionChainRepairResources` sets options `ejector_repair_times` and `ejector_repair_resources` to `true`, and option `ejector_limit_node` to `NULL`. Setting `ejector_repair_times` to `false` here would be a reasonable alternative; it would prevent repairs from trying meet moves in their quest to improve task assignments.

12.7.2. Correlation problems involving demand defects

Section 9.8 discusses the problem of correlated monitors, and how it can be solved by grouping. Demand monitors obviously correlate with avoid clashes monitors: when there is a clash, there will be both an avoid clashes defect and an ordinary demand defect. They also correlate with avoid unavailable times, limit busy times, and limit workload monitors: when there is a hard avoid unavailable times defect, for example, there will also be a demand defect. This section explores several ways of handling these correlations, beginning with grouping.

Group the correlated monitors. Grouping is the ideal solution for correlation problems, but it does not work here. There are two reasons for this.

First, although every avoid clashes defect has a correlated ordinary demand defect, unless the resource involved is preassigned there is no way to predict which monitors will be correlated, since that depends on which resource is assigned to the demand monitors' tasks.

Second, grouping workload demand monitors with the resource monitors they are derived from has a subtle flaw. A demand defect is really the whole set of demand monitors that compete for the insufficient supply. (These sets are quite unpredictable and cannot themselves be grouped.) A workload demand defect shows up, not in the workload demand monitor itself, but in an ordinary demand monitor that competes with it. This is because the last demand tixel to change its domain and require rematching is the one that misses out on the available supply, and workload demand monitors never change their domains. So this grouping still leaves two correlated defects ungrouped: the group monitor and the unmatched ordinary demand monitor.

If grouping does not work, then one of the correlated monitors has to be detached or otherwise ignored. There are several ways to do this.

Detach demand monitors. This does not work, because no-one notices that six Science meets are scheduled simultaneously when there are only five Science laboratories, and the resulting time assignment is useless.

Attach demand monitors but exclude them from continue group monitors. This prevents correlated monitors from appearing on defect lists, but both costs continue to be added to the solution cost, so removing the resource defect alone does not produce an overall improvement. The chain terminates in failure; the ejector cannot see that repairing the resource defect could work.

Attach demand monitors but exclude them from the solution and continue group monitors; add a limit monitor holding them. Then other monitors will be repaired, but no chain which increases the total number of demand defects will be accepted. This has two problems.

First, it can waste time constructing chains which fall at the last hurdle when it is discovered

that they increase demand defects. This is particularly likely during time repair: the six Science meets problem could well occur and pass unnoticed for a long time.

Second, although it prevents demand defects from increasing, it does not repair them. This rules it out for time repair, which is largely about repairing demand defects, but it may not matter for resource repair. Resource repair cannot reduce the number of demand defects unless it moves meets: merely assigning resources reduces the domains of demand tixels, which cannot reduce demand defects. Even moving meets is unlikely to reduce demand defects during resource repair, since many of those moves will have been tried previously, during time repair.

Detach correlated resource monitors. Instead of detaching demand monitors, detach the resource monitors they correlate with. Each kind of resource monitor is considered below. The main danger here is *inexactness*: if some detached resource monitor is not modelled exactly by the demand monitors that replace it, then some defects go undetected and unrepaired.

Detach all avoid clashes monitors. For every avoid clashes defect there is an ordinary demand defect. The only inexactness is that avoid clashes monitors may have any weights, whereas demand monitors have equal weight, usually 1 (hard). But avoid clashes constraints usually have weight 1 (hard) or more, so this does not seem to be a problem in practice, given that, as the specification says, hard constraint violations should be few in good solutions.

Detach avoid unavailable times monitors that give rise to workload demand monitors. These are monitors with weight at least 1 (hard). The modelling here is exact apart from any difference in hard weight, so again there is no problem in practice.

Detach limit busy times monitors that give rise to workload demand monitors. These are monitors with weight at least 1 (hard) which satisfy the subset tree condition (Section 7.4.2). Apart from the possible difference in hard weight, this is exact except for one problem: limit busy times constraints can impose both a lower and an upper limit on resource availability in some set of times, and workload demand monitors do not model lower limits at all.

This can be fixed by (conceptually) breaking each limit busy times monitor into two, an underload monitor and an overload monitor, and detaching the overload monitor but not the underload monitor. KHE expresses this idea in a different way, chosen because it also solves the problem presented by limit workload monitors, to be discussed in their turn.

Limit busy times monitors have two attributes, `Minimum` and `Maximum`, such that less than `Minimum` busy times is an underload, and more than `Maximum` is an overload. Add a third attribute, `Ceiling`, such that `Ceiling >= Maximum`, and specify that, with higher priority than the usual rule, when the number of busy times exceeds `Ceiling` the deviation is 0.

Function `KheLimitBusyTimesMonitorSetCeiling` (Section 6.6.5) may be called to set the ceiling. Setting it to `INT_MAX` (the default value) produces the usual rule. Setting it to `Maximum` is equivalent to detaching overload monitoring.

Detach limit workload monitors that give rise to workload demand monitors. Limit workload monitors are similar to limit busy times monitors whose set of times is the entire cycle. However, the demand monitors derived from a limit workload monitor do not necessarily model even the upper limit exactly (Section 7.4.1). This problem can be solved as follows.

Consider a resource with a hard limit workload monitor and some hard workload demand monitors derived from it, and suppose that all of these monitors are attached. As the resource's workload increases, it crosses from a 'white region' of zero cost into a 'grey region' where the

limit workload monitor has non-zero cost but the workload demand monitors do not, and then into a 'black region' where both the limit workload monitor and the workload demand monitors have non-zero cost. This black region is the problem.

The problem is solved by adding a `Ceiling` attribute to limit workload monitors, as for limit busy times monitors. Function `KheLimitWorkloadMonitorSetCeiling` (Section 6.6.6) sets the ceiling. As before, the default value is `INT_MAX`. The appropriate alternative value is not `Maximum`, but rather a value which marks the start of the black region, so that the limit workload monitor's cost drops to zero as the workload crosses from the grey region to the black region. In this way, all workload overloads are reported, but by only one kind of monitor at any one time.

There is one anomaly in this arrangement: a repair that reduces workload from the black region to the grey region does not always decrease cost. This is a pity but it is very much a second-order problem, given that the costs involved are all hard costs, so that in practice repairs are wanted that reduce them to zero. What actually happens is that one repair puts a resource above the white zone, and this stimulates a choice of next repair which returns it to the white zone. Repairs which move between the grey and black zones are possible but are not likely to lie on successful chains anyway, so it does not matter much if their handling is imperfect.

The appropriate value for `Ceiling` is the number of times in the cycle minus the total number of workload demand monitors for the resource in question, regardless of their origin. When the resource's workload exceeds this value, there will be at least one demand defect, and it is time for the limit workload monitor to bow out.

To summarize all this: there is some choice during resource repair, but detaching resource monitors (with ceilings) always works, and it is the only method that works during time repair.

12.7.3. Primary grouping and detaching

To install and remove the primary groupings used by ejection chains, call

```
void KheEjectionChainPrepareMonitors(KHE_SOLN soln);
void KheEjectionChainUnPrepareMonitors(KHE_SOLN soln);
```

This includes detaching some resource monitors, as in the plan evolved in Section 12.7.2. This section explains exactly what `KheEjectionChainPrepareMonitors` does.

`KheEjectionChainPrepareMonitors` partitions the events of `soln`'s instance into classes, placing events into the same class when following the fixed assignment paths out of their meets proves that their meets must run at the same times. It then groups event monitors as follows.

Split events and distribute split events monitors. For each class, it groups together the split events and distribute split events monitors that monitor the events of that class. It gives sub-tag `KHE_SUBTAG_SPLIT_EVENTS` to any group monitors it creates. There is also a `KHE_SUBTAG_DISTRIBUTE_SPLIT_EVENTS` subtag, but it is not used.

Assign time monitors. For each class, it groups the assign time monitors that monitor the events of that class, giving sub-tag `KHE_SUBTAG_ASSIGN_TIME` to any group monitors.

Prefer times monitors. Within each class, it groups those prefer times monitors that monitor events of that class whose constraints request the same set of times, giving sub-tag `KHE_SUBTAG_PREFER_TIMES` to any group monitors.

Spread events monitors. For each spread events monitor, it finds the set of classes that hold the events it monitors. It groups attached spread events monitors whose sets of classes are equal, giving sub-tag `KHE_SUBTAG_SPREAD_EVENTS` to any group monitors. Strictly speaking, only monitors whose constraints request the same time groups with the same limits should be grouped, but that check is not currently being made.

Link events monitors. Like split events monitors, these are usually handled structurally, so it does nothing with them. They usually have provably zero fixed cost, so are already detached.

Order events monitors. For each order events monitor, it finds the sequence of classes that hold the two events it monitors. It groups attached order events monitors whose sequences of classes are equal, giving sub-tag `KHE_SUBTAG_ORDER_EVENTS` to any group monitors. Strictly speaking, only monitors whose constraints request the same event separations should be grouped, but that check is not currently being made.

Next, `KheEjectionChainPrepareMonitors` partitions the event resources of `soln`'s instance into classes, placing event resources into the same class when following the fixed assignment paths out of their tasks proves that they must be assigned the same resources. It then groups event resource monitors as follows.

Assign resource monitors. For each class, it groups the assign resource monitors of that class's event resources, giving sub-tag `KHE_SUBTAG_ASSIGN_RESOURCE` to any group monitors.

Prefer resources monitors. Within each class, it groups those prefer resources monitors that monitor the event resources of that class whose constraints request the same set of resources, giving sub-tag `KHE_SUBTAG_PREFER_RESOURCES` to any group monitors.

Avoid split assignments monitors. There seems to be no useful primary grouping of these monitors, so nothing is done with them. They may be handled structurally, in which case they will have provably zero fixed cost and will be already detached.

Students who follow the same curriculum have the same timetable. So for each resource type `rt` such that a call to `KheResourceTypeDemandIsAllPreassigned(rt)` (Section 3.5.1) shows that its resources are all preassigned, `KheEjectionChainPrepareMonitors` groups the resource monitors of `rt`'s resources as follows.

Avoid clashes monitors. It groups those avoid clashes monitors derived from the same constraint whose resources attend the same events, giving sub-tag `KHE_SUBTAG_AVOID_CLASHES` to any group monitors.

Avoid unavailable times monitors. It groups avoid unavailable times monitors derived from the same constraint whose resources attend the same events, giving any group monitors sub-tag `KHE_SUBTAG_AVOID_UNAVAILABLE_TIMES`.

Limit idle times monitors. It groups limit idle times monitors derived from the same constraint whose resources attend the same events, giving sub-tag `KHE_SUBTAG_LIMIT_IDLE_TIMES` to any group monitors.

Cluster busy times monitors. It groups cluster busy times monitors derived from the same constraint whose resources attend the same events, giving any group monitors sub-tag `KHE_SUBTAG_CLUSTER_BUSY_TIMES`.

Limit busy times monitors. It groups limit busy times monitors derived from the same constraint whose resources attend the same events, giving sub-tag `KHE_SUBTAG_LIMIT_BUSY_TIMES` to any group monitors.

Limit workload monitors. It groups limit workload monitors derived from the same constraint whose resources attend the same events, giving sub-tag `KHE_SUBTAG_LIMIT_WORKLOAD` to any group monitors.

Fixed assignments between meets are taken into account when deciding whether two resources attend the same events. As far as resource monitors are concerned, it is when the resource is busy that matters, not which meets it attends.

`KheEjectionChainPrepareMonitors` also treats some resource monitors according to the plan from Section 12.7.2, whether they are grouped or not:

- It detaches all attached avoid clashes monitors.
- For each attached avoid unavailable times monitor `m` for which a workload demand monitor with originating monitor `m` is present (all hard ones, usually), it detaches `m`.
- For each attached limit busy times monitor `m` for which a workload demand monitor with originating monitor `m` is present (all hard ones satisfying the subset tree condition of Section 7.4.2, usually), if `m`'s lower limit is 0 it detaches `m`, otherwise it sets `m`'s `ceiling` attribute to its `maximum` attribute, by calling `KheLimitBusyTimesMonitorSetCeiling`.
- For each attached limit workload monitor `m` for which a workload demand monitor with originating monitor `m` is present (all hard ones, usually), it sets `m`'s `ceiling` attribute to the cycle length minus the number of workload demand monitors for `m`'s resource (regardless of origin), or 0 if this is negative, by calling `KheLimitWorkloadMonitorSetCeiling`.

Section 12.7.2 has the reasoning.

Finally, `KheEjectionChainPrepareMonitors` groups demand monitors as follows. If a limit monitor containing these monitors is wanted, a separate call is needed (Section 12.7.4).

Ordinary demand monitors. For each set of meets such that the fixed assignment paths out of those meets end at the same meet, it groups the demand monitors of those meets' tasks, giving sub-tag `KHE_SUBTAG_MEET_DEMAND` to any group monitors. The reasoning is that the only practical way to repair an ordinary demand defect is to change the assignment of its meet (or some other clashing meet), which will affect all the demand monitors grouped with it here.

Workload demand monitors. These remain ungrouped. As explained in Section 12.7.2, workload demand defects appear only indirectly, as competitors of ordinary demand defects.

12.7.4. Secondary groupings

Section 9.8 introduces the concept of secondary groupings. The three ejection chain functions need secondary groupings built on primary groupings for their start group monitors (but not their continue group monitors, since they use the solution object for that), and other secondary groupings for their limit monitors. These are the subject of this section.

`KheEjectionChainNodeRepairTimes` uses the group monitor returned by

```
KHE_GROUP_MONITOR KheNodeTimeRepairStartGroupMonitorMake(KHE_NODE node);
```

as its start group monitor. The result has sub-tag `KHE_SUBTAG_NODE_TIME_REPAIR`. Its children are monitors, or primary groupings of monitors where these are already present, of two kinds.

First are all assign time, prefer times, spread events, order events, and ordinary demand monitors that monitor the meets of `node` and its descendants, plus any meets whose assignments are fixed, directly or indirectly, to them. Second are all resource monitors. Only preassigned resources are assigned during time repair, but those assignments may cause resource defects which can only be repaired by changing time assignments, just because the resources involved are preassigned.

`KheEjectionChainLayerRepairTimes` chooses one of the group monitors returned by

```
KHE_GROUP_MONITOR KheLayerTimeRepairStartGroupMonitorMake(
  KHE_LAYER layer);
KHE_GROUP_MONITOR KheLayerTimeRepairLongStartGroupMonitorMake(
  KHE_LAYER layer);
```

as its start group monitor, depending on option `time_layer_repair_long`. The result has sub-tag `KHE_SUBTAG_LAYER_TIME_REPAIR`, with the same children as before, only limited to those that monitor the meets and resources of `layer`, or (if `time_layer_repair_long` is `true`) of layers whose index number is less than or equal to `layer`'s.

`KheEjectionChainRepairResources` uses the group monitor returned by

```
KHE_GROUP_MONITOR KheTaskingStartGroupMonitorMake(KHE_TASKING tasking);
```

for its start group monitor. The result has sub-tag `KHE_SUBTAG_TASKING`, and its children are the following monitors (or primary groupings of those monitors, where those already exist): the assign resource, prefer resources, and avoid split assignments monitors, and the six resource monitors that monitor the tasks and resources of `tasking`. If the tasking is for a particular resource type, only monitors of entities of that type are included.

To allow an ejection chain to unassign meets temporarily but prevent it from leaving meets unassigned in the end, a limit monitor is imposed which rejects chains that allow the total cost of assign time defects to increase. This monitor is created by calling

```
KHE_GROUP_MONITOR KheGroupEventMonitors(KHE_SOLN soln,
  KHE_MONITOR_TAG tag, KHE_SUBTAG_STANDARD_TYPE sub_tag);
```

passing `KHE_ASSIGN_TIME_MONITOR_TAG` and `KHE_SUBTAG_ASSIGN_TIME` as tag parameters.

To prevent the number of unmatched demand tixels from increasing, when that is requested by the `resource_invariant` option, the group monitor returned by function

```
KHE_GROUP_MONITOR KheAllDemandGroupMonitorMake(KHE_SOLN soln);
```

is used as a limit monitor. Its sub-tag is `KHE_SUBTAG_ALL_DEMAND`, and its children are all ordinary and workload demand monitors. Primary groupings are irrelevant to limit monitors, so these last two functions take no account of them.

12.7.5. Augment functions

The augment functions passed to the ejector are private to KHE. They are open to inspection in the source code as usual. This section explains what they do in detail.

The augment functions consult several options. Three of them, `ejector_repair_times`,

ejector_limit_node, and ejector_repair_resources, are particularly important because they limit the scope of repairs. They cannot be set by the user—or rather, they can, but that would be futile because they are reset within the main functions. Any repair which assigns or moves a meet first consults ejector_repair_times, and only proceeds if it is true. It tries moving each ancestor of the meet, since moving an ancestor will also move the original meet; but if ejector_limit_node is non-NULL, it omits moves of meets lying within nodes which are not proper descendants of ejector_limit_node. Any repair which assigns or moves a task first consults ejector_repair_resources, and only proceeds if it is true.

Here is the full list of repair operations executed by KHE's augment functions.

Node swaps, which use KheNodeMeetSwap (Section 10.2.1) to swap the assignments of the meets of two nodes. If the ejector_nodes_before_meets option is true, then if node swaps are tried at all, they are tried before (rather than after) meet moves.

Basic and ejecting meet assignments and moves and Kempe meet moves, which move meets (Section 10.2.2). Where it is stated that a Kempe meet move is tried, it is in fact tried only when the ejector_use_kempe_moves option (Section 8.4.5) is KHE_OPTIONS_KEMPE_YES, or is KHE_OPTIONS_KEMPE_AUTO and the meet to be moved lies in at least one layer whose duration is at least 80% of the duration of the cycle. Where it is stated that an ejecting meet assignment or move is tried, it is in fact tried only when the ejector_ejecting_not_basic option is true, otherwise a basic meet assignment or move is tried instead.

Fuzzy meet moves, which move meets in a more elaborate way (Section 10.7.4). These are not mentioned below, but they are tried after Kempe and ejecting meet moves, although only when the ejector_use_fuzzy_moves option is true and the current depth is 1.

Split moves, which split a meet into two and Kempe-move one of the fragments, and *merge moves*, which Kempe-move one meet to adjacent to another and merge the two fragments. As well as being used to repair split defects, split moves are used similarly to fuzzy meet moves: although not mentioned below, they are tried after Kempe and ejecting meet moves, but only when the ejector_use_split_moves option is true and the current depth is 1. These Kempe meet moves are not influenced by the ejector_use_kempe_moves option.

Ejecting task assignments and moves, which assign or move a task to a given resource and then unassign any clashing tasks (Section 11.7).

Ejecting task-set moves, which use ejecting task moves to move a set of tasks to a common resource, succeeding only if all of the moves that change anything succeed.

Meet-and-task moves, which Kempe-move a meet at the same time as moving one of its tasks, succeeding only if both moves succeed.

Each repair is enclosed in calls to KheEjectorRepairBegin and KheEjectorRepairEnd as usual. In the more complex cases, such as the last two on the list above, the success argument of KheEjectorRepairEnd is set to true only if all parts of the repair succeed. Some of the more complex repairs are tried only when the current depth is 1, that is, when the defect being repaired is truly present, not introduced by some other repair.

Some alternative repairs are naturally tried one after another. The ejecting task moves of a given task to each resource in its domain is one example. Here are three less obvious, but nevertheless very useful sequences of alternative repairs.

A *Kempe/ejecting meet move* is a sequence of one or two alternative repairs, first a Kempe

meet move, then an ejecting meet move with the same parameters. The ejecting meet move is omitted when the Kempe meet move reports that it did only what a basic meet move would have done, since in that case the ejecting move is identical to the Kempe move. This sequence is similar to making an ejecting move and then, on the next repair, favouring a particular reassignment of the ejected meet(s) which is likely to work well. Fuzzy and split moves may follow the Kempe and ejecting meet moves, as explained above.

A *resource underload repair* for resource r and time group g is a sequence of alternative repairs which aim to increase the number of times r is busy within g. Unless g is the whole cycle, for each task assigned r whose *overlap* (the number of times it is running within g) is less that its duration, it tries all ejecting meet moves of the task's meet which increase its overlap. After that it tries an ejection tree repair like the one described below for cluster busy times defects, which aims to empty out the entire time group—a quite different way to remove the defect.

A *resource overload repair* for resource r and time group g is a sequence of alternative repairs which aims to decrease the number of times r is busy within g. First, for each task assigned but not preassigned r whose overlap is non-zero, it tries all ejecting task moves of the task to its domain's resources. Then, for each task assigned (including preassigned) r whose overlap is non-zero, it tries all ejecting meet moves of the task's meet which decrease the overlap.

Wherever possible, sequences of alternative repairs change the starting point of the traversal of the alternatives on each call. For example, when trying alternative resources, the code is

```
for( i = 0;  i < KheResourceGroupResourceCount(rg);  i++ )
{
  index = (KheEjectorCurrAugmentCount(ej) + i) %
    KheResourceGroupResourceCount(rg);
  r = KheResourceGroupResource(rg, index);
  ... try a repair using r ...
}
```

The first resource tried depends on the number of augments so far, an essentially random number. This simple idea significantly decreases final cost and run time.

Following is a description of what each augment function does when given a non-group monitor with non-zero cost to repair. When given a group monitor with non-zero cost, since the elements of a group all monitor the same thing in reality, the augment function takes any individual defect from the group and repairs that defect.

Split events and distribute split events defects. Most events are split into meets of suitable durations during layer tree construction, but sometimes the layer tree does not remove all these defects, or a split move introduces one. In those cases, the split analyser (Section 9.7.1) from the `options` object is used to analyse the defects and suggest splits and merges which correct them. For each split suggestion, for each meet conforming to the suggestion, a repair is tried which splits the meet as suggested. For each merge suggestion, for each pair of meets conforming to the suggestion, four combined repairs are tried, each consisting of, first, a Kempe meet move which bring the two meets together, and second, the merge of the two meets. The four Kempe moves move the first meet to immediately before and after the second, and the second to immediately before and after the first.

Assign time defects. For each monitored unassigned meet, all ejecting meet moves to a

parent meet and offset that would assign the meet to a time within its domain are tried.

Prefer times defects. For each monitored meet assigned an unpreferred time, all Kempe/ejecting meet moves to a parent meet and offset giving a preferred time are tried.

Spread events defects. For each monitored meet in an over-populated time group, all Kempe/ejecting moves of the meet to a time group that it would not over-populate are tried; and for each under-populated time group, for each meet whose removal would not under-populate its time group, all Kempe/ejecting moves of it to the under-populated time group are tried.

Link events defects. These are not repaired; they are expected to be handled structurally.

Order events defects. These are currently ignored. It will not be difficult to find suitable meet moves in the future.

Assign resource defects. For each monitored unassigned task, all ejecting assignments of the task to resources in its domain are tried. Then if the `ejector_repair_times` option permits, all combinations of a Kempe meet move of the enclosing meet and an ejecting assignment of the task to resources in its domain are tried.

Prefer resources defects. For each monitored task assigned an unpreferred resource, all ejecting moves of the task to preferred resources are tried.

Avoid split assignments defects. For each resource participating in a split assignment there is one repair: all involved tasks assigned that resource are unassigned, all involved tasks' domains are restricted to the other participating resources, and a set of ejection chains is tried, each of which attempts to reassign one of the unassigned tasks. The repair succeeds only if all these chains succeed, making an ejection tree, not a chain. This is expensive and unlikely to work, so it is only tried when the defect is a main loop defect or only one task needs to be unassigned.

Avoid clashes defects. Avoid clashes monitors are detached during ejection chain repair, since their work is done by demand monitors. So there are no avoid clashes defects.

Avoid unavailable times defects. A resource overload repair (see above) for the monitored resource and the unavailable times is tried.

Limit idle times defects. For each task assigned the monitored resource at the start or end of a 'day' (actually, a time group being monitored), each Kempe/ejecting meet move of that task's meet such that the initial meet move reduces the number of idle times for that resource is tried. Calculating this condition is not trivial, but the augment function does it exactly. Task moves could help to repair limit idle times defects for unpreassigned resources, but in current data sets the resources involved are usually preassigned, so task moves are not currently being tried.

After the repairs just given are tried, if the repair has depth 1 (if the defect was not created by a previous repair on the current chain), a complex repair is tried which eliminates all idle times for one resource on one day. Take the meets assigned that resource on that day. A retimetabling of those meets on that day with no clashes and no idle times is defined by a starting time for the first meet and a permutation of the meets (their chronological order in the assignment). If there are k meets and s starting times that don't put the last meet off the end of the day, then there are $s \cdot k!$ retimetablings in total. In practice this is a moderate number. For safety, only a limited number of retimetablings is tried, by switching to a single permutation at each new node after a fixed limit (currently 1000) is reached.

Cluster busy times defects. If the resource is busy in too few monitored time groups, all ejecting meet moves are tried which move a meet which is not the only meet in a monitored time

group (either because every monitored time group it overlaps with overlaps with at least one other meet, or because it does not overlap with any monitored time groups) to a monitored time group in which it is. If the resource is busy in too many monitored time groups, then for each monitored time group `tg` containing at least one meet, if the depth is 1 or `tg` contains exactly one meet, all the meets in `tg` are unassigned, and `tg` and all monitored time groups containing no meets are removed from the domains of all meets assigned the resource. This is an ejection tree repair if more than one meet is unassigned: all of the unassigned meets must be reassigned for success. The `on_success_fn` parameter of `KheEjectorRepairEndLong` is used to ensure that the domains are restored on success as well as on failure.

Limit busy times defects. For each set of times when the resource is underloaded (resp. overloaded), a resource underload (resp. overload) repair of the resource at those times is tried.

Limit workload defects. If the resource is overloaded, a resource overload repair is tried, taking the set of times to be the entire cycle. There is currently no repair for underloads.

Ordinary and workload demand defects. If the defect has a workload demand monitor competitor (possibly itself, although workload demand monitors rarely fail to match), a resource overload repair is tried for the workload demand monitor's resource and the domain of its originating monitor. If the defect has no workload demand defect competitor, all ejecting meet moves of competitor meets to anywhere different are tried; but meets within vizier nodes are not moved in this case, since that would only shift the problem to a different time.

Appendix A. Modules Packaged with KHE

This Appendix documents several modules packaged with KHE and used by it behind the scenes. By including their header files the user may also use these modules.

A.1. The M module

M is a C module consisting of header file `m.h` and implementation file `m.c`. These are stored and compiled with KHE, but they can be used separately. M offers macros and functions for memory allocation, assertions, and variable-length typed generic arrays and symbol tables. The latter come in two forms, one with keys of type `char *`, the other with keys of type `wchar_t *`.

M itself contains no calls to synchronization operations, but it has been written with multi-threading in mind, and it is thread-safe in the following sense. It is safe for query operations (retrievals and traversals on an unchanging array or symbol table) to occur at the same time on the same object within different threads. It is not safe for update operations (insertions and deletions) to occur at the same time as either query or update operations within different threads. This is the most that can be hoped for without explicit synchronization.

KHE uses M extensively behind the scenes. It is useful, too, when writing helper functions and solvers, which is why it is documented here. To use M, simply include `m.h`. Including `khe.h` does not automatically include `m.h` as well.

A.1.1. Memory allocation

M offers macro

```
MMake(x);
```

which assumes that `x` is a variable of pointer type, and initializes it to point to some new memory of the appropriate size. For example,

```
struct { int x; int y } pt;
MMake(pt);
```

sets `pt` to point to two new words of memory. Conversely, for any pointer `x` set by calling `MMake(x)` one may call

```
MFree(x);
```

to free the memory pointed to by `x`. The two macros are simple wrappers for the `malloc()` and `free()` system calls; `MMake` is more convenient than `malloc()`, since it works out the size for you, but `MFree` is identical with `free()` and is included only for completeness.

A.1.2. Assertions

M also offers a useful assert function:

```
void MAssert(bool cond, char *fmt, ...);
```

If `cond` is `true`, this does nothing; but if it is `false`, it uses `fprintf` to print a message made from `fmt` and the following parameters onto `stderr`, then aborts.

A.1.3. Variable-length arrays

M also offers variable-length arrays. These are not just arrays of void pointers. Instead, like C's native arrays, they are strongly typed: the C compiler will report an error if there is a type mismatch. Each array may have elements of any one type, and that type may have any width.

The type of a variable-length array must be declared using a `typedef`. For example, the following declarations already appear within `m.h`:

```
typedef MARRAY(bool)          ARRAY_BOOL;
typedef MARRAY(char)          ARRAY_CHAR;
typedef MARRAY(wchar_t)       ARRAY_WCHAR;
typedef MARRAY(short)         ARRAY_SHORT;
typedef MARRAY(int)           ARRAY_INT;
typedef MARRAY(int64_t)       ARRAY_INT64;
typedef MARRAY(void *)        ARRAY_VOIDP;
typedef MARRAY(char *)        ARRAY_STRING;
typedef MARRAY(wchar_t *)     ARRAY_WSTRING;
```

To gain access to `wchar_t` and `int64_t`, `m.h` includes standard header files `<wchar.h>` and `<stdint.h>`. Use of `long` just leads to trouble, in the author's experience, since its width varies between 32-bit and 64-bit platforms, so `int64_t`, a standard 64-bit signed integral type, appears here instead. Create your own array type by placing any type at all between the parentheses.

A variable of any of these types is a record (not a pointer to a record) holding three fields: the current length, the current capacity, and a typed pointer to memory holding the elements. If one array is assigned to another, the two arrays will have independent length and capacity fields, yet share their content. This is only safe when the original is not used afterwards, or the array remains constant. It was done this way because, in the author's experience, an extensible array is best kept private to one class or one function body; and in that case, there is no problem in having its record lie directly in the class object or on the call stack, rather than in separately allocated memory at the end of a pointer, and it is more efficient that way.

Although the record fields can be accessed directly by the user, they should not be. Instead, only the following operations (which are generic macros for the most part) should be called. The notation `ARRAY_X` stands for a variable-length array whose elements have type `X`.

```
void MArrayInit(ARRAY_X a);
```

Initialize `a` to the empty array. This must be called before any other operation on `a`. Macro

```
void MArrayFree(ARRAY_X a)
```

frees the memory used to hold the elements of `a`. It does not free `a` itself; `a` is not a pointer.

```
int MArraySize(ARRAY_X a);
```

Return the number of elements currently stored in `a`. An integer `i` is a legal index of `a` if `0 <= i < MArraySize(a)`, as usual in C. Array bounds are not checked by any M operation.

```
void MArrayClear(ARRAY_X a);
```

Clear the array, that is, set its current number of elements to 0.

```
void MArrayAddLast(ARRAY_X a, X x);
void MArrayInsert(ARRAY_X a, int i, X x);
void MArrayAddFirst(ARRAY_X a, X x);
```

Add `x` to the end of `a`, insert it so that its index afterwards is `i`, or insert it at the front, shifting all higher elements up one place to make room. The array will be resized if necessary.

```
X MArrayGet(ARRAY_X a, int i);
X MArrayFirst(a);
X MArrayLast(a);
```

Return the `i`'th element of `a`, or the first, or the last.

```
void MArrayPut(ARRAY_X a, int i, X x);
X MArrayPreInc(ARRAY_X a, int i);
X MArrayPostInc(ARRAY_X a, int i);
X MArrayPreDec(ARRAY_X a, int i);
X MArrayPostDec(ARRAY_X a, int i);
```

Replace the existing `i`'th element of `a` with `x`, or carry out the usual pre- and post-increment and decrement operations on the `i`'th element of `a`, returning the usual results. The last four apply only to arrays of integral types.

```
void MArrayFill(ARRAY_X a, int len, X x);
```

If the length of `a` is less than `len`, increase it to `len` by repeatedly adding the value `x` to the end; otherwise do nothing. There is no way, using this or any other M operation, to cause any legal index of `a` to contain an undefined value. `MArrayFill` is a macro that evaluates `x` repeatedly.

```
X MArrayRemove(ARRAY_X a, int i);
X MArrayRemoveFirst(ARRAY_X a);
X MArrayRemoveLast(ARRAY_X a);
```

Remove the element at index `i`, or the first element, or the last element, shifting any higher elements down one place to close up the gap, and returning the removed element as result.

```
void MArrayDropLast(ARRAY_X a);
void MArrayDropFromEnd(ARRAY_X a, int n);
```

Remove the last, or the last `n`, elements of `a`.

```
X MArrayRemoveAndPlug(ARRAY_X a, int i);
X MArrayRemoveFirstAndPlug(ARRAY_X a);
```

Remove the element at index `i`, or the first element, but plug the gap quickly by moving the last element into it instead of shifting; or if the removed element is the last element, just remove it. Return the original last element, possibly but not usually the deleted element.

```
void MArrayAppend(ARRAY_X dest, ARRAY_X source, int i);
```

Append the elements of `source` to the end of `dest`, leaving `source` unchanged. Parameter `i` is a variable used as an external cursor when scanning `source`.

```
void MArraySwap(ARRAY_X a, int i, int j, X tmp);
```

Swap the elements of `a` at positions `i` and `j`. Parameter `tmp` is a variable used to hold an element temporarily while swapping.

```
void MArrayWholeSwap(ARRAY_X a, ARRAY_X b, ARRAY_X tmp);
```

Swap two whole arrays, that is, swap their structs.

```
void MArraySort(ARRAY_X a, int(*compar)(const void *, const void *));
```

Sort `a` by means of a call to `qsort`, using `compar` as the comparison function.

```
void MArraySortUnique(ARRAY_X a, int(*compar)(const void *, const void *));
```

Similar to `MArraySort`, except that after sorting, elements are removed until no two adjacent elements return 0 when compared using `compar`. If this is done purely for uniqueifying, it is common to implement `compar` as a mere subtraction of two pointers. However, on a 64-bit architecture this yields a 64-bit integer, and merely returning this cast to `int`, the return type of `compar`, does not work. Use a conditional expression returning `-1`, `0`, or `1` instead.

```
bool MArrayContains(ARRAY_X a, X x, int *pos);
```

If `a` contains `x`, return `true` and set `*pos` to the first occurrence of `x` within `a`; otherwise return `false`, leaving `*pos` unchanged. The implementation uses `memcmp` for the individual comparisons, ensuring that elements of any width are handled correctly.

```
MArrayForEach(ARRAY_X a, X *x, int *i)
MArrayForEachReverse(ARRAY_X a, X *x, int *i)
```

These macros are iterators which iterate over the elements of `a`, in forward or reverse order. During each iteration, `*x` is one element of `a` and `*i` is the index of `*x` in `a`. For example,

```
MArrayForEach(strings, &str, &i)
  fprintf(stdout, "string %d: %s\n", i, str);
```

prints the elements of array `strings`. The use of `&` could be avoided, since `MArrayForEach` is a macro; it has been included as a reminder that `MArrayForEach` assigns values to `str` and `i` (it would have been necessary if `MArrayForEach` had been a function). Both macros expand to

```
for( ... ; ... ; ... )
```

and may be used syntactically in any way that this construct may be.

A.1.4. String factories

One handy use for variable-length arrays is for building up strings piece by piece, similarly to `open_memstream` from POSIX-2008. The growing string is held in a variable-length array called a *string factory*, added to as appropriate, and retrieved from the factory at the end.

M's string factories come in two forms, one with elements of type `char` called `ARRAY_CHAR`, the other with elements of type `wchar_t` called `ARRAY_WCHAR`. Both these types appear in the list of pre-declared array types given above. We'll start with `ARRAY_CHAR`.

```
void MStringInit(ARRAY_CHAR ac);
```

Initialize string factory `ac`. This is just a synonym for `MArrayInit`. While the string is being constructed, the array just contains its elements, with no terminating `'\0'`.

```
void MStringAddChar(ARRAY_CHAR ac, char ch);
void MStringAddInt(ARRAY_CHAR ac, int i);
void MStringAddString(ARRAY_CHAR ac, char *s);
```

Add `ch`, `i` (formatted into a character sequence), or `s` to the end of the growing string.

```
void MStringPrintf(ARRAY_CHAR ac, size_t maxlen,
  const char *format, ...);
```

Add the result of `snprintf(-, maxlen, format, ...)` to the end of the growing string.

```
char *MStringVal(ARRAY_CHAR ac);
```

Add a `'\0'` to the end of `ac`, then return the string held in the factory. After you are finished with the result of `MStringVal`, you can reclaim memory by calling `MArrayFree(ac)` as usual, or (equivalently, as it turns out) by calling `free` on the result of `MStringVal`.

There is also a separate function for copying a string into heap memory:

```
char *MStringCopy(char *s);
```

This is equivalent to making a string factory, adding `s` to it, and returning the value; or if `s` is `NULL` it simply returns `NULL`. A returned non-`NULL` value may be freed by a call to `free`.

Here are the `wchar_t` versions of these functions:

```
void MWStringInit(ARRAY_WCHAR awc);
void MWStringAddChar(ARRAY_WCHAR awc, wchar_t ch);
void MWStringAddInt(ARRAY_WCHAR awc, int i);
void MWStringAddString(ARRAY_WCHAR awc, wchar_t *s);
void MWStringPrintf(ARRAY_WCHAR awc, size_t maxlen,
  const wchar_t *format, ...);
wchar_t *MWStringVal(ARRAY_WCHAR awc);
wchar_t *MWStringCopy(wchar_t *s);
```

They work in exactly the same way, with just the obvious changes to names and types.

A.1.5. Symbol tables

A symbol table is a set of *entries*, each of which consists of two parts, a *key* which is a string, and a *value*. In any one table the values must all have the same type, declared by the user (the C compiler checks this). The declared type is arbitrary, and may have any width. Each table may contain any number of entries. The basic operations are to insert an entry and to retrieve the value of an entry by giving its key.

As for arrays, and for the same reasons, M's symbol tables are records, not pointers to records. The implementation uses a linear probing hash table which doubles in size when it reaches 80% capacity. This kind of hash table is essentially just an array (actually two arrays, one for keys, one for values). At any moment, some of its elements contain entries, others do not.

The hardest part of implementing M was to find a way to resize the generic array of values without falling foul of the C language's strict aliasing rule, which states that pointers of different types should never point to overlapping memory. My solution invokes the part of the rule that says that pointers of type `char *` are exempt from the rule, which is not a confidence-inspiring state of affairs. If you find corrupted values in your symbol tables, try turning optimization off. If that fixes the problem, then strict aliasing was the problem (please let me know).

M's symbol tables come in two forms, one with keys of type `char *` called `MTABLE`, the other with keys of type `wchar_t *` called `MWTABLE`. (File `m.h` also defines types `MTABLE_U` and `MWTABLE_U`, but they are for use behind the scenes. Do not use these names.) `MTABLE` is documented here. For wide character string tables, replace `MTABLE` with `MWTABLE`, `MTable` with `MWTable`, and `char *` with `wchar_t *` in what follows.

To define a symbol table type whose values have type `PERSON`, say, write this:

```
typedef MTABLE(PERSON) TABLE_PERSON;
```

In the following definitions (most of which are implemented by macros), type `TABLE_X` stands for any type defined by a typedef like the one just given, and `X` stands for the type (`PERSON` or whatever) between the parentheses in that typedef.

```
void MTableInit(MTABLE_X table);
```

Initialize `table` to a new table, currently empty.

```
void MTableFree(MTABLE_X table);
```

Free `table` (that is, free its arrays of keys and values).

```
void MTableInsert(MTABLE_X table, char *key, X value);
```

Insert a new entry with the given key and value into `table`. It is not an error if there is already an entry with the same key in `table`; in that case, the table simply stores both.

```
bool MTableInsertUnique(MTABLE_X table, char *key, X value, X *other);
```

If there is no entry with the given key in `table`, insert an entry with the given key and value and

return `true`. Otherwise, change nothing, set `*other` to the value of an existing entry with this key, and return `false`.

```
void MTableClear(MTABLE_T table);
```

Delete every entry from `table`.

```
int MTableHash(wchar_t *key)
```

Return the hash code (before reduction modulo the table size) used when searching for `key`.

Retrieval comes in two forms. The first is the 'contains' form, which merely reports whether an entry with the given key is present:

```
bool MTableContains(MTABLE_X table, char *key, int *pos);
bool MTableContainsHashed(MTABLE_X table, int hash_code, char *key,
  int *pos);
bool MTableContainsNext(MTABLE_X table, int *pos);
```

`MTableContains` returns `true` if `table` contains an entry with the given key, setting `*pos` to its position in the table, or `false` if there is no such entry, in which case `*pos` is an empty position in the table. `MTableContainsHashed` is the same, except that it assumes that `hash_code` is the hash code of `key` as returned by `MTableHash`; passing it saves time when searching for the same key in several tables. `MTableContainsNext` assumes that `*pos` is a non-empty position of `table`; it searches the table beyond that point (wrapping around to the front if necessary) for an entry with the same key as the one at that point. Like `MTableContains`, it returns `true` or `false` depending on whether it found such an entry, and it changes `*pos` to its new position, or an empty position.

The second form of retrieval is the 'retrieve' form. It returns the value associated with the given key, as well as saying whether the key is present:

```
bool MTableRetrieve(TABLE_X table, char *key, X *value, int *pos);
bool MTableRetrieveHashed(TABLE_X table, int hash_code, char *key,
  X *value, int *pos);
bool MTableRetrieveNext(TABLE_X table, X *value, int *pos);
```

Apart from returning the value in `*value` when an entry is found, these functions are the same as the corresponding 'contains' versions.

```
void MTableDelete(MTABLE_X table, int pos);
```

Delete the entry of `table` at position `pos`. Here `pos` must contain an entry; for example, it could be the position returned by a successful call to `MTableRetrieve`.

```
void MTableForEachWithKey(MTABLE_X table, char *key, X *value,
  int *pos);
void MTableForEachWithKeyHashed(MTABLE_X table, int hash_code,
  char *key, X *value, int *pos);
```

These are iterator macros which visit every entry with a given key. For example, to visit every person called `"fred"` in table `people`, the code is

```
MTableForEachWithKey(people, "fred", &person, &pos)
{
   ... visit person ...
}
```

On each iteration, this code sets `person` to a person with name `"fred"`, and `pos` to the position of that person in the table. `MTableForEachWithKeyHashed` is the same except that the user supplies the hash code as well, as for `MTableRetrieveHashed`.

A similar iterator macro visits every entry of the table:

```
void MTableForEach(MTABLE_X table, char **key, X *value, int *pos);
```

The entries will be visited in an essentially random order, as usual with hash tables. For example, the following code will count the number of entries in `table`:

```
count = 0;
MTableForEach(table, &key, &value, &pos)
  count++;
```

This number is rarely needed by applications so it is not maintained automatically.

Another fairly useless number is

```
int MTableSize(MTABLE_X table);
```

which is the current size of the hash table array. This will be somewhat larger than the current number of entries. And here are a few final macros, of minor interest:

```
bool MTableOccupiedPos(MTABLE_X table, int pos);
char *MTableKey(MTABLE_X table, int pos);
X MTableValue(MTABLE_X table, int pos);
void MTableSetValue(MTABLE_X table, int pos, X value);
```

`MTableOccupiedPos` returns `true` when position `pos` of `table` contains an entry. `MTableKey` and `MTableValue` return the key and value of the entry at position `pos`; they are undefined if there is no entry at `pos`. `MTableSetValue` changes the value of the entry at position `pos`. For example, assuming that the table contains at least one entry with key `"fred"`, the code

```
MTableContains(table, "fred", &pos);
MTableSetValue(table, pos, new_value);
```

changes the value of the first such entry to `new_value`.

A.2. Variable-length bitsets

KHE comes with a C module called LSet for managing variable-length sets of smallish unsigned integers implemented as bit vectors. The module consists of header file `khe_lset.h` and implementation file `khe_lset.c`. These are stored and compiled with KHE, but they can also be used separately. KHE uses LSet extensively behind the scenes (all its time groups, resource groups, and event groups are represented both as arrays of elements and LSets of element index

numbers), and it is also occasionally useful when writing helper functions and solvers, which is why it is documented here. To use LSet, simply include `khe_lset.h`. Including `khe.h` does not automatically include `khe_lset.h` as well.

File `khe_lset.h` begins with these two type definitions:

```
typedef struct lset_rec *LSET;
typedef MARRAY(LSET) ARRAY_LSET;
```

The first defines the type of an LSet, and the second defines an array of LSets, as usual.

Internally, an LSet is represented by a pointer to a `struct` containing a length followed by the bit vector itself. When an element needs to be added that would overflow the currently allocated memory, the whole LSet is freed and a new one is returned. This is not particularly convenient for the user of LSet but it is the most efficient way.

Functions

```
LSET LSetNew(void);
void LSetFree(LSET s);
```

create a new, empty LSet and free an LSet;

```
LSET LSetCopy(LSET s);
```

creates a fresh new LSet with the same value as `s`. Function

```
void LSetShift(LSET s, LSET *res, unsigned int k,
  unsigned int max_nonzero);
```

takes two existing LSets, `s` and `*res`, and replaces the current value of `*res` by `s` with `k` added to each of its elements, except that elements which would thereby have value greater than `max_nonzero` are omitted. The old `*res` will be freed and a new one allocated if necessary. This arcane function is used behind the scenes to calculate shifted time domains. Function

```
void LSetClear(LSET s);
```

clears `s` back to the empty set, and

```
void LSetInsert(LSET *s, unsigned int i);
void LSetDelete(LSET s, unsigned int i);
```

insert element `i` (changing nothing if `i` is already present) and delete it (changing nothing if `i` is already absent). The value of `i` is arbitrary but very large values are obviously undesirable, since the bit vectors then become very large.

```
void LSetAssign(LSET *target, LSET source);
```

replaces the current value of `*target` with the value of `source`, reallocating `*target` if necessary. The value is a copy, there is no sharing anywhere in the LSet module.

The next three functions implement the set operations of union, intersection, and difference, replacing their first parameter's value with the result of the operation:

```
void LSetUnion(LSET *target, LSET source);
void LSetIntersection(LSET target, LSET source);
void LSetDifference(LSET target, LSET source);
```

The usual Boolean operations are available on LSets:

```
bool LSetEmpty(LSET s);
bool LSetEqual(LSET s1, LSET s2);
bool LSetSubset(LSET s1, LSET s2);
bool LSetDisjoint(LSET s1, LSET s2);
bool LSetContains(LSET s, unsigned int i);
```

These return `true` when `s` is empty, when `s1` and `s2` are equal, when `s1` is a subset of `s2`, when `s1` and `s2` are disjoint, and when `s` contains `i`. Functions

```
unsigned int LSetMin(LSET s);
unsigned int LSetMax(LSET s);
```

return the smallest and largest elements of `s` respectively, using an efficient table lookup on the first or last non-zero byte. Both functions abort if `s` is empty. Function

```
int LSetLexicalCmp(LSET s1, LSET s2);
```

returns a negative, zero, or positive result depending on whether `s1` is lexicographically less than, equal to, or greater than `s2`. Function

```
void LSetExpand(LSET s, ARRAY_SHORT *add_to)
```

assumes that `*add_to` is an initialized array, and adds the elements of `s` to the array in increasing order by repeated calls to `MArrayAddLast`. Function

```
char *LSetShow(LSET s);
```

returns a display of `s` in static memory (so it is not thread-safe, but it does keep four separate buffers, allowing it to be called several times in one line of debug output). Finally,

```
void LSetTest(FILE *fp);
```

tests the module and prints its results onto file `fp`.

A.3. Priority queues

When a solver needs to visit things in priority order, it is easiest to just put them in an array and sort them. Occasionally, however, their priorities change as solving proceeds, and then, since resorting after every change is not efficient, a priority queue is needed.

KHE comes with a C priority queue module called PriQueue, consisting of header file `khe_priqueue.h` and implementation file `khe_priqueue.c`. These are stored and compiled with KHE, but can also be used separately. To use PriQueue, simply include `khe_priqueue.h`. Including `khe.h` does not automatically include `khe_priqueue.h` as well. The implementation uses a Floyd-Williams heap with back indexes. Each operation takes $O(\log(n))$ time at most.

File `khe_priqueue.h` begins with these type definitions:

```
typedef struct khe_priqueue_rec *KHE_PRIQUEUE;

typedef int64_t (*KHE_PRIQUEUE_KEY_FN)(void *entry);
typedef int (*KHE_PRIQUEUE_INDEX_GET_FN)(void *entry);
typedef void (*KHE_PRIQUEUE_INDEX_SET_FN)(void *entry, int index);
```

The first defines the type of a PriQueue as a pointer to a private record in the usual way. The others define the types of callback functions stored within the PriQueue and called by it.

An *entry* is one element of a priority queue. PriQueue is generic: its entries are represented by void pointers and may have any type consistent with that. Each entry has a *key*, which is its priority in the priority queue, and an *index*, which is used internally by PriQueue to point to its position in the priority queue. A typical entry type would look like this:

```
typedef struct my_entry_rec {
  int64_t       key;                      /* PriQueue key */
  int           index;                    /* PriQueue index */
  ...
} *MY_ENTRY;
```

where `...` stands for other fields. PriQueue needs to retrieve the key, and to retrieve and set the index, which is what the three callback functions are for. Here they are for type `MY_ENTRY`:

```
int64_t MyEntryKey(void *entry)
{
  return ((MY_ENTRY) entry)->key;
}

int MyEntryIndex(void *entry)
{
  return ((MY_ENTRY) entry)->index;
}

void MyEntrySetIndex(void *entry, int index)
{
  ((MY_ENTRY) entry)->index = index;
}
```

PriQueue sets the value of an entry's index field to a positive integer during an insertion, and to zero during a deletion. Accordingly, the user should initialize it to zero, and then it can be used to determine whether the entry is currently in a priority queue or not.

To create a new PriQueue, call

```
KHE_PRIQUEUE KhePriQueueMake(KHE_PRIQUEUE_KEY_FN key,
  KHE_PRIQUEUE_INDEX_GET_FN index_get,
  KHE_PRIQUEUE_INDEX_SET_FN index_set);
```

For the example above, the call would be

```
KhePriQueueMake(&MyEntryKey, &MyEntryIndex, &MyEntrySetIndex);
```

Initially the queue is empty. To delete a priority queue when it is no longer needed, call

```
void KhePriQueueDelete(KHE_PRIQUEUE p);
```

To test whether a priority queue is empty or not, call

```
bool KhePriQueueEmpty(KHE_PRIQUEUE p);
```

To insert an entry, call

```
void KhePriQueueInsert(KHE_PRIQUEUE p, void *entry);
```

making sure that the entry's key is defined beforehand; the index need not be, since it will be set by PriQueue. Functions

```
void *KhePriQueueFindMin(KHE_PRIQUEUE p);
void *KhePriQueueDeleteMin(KHE_PRIQUEUE p);
```

return an entry with minimum key, assuming that `p` is not empty, and `KhePriQueueDeleteMin` removes the entry from the queue at the same time. Function

```
void KhePriQueueDeleteEntry(KHE_PRIQUEUE p, void *entry);
```

deletes `entry` from `p`; it must lie in `p`.

To update the priority of an entry, first change its key and then call

```
void KhePriQueueNotifyKeyChange(KHE_PRIQUEUE p, void *entry);
```

to inform `p` that it has changed. This will change `entry`'s order in the queue, moving it forwards or backwards as required. Finally,

```
void KhePriQueueTest(FILE *fp);
```

tests the module and prints its results onto file `fp`.

A.4. XML handling with KML

KML is a C module for reading and writing XML. It consists of a header file called `kml.h`, and an implementation file called `kml.c`. These are stored and compiled with the KHE module, and `khe.h` includes `kml.h`. They can also be abstracted from it and used separately, although `kml.c` does depend on the `M` memory module (Appendix A.1).

KHE uses KML to read and write XML. The KHE user encounters KML in exactly one place: when reading an archive, an object of type `KML_ERROR` is returned if there is a problem.

A.4.1. Representing XML in memory

Type `KML_ELT` represents one node in an XML tree structure, including its label, attributes, and children. The operations for querying a `KML_ELT` object are

```
int KmlLineNum(KML_ELT elt);
int KmlColNum(KML_ELT elt);
char *KmlLabel(KML_ELT elt);
KML_ELT KmlParent(KML_ELT elt);
int KmlAttributeCount(KML_ELT elt);
char *KmlAttributeName(KML_ELT elt, int index);
char *KmlAttributeValue(KML_ELT elt, int index);
int KmlChildCount(KML_ELT elt);
KML_ELT KmlChild(KML_ELT elt, int index);
bool KmlContainsChild(KML_ELT elt, char *label, KML_ELT *child_elt);
char *KmlText(KML_ELT elt);
```

`KmlLineNum` and `KmlColNum` return a line number and column number stored in the element, presumably recording its position in some input file somewhere. `KmlLabel` returns the label of the element, and `KmlParent` returns its parent element in the tree structure, or `NULL` if none.

`KmlAttributeCount` returns the number of `elt`'s attributes, and `KmlAttributeName` and `KmlAttributeValue` return its `index`'th attribute's name and value. The first attribute has index 0. Negative indexes are allowed: `-1` means the last attribute, `-2` the second last, and so on.

`KmlChildCount` returns the number of children, and `KmlChild` returns the `index`'th child, again counting from 0 with negative indices allowed. `KmlContainsChild` returns `true` if `elt` contains a child with the given label, setting `*child_elt` to the first such child if so. `KmlText` returns the text which is the body of the element, possibly `NULL`.

There are operations for constructing `KML_ELT` objects directly:

```
KML_ELT KmlMakeElt(int line_num, int col_num, char *label);
void KmlAddAttribute(KML_ELT elt, char *name, char *value);
void KmlAddChild(KML_ELT elt, KML_ELT child);
void KmlDeleteChild(KML_ELT elt, KML_ELT child);
void KmlAddText(KML_ELT elt, char *text);
```

The first creates a new element with the given line number, column number, and label; the second adds an attribute; the next two add and delete a child; and the last adds text. `KmlAddText` actually stores a malloced copy of the content of the `text` parameter. It may be called repeatedly on one `elt`, in which case the successive texts are concatenated.

When a `KML_ELT` object is just an intermediate representation on the path from an XML file to the user's data structure, it is no longer needed after the user's data structure is built. The memory occupied by it may be returned to the memory allocator for re-use by calling

```
void KmlFree(KML_ELT elt, bool free_attribute_values, bool free_text);
```

This frees `elt` and its descendants. Setting `free_attribute_values` causes all attribute value strings to be freed, and setting `free_text` to `true` causes all text to be freed. There are no options for freeing the strings which label elements and name attributes, because there are usually shared. These strings therefore leak.

The values passed to `KmlAddAttribute` are stored without copying. If they are literal strings, or if they are transferred to the user's data structure without copying, then freeing them

is not safe. On the other hand, the strings passed to `KmlAddText` are always copied into malloced memory, so they are safe to free provided they are not transferred without copying into the user's data structure.

The following functions are useful when sorting out what to free:

```
char *KmlExtractAttributeValue(KML_ELT elt, int index);
char *KmlExtractText(KML_ELT elt);
```

They do what the corresponding operations without the word `Extract` do, but they also clear that part of the KML object: `KmlExtractAttributeValue` returns the attribute value but then sets it to `NULL` within `elt`, and so on. Extracting those string parts of the KML tree that are used within the user's data structure makes them immune from freeing later.

In the trees returned by `KmlReadFile` and `KmlReadString` below, all attribute values and text lie in malloced memory. The best policy in that case is to extract those strings that are to be kept, and free the rest by setting the `bool` parameters of `KmlFree` to `true`. `KheArchiveRead` does this, for example.

A.4.2. Error handling and format checking

KML does not print any error messages; instead it reports an error by returning an object of type `KML_ERROR`, containing the line number and column number of the point of error, plus a message explaining what the problem was:

```
int KmlErrorLineNum(KML_ERROR ke);
int KmlErrorColNum(KML_ERROR ke);
char *KmlErrorString(KML_ERROR ke);
```

These objects can form the basis of error messages printed by the user. Most of the error strings actually encountered are generated by the Expat parser which KML uses when reading a file.

KML's operations for reading a file check only for well-formedness, not for conformance to a legal document type definition, nor for high-level semantic constraints. During the conversion from `KML_ELT` to the user's own data structure, other errors may be uncovered, and it is convenient to be able to report those as objects of type `KML_ERROR` also. Accordingly, operation

```
KML_ERROR KmlErrorMake(int line_num, int col_num, char *fmt, ...);
```

is provided. It creates a new object of type `KML_ERROR`, initializes it with the given line number, column number, and formatted text (as for `printf`), and returns it. There is also

```
KML_ERROR KmlVErrorMake(int line_num, int col_num, char *fmt, va_list ap);
```

which is to `KmlErrorMake` what `vprintf` is to `printf`, and

```
bool KmlError(KML_ERROR *ke, int line_num, int col_num, char *fmt, ...);
```

which is like `KmlErrorMake` except that it sets `*ke` to the object it makes, and always returns `false`. This is convenient for uses such as

```
if( bad_thing_discovered )
  return KmlError(ke, line_num, col_num, "bad %s thing", str);
```

which bails out of a function that returns a boolean indicating whether all is well.

To check whether a `KML_ELT` object conforms to a document type definition, call:

```
bool KmlCheck(KML_ELT elt, char *fmt, KML_ERROR *ke);
```

If `elt` conforms to the definition expressed by `fmt`, then `true` is returned; otherwise, `false` is returned and `*ke` is set to an object recording the nature of the error, including a line and column number taken from either `elt` itself or one of its children, as appropriate.

Parameter `fmt` describes the attributes and children of `elt`—not the label of `elt`, which will have already been checked by the time `elt` is examined, nor the children's children, which may be checked by the user during a recursive traversal of `elt`'s children. For example,

```
"+Reference : #Value"
```

says that `elt` has an optional attribute whose name is `Reference`, and exactly one child whose label is `Value` and whose body must contain text denoting an integer (no children). The part before the colon specifies attributes, and the part after it (if there is a colon at all) specifies children. An initial `+` means optional, and an initial `*` means zero or more; neither means exactly one. After that, an initial `$` means text (no children), and an initial `#` means text representing an integer (again, no children); neither means that there may be children. Here is a longer example:

```
"Reference : +#Duration +Time +Resources"
```

The element must have exactly one attribute, `Reference`. It has up to three children, an optional integer `Duration`, followed by an optional `Time`, and finally an optional `Resources`. As mentioned, the structure of the children may be checked by subsequent calls to `KmlCheck`.

A.4.3. Reading XML files

To read an XML file, call

```
bool KmlReadFile(FILE *fp, KML_ELT *res, KML_ERROR *ke,
  char *end_label, char **leftover, int *leftover_len, FILE *echo_fp);
```

`KmlReadFile` reads `fp`, which must be open for reading UTF-8. If legal XML is found, `*res` is set to a new `KML_ELT` object representing that XML, and `true` is returned. The operations of Appendix A.4.1 may then be used to traverse `*res`. Otherwise, `*ke` is set to an error object recording the file position and nature of the first error (reading stops there), and `false` is returned.

If `end_label` is `NULL`, `KmlReadFile` interprets the entire file, starting from `fp`'s current position, as XML. If `end_label` is non-`NULL`, it must be a string of length at least 1 and at most 1024, and `KmlReadFile` stops reading `fp` immediately after its first occurrence, or else at the end of the file. For example, `"</HighSchoolTimetableArchive>"` is a suitable `end_label`.

For efficiency on large files, `KmlReadFile` reads `fp` one chunk at a time. When `end_label` is non-`NULL`, `KmlReadFile` correctly handles the case of the first occurrence of `end_label` straddling two chunks, but the last chunk it reads will usually contain some of the characters that

follow `end_label`, and it has no way of pushing them back onto `fp`. So in that case, `leftover` and `leftover_len` must also be non-`NULL`, and when `KmlReadFile` returns, `*leftover` points into the last chunk immediately after `end_label`, and `*leftover_len` contains the number of characters (possibly 0) from that point to the end of the chunk. The unconsumed remnant of `fp` consists of `*leftover_len` characters starting at `*leftover`, plus whatever still lies in `fp`. A `'\0'` will not usually follow the leftover characters.

If `echo_fp` is non-`NULL`, it must be open for writing UTF-8, and `KmlReadFile` echoes every character it reads to `echo_fp`, including leftover characters. This is useful for debugging.

When `end_label` is non-`NULL`, `KmlReadFile` cannot free the single 1024-byte buffer of malloced memory it obtains and uses repeatedly to hold each chunk, since `leftover` points into it. The caller cannot free it either, so it must leak.

It is also possible to read XML by scanning a string:

```
bool KmlReadString(char *str, KML_ELT *res, KML_ERROR *ke);
```

`KmlReadString` is like `KmlReadFile` except that `str` is read rather than `fp`, and the XML is expected to occupy the entire string.

Some XML files are so large that they need to be read one piece at a time. For this there is

```
bool KmlReadFileIncremental(FILE *fp, KML_ELT *res, KML_ERROR *ke,
  char *end_label, char **leftover, int *leftover_len, FILE *echo_fp,
  KML_ELT_FN elt_fn, void *impl, int max_depth);
```

The first seven parameters are as for `KmlReadFile`. Parameter `elt_fn` is a callback function which gives sneak previews of the `KML_ELT` objects that `KmlReadFileIncremental` is creating. The user defines `elt_fn` like this:

```
void elt_fn(KML_ELT elt, KML_READ_INFO ri)
{
  ...
}
```

When `KmlReadFileIncremental` finishes reading an XML element and everything in it, and creating the corresponding `KML_ELT` object and its descendants, it calls `elt_fn` with `elt` set to that object, and `ri` set to an object containing other information, obtainable by calling

```
void *KmlReadImpl(KML_READ_INFO ri);
int KmlReadMaxDepth(KML_READ_INFO ri);
int KmlReadCurrDepth(KML_READ_INFO ri);
```

These return the values of the `impl` and `max_depth` parameters passed to the original call to `KmlReadFileIncremental`, and the depth of the current element. There is also

```
void KmlReadFail(KML_READ_INFO ri, KML_ERROR ke);
```

A call on this from within `elt_fn` or within functions called by it, directly or indirectly, causes an immediate return from `KmlReadFileIncremental` with value `false` and the given `ke`. It is implemented using the `setjmp` and `longjmp` functions; the jump context is stored in `ri`.

Parameter `max_depth` of `KmlReadFileIncremental` limits the callbacks to those whose depth parameter is at most `max_depth`. For example, setting `max_depth` to `-1` produces no callbacks at all; setting it to `0` produces a callback only on the outermost element; and so on. Limiting depth is an efficient way to avoid most callbacks on insignificant quantities of data.

The main use for incremental reading is to grab part of the object tree, process it, and reclaim the memory used by it. To reclaim its memory, place this at the end of the callback function:

```
KmlDeleteChild(KmlEltParent(elt), elt);
KmlFree(elt, ...);
```

When `elt` is the root of the whole document tree, this will cause `KmlReadFileIncremental` to report error `"0 outer units in input file"`. It works well on all other elements, however. `KmlReadFileIncremental` calls `KmlAddChild` long before the callback, and will not notice if `elt` is deleted and freed. It does not touch a non-root `elt` after passing it to the callback.

A.4.4. Writing XML files

Writing an XML file begins with the creation of a `KML_FILE` object, by calling

```
KML_FILE KmlMakeFile(FILE *fp, int initial_indent, int indent_step);
```

Pointer type `KML_FILE`, defined in `kml.h`, represents an XML file open for writing (never reading). It holds a file pointer and a few attributes describing the state of the write, including a current indent, used to produce neatly indented XML. File `fp` must be open for writing UTF-8 characters; `initial_indent` is the initial indent, typically 0, and `indent_step` is the number of spaces to indent at each level, typically 2 or 4.

When reading an XML file using KML it is necessary to first read the file into a `KML_ELT` object, and then build the user data structure that is really wanted, while traversing the `KML_ELT` object. The reverse procedure may be used for writing, by calling

```
void KmlWrite(KML_ELT elt, KML_FILE kf);
```

`KmlWrite` writes `elt` and its attributes and children recursively to `kf`. But it is also possible to write directly to a file while traversing the user's data structure, without using `KML_ELT` objects. To do this, the operations are

```
void KmlBegin(KML_FILE kf, char *label);
void KmlAttribute(KML_FILE kf, char *name, char *value);
void KmlPlainText(KML_FILE kf, char *text);
void KmlFmtText(KML_FILE kf, char *fmt, ...);
void KmlEnd(KML_FILE kf, char *label);
```

`KmlBegin` begins an object with the given label, and `KmlEnd` ends it. KML does not check that the labels match, even though they must. Immediately after calling `KmlBegin`, any number of calls to `KmlAttribute` are allowed; each adds one attribute, with the given name and value, to the object just begun. After that, `KmlPlainText` may be called to add some text as the body of the object, or `KmlFmtText` to add some formatted text as the body (where `fmt` and the following parameters are suitable for passing on to `fprintf`). `KmlPlainText` prints the characters `&<>'"`

in their escape sequence forms (`&` and so on); `KmlFmtText` does not, so it is best limited to tasks that cannot generate such characters (printing numbers, etc.). Alternatively, any number of nested calls to `KmlBegin` ... `KmlEnd` may precede the matching `KmlEnd`, to add children.

For convenience, three operations are offered which write an entire element in one call:

```
void KmlEltAttribute(KML_FILE kf, char *label, char *name, char *value);
void KmlEltPlainText(KML_FILE kf, char *label, char *text);
void KmlEltFmtText(KML_FILE kf, char *label, char *fmt, ...);
```

These are simple combinations of the functions above, only writing on one line (except newlines in text). `KmlEltAttribute` writes an object with the given label and attribute, but no body. `KmlEltPlainText` and `KmlEltFmtText` write an object with the given label, no attributes, and a plain or formatted text body. A few other such functions are available, for which see `kml.h`.

Appendix B. Implementation Notes

This appendix documents aspects of the implementation of KHE. It is included mainly for the author's own reference; it is not needed for using KHE.

B.1. Source file organization

The KHE platform is organized in object-oriented style, with one C source file for each major type. A type's internals are visible only within its file, so that all access to them is via functions. Headers for some of these functions appear in `khe.h`, making them available to the end user. Headers for others appear in `khe_interns.h`, making them available only to the platform.

Although this section applies to all source files, it is motivated by the problems of organizing the source files of types defining parts of solutions. Some of these are quite large. For example, `khe_meet.c`, which holds the internals of type `KHE_MEET`, is over 5000 lines.

There is a canonical order for the types representing parts of solutions: `KHE_SOLN`, `KHE_MEET`, `KHE_MEET_BOUND`, `KHE_TASK`, `KHE_TASK_BOUND`, `KHE_MARK`, `KHE_PATH`, `KHE_NODE`, `KHE_LAYER`, `KHE_ZONE`, `KHE_TASKING`. The idea is to handle these types in this order whenever appropriate—in this Guide for example.

Source files are organized internally by dividing them into *submodules*, which are segments of the files separated by comments. Each submodule handles one aspect of the type. Here is a generic list of the submodules appearing in any one file, in their order of appearance:

Type declaration
Simple attributes (back pointers, visit numbers, etc.)
Creation and deletion
Relations with objects of the same type (copy, split, etc.)
Relations with objects of different types
File reading and writing
Debug

Simple attributes are easily handled attributes that are not closely related to any following categories. They may appear in separate submodules, or be grouped into one submodule. Each relation is one submodule (counting opposite operations, such as split and merge, as part of one relation), except that a large relation may be broken into several submodules. Relations with different types appear in the canonical order defined above.

An attempt has been made to keep the submodules in the same order as their functions are presented in this Guide, except for debugging. Some submodules have no defined position according to this rule, because they are present only to support other submodules, and offer no functions to the end user. Those are placed where they seem to fit best.

B.2. Relations between objects

This section explains how KHE maintains relations between objects. Not every relation is maintained as explained here, but it is the author's aim to achieve that in time.

The most common relation, by far, is the *one-to-many* relation, in which one object is related to any number of objects of the same or another type: one node contains any number of meets, one meet contains any number of tasks, one meet is assigned any number of meets, and so on.

Let `KHE_A` be the type of the entity that there is one of, and `KHE_B` be the type of the entity that there are many of. KHE implements the relation by placing one attribute, of type `ARRAY_KHE_B`, in `KHE_A`, holding the many `KHE_B` objects related to `KHE_A`, and two in `KHE_B`:

```
KHE_A   a;
int     a_index;
```

holding the one `KHE_A` object related to this object, and this object's index in that object's array. Any attributes of the relation, such as the offset attribute of the meet assignment relation, appear alongside these two. In the `KHE_A` class file, functions

```
void KheAAddB(KHE_A a, KHE_B b);
void KheADeleteB(KHE_A a, KHE_B b);
```

are defined which add and delete elements of the relation, as well as the usual `KheABCount` and `KheAB` functions which iterate over the array. In the `KHE_B` class file, functions

```
KHE_A KheBA(KHE_B b);
void KheBSetA(KHE_B b, KHE_A a);
int KheBAIndex(KHE_B b);
void KheBSetAIndex(KHE_B b, int a_index);
```

get and set the `a` and `a_index` attributes of `b`, supporting constant time deletions. Instead of searching for `b` in `a`'s array, `a_index` is used to find it directly. It is overwritten by the entity at the end of the array, whose index is then changed. This assumes that the order of the array's elements may be arbitrary, as is usually the case. The setter functions are private to the platform.

This plan allows a `KHE_B` object to be unrelated to any `KHE_A` object (just set its `a` attribute to `NULL`), but does not support *many-to-many* relations, where a `KHE_B` object may be related to any number of `KHE_A` objects. On the rare occasions when KHE needs this kind of relation, it adapts the familiar edge lists implementation of graphs: it defines a type `KHE_A_REL_B` representing one element of the many-to-many relation, and installs one one-to-many relation from `KHE_A` to `KHE_A_REL_B`, and another from `KHE_B` to `KHE_A_REL_B`. This gives `KHE_A_REL_B` attributes

```
KHE_A   a;
int     a_index;
KHE_B   b;
int     b_index;
```

and places it in arrays in both `entity_a` and `entity_b`. Now the operations for adding and deleting an element of the relation must add or delete two one-to-many relations, as well as creating or deleting one `KHE_A_REL_B` object, which is done using a free list to save time.

B.3. Kernel operations

The promises made in connection with marks and paths, that all operations that change a solution can be undone (except changes to visit numbers), and that undoing a deletion recreates the object at its original address, have significant implications for the implementation.

The KHE platform has an inner layer called the *solution kernel*, or just the *kernel*, consisting of a set of private operations, called *kernel operations*, which change a solution. Each kernel operation has a name of the form `KheEntityKernelOp`, where `Entity` is the data type and `Op` is the operation. It is the kernel operations that are stored in paths. All operations (except operations on visit numbers) change the solution only by calling kernel operations, so if those are correctly done, undone, and redone, all operations will be correctly done, undone, and redone.

For the record, here is the complete list of kernel operations:

```
KheMeetKernelSetBack
KheMeetKernelAdd
KheMeetKernelDelete
KheMeetKernelSplit
KheMeetKernelMerge
KheMeetKernelMove
KheMeetKernelAssignFix
KheMeetKernelAssignUnFix
KheMeetKernelAddMeetBound
KheMeetKernelDeleteMeetBound
KheMeetKernelSetAutoDomain

KheMeetBoundKernelAdd
KheMeetBoundKernelDelete
KheMeetBoundKernelAddTimeGroup
KheMeetBoundKernelDeleteTimeGroup

KheLayerKernelSetBack
KheLayerKernelAdd
KheLayerKernelDelete
KheLayerKernelAddChildNode
KheLayerKernelDeleteChildNode
KheLayerKernelAddResource
KheLayerKernelDeleteResource

KheTaskKernelSetBack
KheTaskKernelAdd
KheTaskKernelDelete
KheTaskKernelSplit
KheTaskKernelMerge
KheTaskKernelMove
KheTaskKernelAssignFix
KheTaskKernelAssignUnFix
KheTaskKernelAddTaskBound
KheTaskKernelDeleteTaskBound

KheTaskBoundKernelAdd
KheTaskBoundKernelDelete

KheNodeKernelSetBack
KheNodeKernelAdd
KheNodeKernelDelete
KheNodeKernelAddParent
KheNodeKernelDeleteParent
KheNodeKernelSwapChildNodesAndLayers
KheNodeKernelAddMeet
KheNodeKernelDeleteMeet

KheZoneKernelSetBack
KheZoneKernelAdd
KheZoneKernelDelete
KheZoneKernelAddMeetOffset
KheZoneKernelDeleteMeetOffset
```

Each `KheEntityKernelOp` function has a companion `KheEntityKernelOpUndo` function. `KheEntityKernelOp` carries out its operation and adds itself to the solution's path, if present. `KheEntityKernelOpUndo` undoes what `KheEntityKernelOp` did, only without removing itself from the solution's path, since it is called by a function that has already done that.

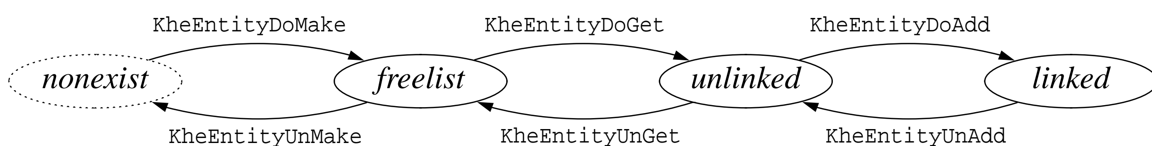
A redo must be identical to the original operation, because both can be inverted by calling `KheEntityKernelOpUndo` and removing one record from the solution path. So there are no `KheEntityKernelOpRedo` functions; `KheEntityKernelOp` functions are called instead.

Some operations come in opposing pairs (split and merge, fix and unfix, and so on), such that doing one is the same as undoing the other, except that a do or redo adds a record to the solution's path, whereas an undo does not. In these cases the implementation contains one private function called `KheEntityDoOp1` and another called `KheEntityDoOp2`, where `Op1` and `Op2` are opposing pairs. These functions carry out the two operations without touching the solution's path. Then `KheEntityKernelOp1`, `KheEntityKernelOp2`, `KheEntityKernelOp1Undo`, and `KheEntityKernelOp2Undo` are each implemented by one call on `KheEntityDoOp1` or `KheEntityDoOp2`, plus an addition to the solution's path if the operation is not `Undo`.

Operations that create and delete objects are awkward, as it turns out, so the rest of this section is devoted to them. The meet split and merge operations are particularly awkward, so we will start with the regular creation and deletion operations, generically named `KheEntityMake` and `KheEntityDelete`, and treat meet splitting and merging afterwards.

Solution objects are recycled through free lists held in the enclosing solution. When a new object is needed, it is taken from the free list, or from the memory allocator if the free list is empty. When an object is no longer needed, it is added to the free list. When the solution is deleted, and only then, the objects on the free list are returned to the memory allocator. Free lists save time handling extensible arrays within objects: the arrays of a free list object remain initialized.

An operation which obtains a new object from a memory allocator or free list cannot be a kernel operation, because then a redo would not re-create the object at its previous memory location. An operation which returns an object to a memory allocator or free list cannot be a kernel operation, because an undo would not re-create the object at its previous memory location. So only the part of `KheEntityMake` which initializes the object and links it into the solution is the kernel operation, and only the part of `KheEntityDelete` which unlinks the object from the solution is the kernel operation. This leads to this picture of the life cycle of a kernel object:



State *nonexist* means that the object does not exist; *freelist* means that it exists on a free list; *unlinked* means that it exists, not on a free list, not linked to the solution, but referenced from somewhere on some path; and *linked* means that it exists and is linked to the solution.

`KheEntityDoMake` obtains a fresh object from the memory allocator and initializes its private arrays. `KheEntityUnMake` does the opposite, returning the memory consumed by the object and its private arrays to the memory allocator.

`KheEntityDoGet` obtains a fresh object from the free list, or from `KheEntityDoMake` if the free list is empty. Either way, the object's arrays are initialized, although not necessarily empty. Objects returned by `KheEntityDoMake` do not actually enter the free list. `KheEntityUnGet` does the opposite, adding the object it is given to the free list. It does not call `KheEntityUnMake`.

`KheEntityDoAdd` initializes the unlinked object it is given, assuming that its private arrays are initialized, although not necessarily empty (it clears them), and links it into the solution.

`KheEntityUnAdd` does the opposite, unlinking the object it is given from the solution.

The kernel operations `KheEntityKernelAdd` and `KheEntityKernelDelete` and their `Undo` companions are each implemented by one call to `KheEntityDoAdd` or `KheEntityUnAdd`, plus an addition to the solution path if the function is not an undo. `KheEntityKernelAdd` and `KheEntityKernelDelete` form an opposing pair, as defined above, except that `KheEntityKernelDelete` may include a call to `KheEntityUnGet` as explained below.

The public function that creates a kernel object, `KheEntityMake`, is `KheEntityDoGet` followed by `KheEntityKernelAdd`. The public function that deletes one, `KheEntityDelete`, begins with kernel operations that help to unlink the object (unassignments and so on), then ends with `KheEntityKernelDelete`.

These functions do not call `KheEntityUnMake`, since kernel objects are returned to the memory allocator only when the entire solution is deleted. The function for deleting a solution first calls user functions which delete all kernel objects and paths. This places all kernel objects on the free list. It then traverses that list, passing each object to `KheEntityUnMake`.

An object can be referenced from the solution and from paths, and there is no simple rule saying when to call `KheEntityUnGet` to add it to the free list. To solve this problem, an integer reference count field is placed in each kernel object, counting the number of references to the object. Not all references are counted. References from paths at points where the object is added or deleted are counted. For example, in a path's record of a meet split or merge, the reference to the second meet is counted, but not the first. So reference counts increase when paths grow or are copied, and decrease when paths shrink or are deleted. Also, `KheEntityDoAdd` adds 1 to the count, and `KheEntityUnAdd` subtracts 1. This summarizes references from the solution generally in one unit of the count.

When the reference count falls to zero, `KheEntityUnGet` is called to return the object to the free list. This could happen during a call to `KheEntityUnAdd`, or when a path shrinks: during a call to `KhePathDelete`, or while undoing, which shrinks the solution's main path.

An *unlinked* object could have come from the free list, and so could contain no useful information. It would be a mistake for `KheEntityDoAdd` to assume that the object it is given has passed through `KheEntityUnAdd` and retains useful information from when it was previously linked. Instead, `KheEntityDoAdd` must initialize every field of the object it is given, assuming that its arrays are initialized, but not that they contain useful information.

An example of getting this wrong would be to try to preserve the list of tasks of a meet in its `tasks` array when it is unlinked, in a mistaken attempt to ensure that they remain available for when the meet is recreated. What really happens is that before deleting the meet, `KheMeetDelete` deletes its tasks, so records of those task deletions appear on the solution path just before the meet deletion. When an undo recreates the meet, it immediately goes on to recreate the tasks, without any need for their preservation in the dormant meet.

A meet split is similar to a creation of the second meet, and a meet merge is similar to a deletion of the second meet. The main new problem is that tasks need to be split and merged too. So separate kernel operations are defined for splitting the meet itself and for splitting one of its tasks, and conversely for merging two meets and for merging two of their tasks. The user operation for meet splitting does a kernel meet split followed by a sequence of kernel task splits, and the user operation for meet merging does the opposite.

The key advantage of doing it this way is that tasks are stored explicitly in paths, and their reference counters take account of this. So the usual method of handling the allocation and deallocation of entities generally, described above, applies without change to the tasks created and deleted by meet splitting and merging.

Meet bounds are related to meets in much the same way as tasks are. Once again, the kernel meet split operation does not make meet bounds for the split-off meet; instead, they are made by separate kernel meet bound creation operations, and thus will be undone before a meet split is undone. Similarly, task

Paths have negligible time cost compared with the operations they record; and their space cost is moderate, provided they are not used to record wandering methods like tabu search. Reference counting as implemented here also costs very little: in time, a few simple steps, only carried out when creating or deleting a kernel object, not each time the object is referenced; and in space, one integer per kernel object.

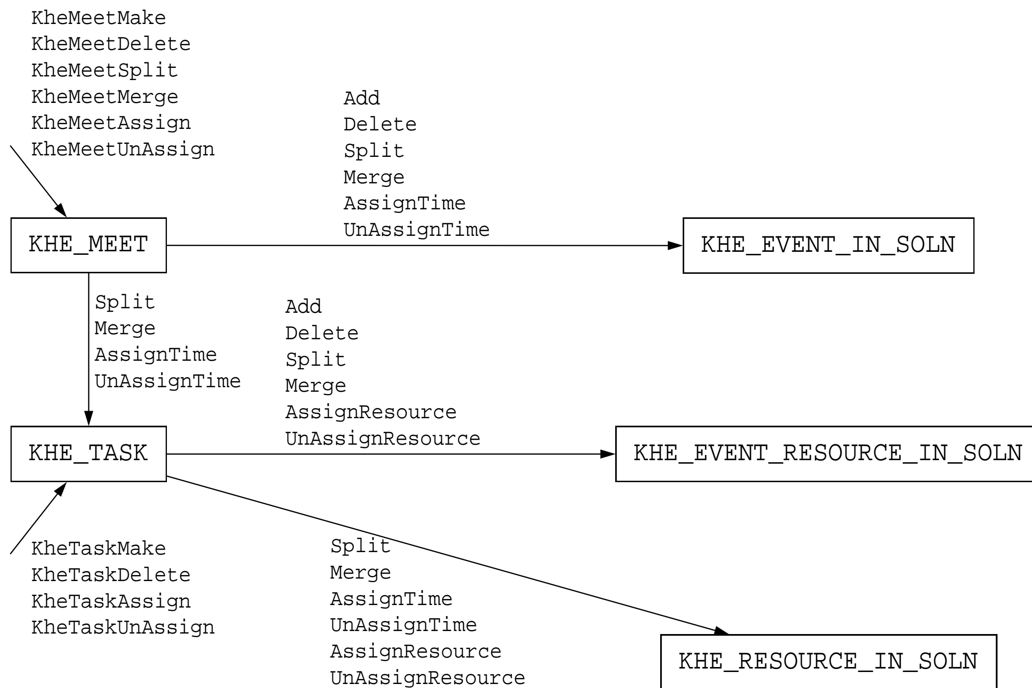
B.4. Monitor updating

When the user executes an operation that changes the state of a solution, KHE works out the revised cost. For efficiency, this must be done incrementally. This section explains how it is done—but just for information: the functions defined here cannot be called by the user.

The monitors are linked into a network that allows state changing operations to flow naturally to where they need to go. Only attached monitors are linked in; detached ones are removed, so that no time is wasted on them. The full list of basic operations that affect cost is

```
KheMeetMake     KheMeetMerge      KheTaskMake
KheMeetDelete   KheMeetAssign     KheTaskDelete
KheMeetSplit    KheMeetUnAssign   KheTaskAssign
                                  KheTaskUnAssign
```

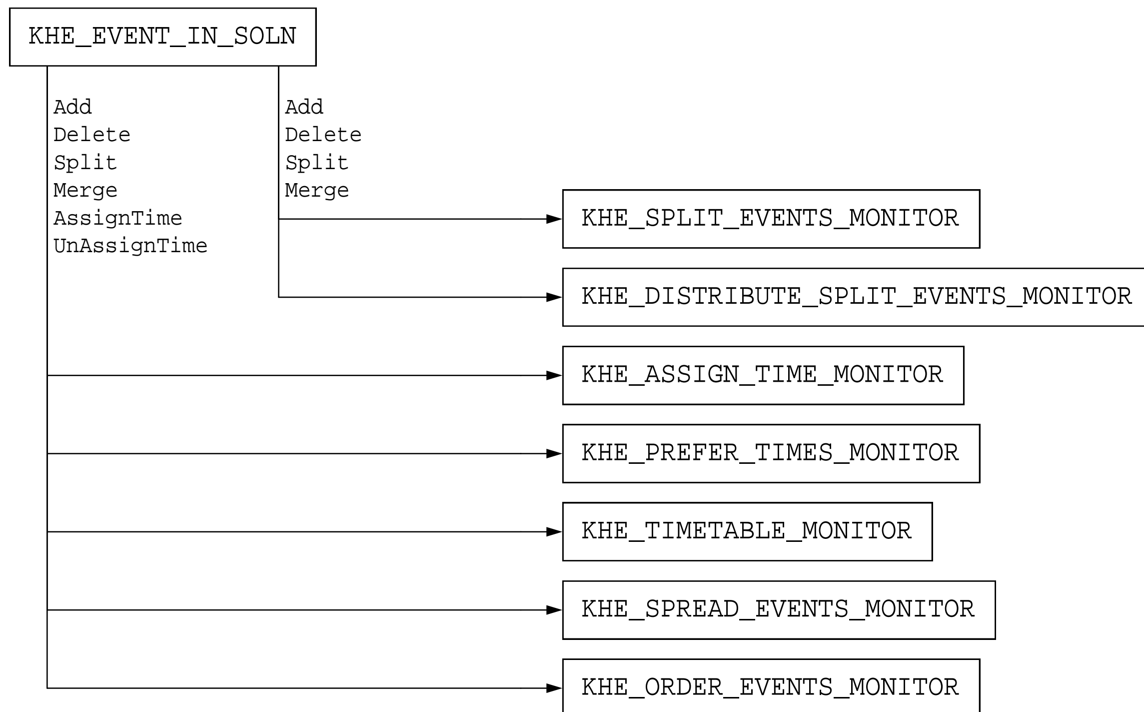
Six originate in `KHE_MEET` objects, four in `KHE_TASK` objects. From there their impulses flow to objects of three private types:



`KHE_EVENT_IN_SOLN` holds information about one event in a solution: the meets derived from it (where `KheEventMeet` gets its values from), a list of 'event resource in solution' objects, one for each of its event resources, and a list of monitors, possibly including a timetable (timetables are monitors). `KHE_EVENT_RESOURCE_IN_SOLN` holds information about one event resource in a solution: the tasks derived from it, and a list of monitors. `KHE_RESOURCE_IN_SOLN` holds information about one resource in a solution: the tasks it is currently assigned to, and a list of monitors, usually including a timetable.

The connections are fairly self-evident. For example, if `KheMeetMake` is called to make a meet derived from a given instance event, then that event's event in solution object needs to know this, and the `Add` operation (full name `KheEventInSolnAddMeet`) informs it. `KheMeetAssign` only generates an `AssignTime` call when the assignment links the meet, directly or indirectly, to a cycle meet, assigning a time to it. Event resource in solution objects are not told about time assignments and unassignments. Calls only pass from a task object `task` to a resource in solution object when `task` is assigned a resource.

The connections leading out of `KHE_EVENT_IN_SOLN` are as follows:



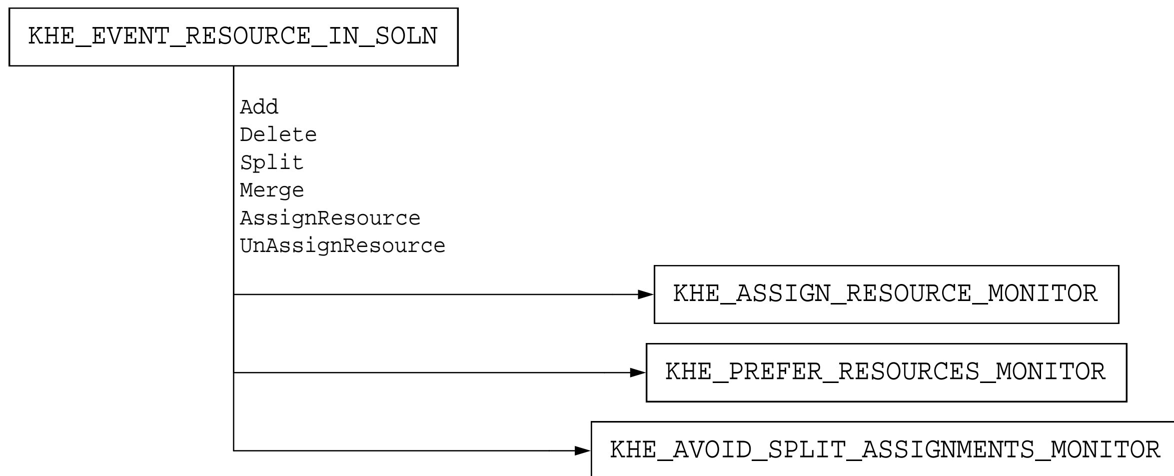
Split events and distribute split events monitors do not need to know about time assignment and unassignment. Based on the calls they receive, they keep track of meet durations and report cost accordingly. Assign time and prefer times monitors are even simpler; they report cost depending on whether the meets reported to them are assigned times or not.

Event timetables are used by link events constraints, which need to know the times when the event's meets are running, ignoring clashes, which is just what timetables offer.

A spread events monitor is connected to the event in solution objects corresponding to each of the events it is interested in. It keeps track of how many meets from those events collectively have starting times in each of its time groups, and calculates deviations accordingly. Spread events monitors are not attached to timetables because, although their monitoring is similar, there are significant differences: spread events monitor time groups come with upper and lower limits, making them not sharable in general, and the quantity of interest is the number of distinct meets that intersect each time group, not the number of busy times calculated by the time group monitors attached to timetables.

An order events monitor is connected to the two event in solution objects corresponding to the two events it is interested in. These keep track of the events' meets, including their number, and the monitor itself keeps track of the number of unassigned meets. So determining whether both events have at least one meet, and whether there are no unassigned meets, take constant time. If both conditions are satisfied, the monitor traverses both sets of meets to calculate the deviation and cost when a meet is added, deleted, or assigned a time. (In practice, events subject to order events constraints do not split, so this too takes constant time.) The other operations are faster: unassigning a time produces cost 0, and splitting and merging do not change the cost.

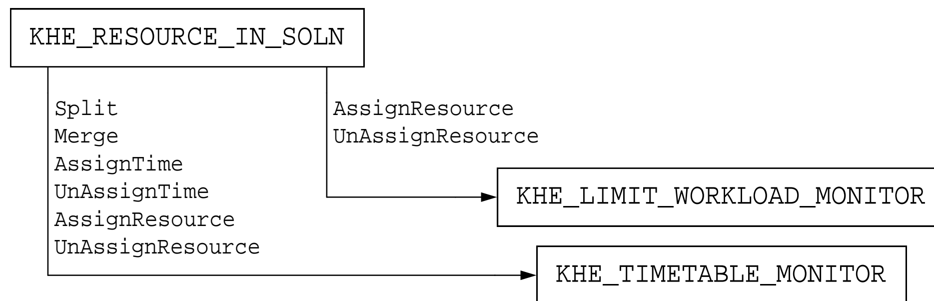
The connections leading out of `KHE_EVENT_RESOURCE_IN_SOLN` are



None of these monitors cares about time assignments and unassignments. Assign resource monitors and prefer resources monitors are very simple, reporting cost depending on whether the tasks passed to them are assigned or not.

An avoid split assignments monitor is connected to one event resource in solution object for each event resource in its point of application. It keeps track of a multiset of resources, one element for each assignment to each task it is monitoring, and its cost depends on the number of distinct resources in that multiset.

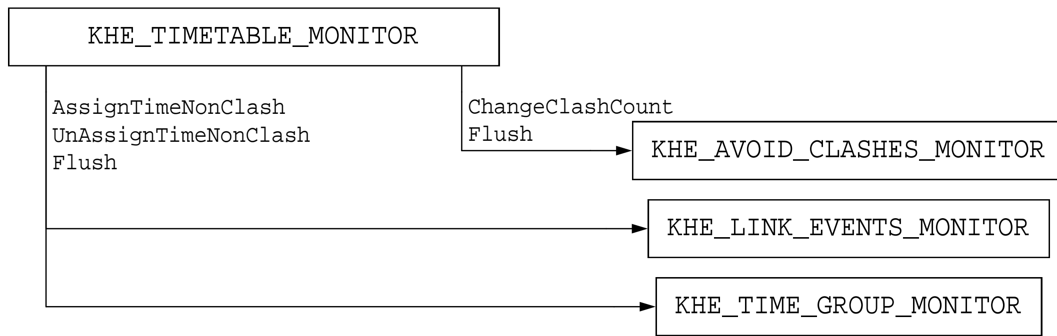
The connections leading out of `KHE_RESOURCE_IN_SOLN` are



Limit workload constraints do not need to know about time assignments, evidently, but they also do not need to know about splits and merges, since these do not change the total workload.

Calculating workloads is then very simple. Each meet receives a workload when it is created, and when a resource is assigned, the workload limit monitors attached to its resource in solution object are updated, and pass revised costs to the solution.

`KHE_TIMETABLE_MONITOR` receives many kinds of calls, some from `KHE_EVENT_IN_SOLN` and others from `KHE_RESOURCE_IN_SOLN`. However, since it monitors the timetable of a set of meets with assigned times, all these can be mapped to just two incoming operations, which we call `AddMeetAtTime` and `DeleteMeetAtTime`. For example, a split maps to one `DeleteMeetAtTime` and two `AddMeetAtTime` calls. The outgoing operations are



An avoid clashes monitor is notified whenever the number of meets at any one time increases to more than 1 or decreases from more than 1 (operation `ChangeClashCount` above). It uses these notifications to maintain its deviation. It updates the solution when a `Flush` is received from the timetable at the end of the operation.

The other monitors are attached to the timetable at each time they are interested in, and are notified when one of those times becomes busy (when its number of meets increases from 0 to 1) and when it becomes free (when its number of meets decreases from 1 to 0), by operations `AssignTimeNonClash` and `UnAssignTimeNonClash` above.

A link events monitor is interested in all the times of all the timetables of the events in its point of application. It is notified when any of these times becomes busy or free, and uses that information to maintain, for each time, the number of its events that are busy at each time. Its deviation, also maintained incrementally, is the number of times where some of its events, but not all of them, are running.

A time group monitor monitors one time group within one timetable. It is attached to its timetable at the times of its time group, so is notified when one of those times becomes busy or free. It keeps track of the number of busy and idle times in its time group.

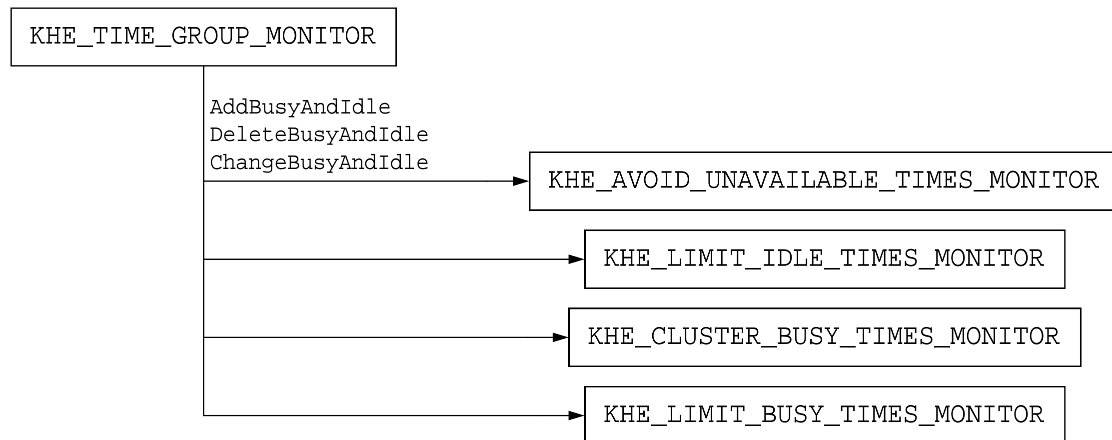
As an optimization, the number of idle times is calculated only when at least one limit idle times monitor is attached to the time group monitor; otherwise the number is taken to be 0. A bit vector V, holding the positions of the busy times in the time group being monitored, is maintained. When the monitor is flushed, the number of idle times of V is calculated as follows. If V is empty, there are no idle times. Otherwise, the number of idle times is

$$\max(V) - \min(V) + 1 - |V|$$

The first three terms give the total number of times from the first busy time to the last inclusive; every non-busy time within that range is an idle time and conversely.

$|V|$ is just the number of busy times, always maintained by the time group monitor, so it is readily available. The calculation of $\min(V)$ and $\max(V)$ on a bit vector is a well-known problem which never seems to attract adequate hardware support. KHE's bit vector module calculates $\min(V)$ by a linear search for the first non-zero word of the bit vector, followed by a linear search for the first non-zero byte of that word, and finishing with a lookup in a 256-word table, indexed by that byte, which returns the position of the first non-zero bit of that byte. The same method, searching in the other direction, finds $\max(V)$.

Old and new values for the number of busy and idle times are stored, and when a flush is received they are propagated onwards via operation `ChangeBusyAndIdle`:



When a monitor is attached, function `AddBusyAndIdle` is called instead, and when a monitor is detached, function `DeleteBusyAndIdle` is called instead.

An unavailable times monitor is connected to a time group monitor monitoring the unavailable times. It receives an updated number of busy times from `ChangeBusyAndIdle` and reports any change of cost to the solution.

A limit idle times monitor is connected to the time group monitors corresponding to the time groups of its constraint. It receives updated idle counts from each of them, and based on them it maintains its deviation.

A cluster busy times monitor is also connected to the time group monitors corresponding to the time groups of its constraint. It is interested in whether the busy counts it receives from them change from zero to non-zero, or conversely.

A limit busy times monitor is also connected to the time group monitors corresponding to the time groups of its constraint. It receives updated busy counts from each of them, and based on them it maintains its deviation.

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